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Original Article

An Investigation of Mathematics Teachers' Specialized Content Knowledge Related to Basic Concepts about Sets *

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Abstract

This research was carried out in order to examine the specialized content knowledge of mathematics teachers related to teaching basic concepts about sets within the MKT Model. In this study, a holistic single-case study design is used in qualitative research methods. The study group consists of 18 mathematics teachers at different educational background and professional experience in teaching sets selected through the sampling of criterion, which is working in different high schools in one province in the Mediterranean region in the academic year 2016-2017. Research data were obtained using semi-structured interview, observation and document analysis techniques. In the analysis of obtained data, descriptive analysis, content analysis, continuous comparison techniques and special data analysis methods were used together with each data collection tool. The findings of the research show that teachers gave correct answers to the questions asked about the basic concepts of "Why?" in the sets. However, it is clear that the teachers cannot write sufficient reason why the asked questions are "Why?". As a result of the research, it has been determined that the teachers have superficial knowledge about the concepts of "Set, Universal Set, Infinite Set" and "Equal Set". However, it was determined that teachers made explanations with a general mathematical knowledge based on the characteristics of the concept rather than providing persuasive mathematical explanations to the questions asked.

Keywords

Mathematics teacher

Basic concepts in sets

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Introduction

It is thought that the effectiveness of the education system can be possible by increasing the quality of teachers (Aksoy, 2013; Bursalıoğlu, 2021), who are at the focal point of the system. Because teachers are the most influential element among all the elements of the education system, from the curriculum to the school, and an education system is as good as its teachers (Kavcar, 2002). That's why, the low level of success achieved results of PISA (Programme for International Student Assessment) that enables comparing and analyzing student successes in the international area and TIMSS (Trends in International Mathematics and Science Study) have caused that many countries linked the deficiencies in their education system with the qualifications of teachers (Gürlen, Demirkaya, & Doğan, 2019; Simola, 2005). Because the quality of teachers is the most important factor affecting students' learning (Darling-Hammond, 2006; Ferguson, 1991; Olson, 2008; Wenglinsky, 2004). For this reason, many studies have been conducted on what qualifications our teachers, who are the essential element of the education system, should have and what should be done to increase these competences (Andrews, 2012; Başkan, 2001; Caspersen, 2013; Connel, 2009; Kim, 2013; Lee & Lee, 2020; Omare, Imonjeb, & Nyagah, 2020; Qin & Bowen, 2019; Schmidt, Houang, & Cogan, 2011). When we examine these studies, it is seen that teachers' content knowledge, their field-specific teaching skills (pedagogical knowledge), their ability to use technology, and their attitudes and sensitivities come to the fore. In addition, it is known that throughout history, the competencies that teachers should possess have changed in the form of teaching skills, subject knowledge skills and general teaching skills (Begle, 1979; Shulman, 1986). In this context, considering the fact that education and training have a variable structure, it is thought that it is inevitable to constantly research and develop the competencies (subject knowledge, pedagogical content knowledge, attitude, behavior, value, etc.) that teachers should have (An, Kulm, & Wu, 2004; Ball & McDiarmid, 1990; Baştürk, 2009; Şişman, 2009).

The Status and Importance of Sets Subject in Mathematics Education

Due to the concept of “set” is effective in building the axiomatic structure of mathematics and the logic of proof, teaching the sets is important to math education (Gavalas, 2005; Uğurel & Moralı, 2010; Yücesan, 2011). Teaching the numbers (N, Z, Q, R) is an obvious sample of that. The correlations as is the case with the superset and the subset can be gained to the students only by teaching the sets (Özdemir, 2015). In addition, concepts such as “and, or, if and only if” from the propositional logic, “Experiment, Sample Space, Event” from the probability concept, and “Point, Line, Plane” from the basic geometry are directly related to the sets (Bayazit & Aksoy, 2010). In studies conducted with students on the concept of set, it is seen that students experience learning difficulties and have superficial understandings of basic concepts (Baki & Mandacı Şahin, 2004; Gür, 2009; İpek, Albayrak, & Işık, 2009; Kolar & Čadež, 2012; Linchevski & Vinner, 1988; Uğurel & Moralı 2010; Zehir, Işık, & Zehir, 2008). In these studies, it was stated that student misconceptions about the basic concepts in sets are generally due to incorrectly chosen activities, incomplete and erroneous definitions of the concept of sets, and the nature of the set concept. In particular, the difficulties arising from the nature of the concept of infinity show that both teachers and students have difficulties in this regard (Cheung, Rubenson, & Barner, 2017; Fischbein, 2001; Hannula & Pehkonnen, 2006; Monaghan, 2001). Fischbein's (2001) statement regarding the concept of infinity also draws attention to this difficulty arising from epistemological reasons. As Fischbein (2001) says: “What our intelligence finds difficult, even impossible, to grasp is actual infinity: the infinity of the world, the infinity of the number of points in a segment, the infinity of real numbers…”(p. 309). As a matter of fact, Uğurel, Bukova-Güzel, and Kula (2010) stated that teachers' erroneous examples such as "the number of leaves of all trees in the world has infinite elements" contains mathematical errors that may cause misconceptions regarding the concept of infinite sets. In another study conducted by Yazıcı and Kültür (2017) pertain to the concept of the universal set; they observed that teachers had misconceptions like supposing the universal set as a major set which comprises everything in it. It is seen in the results of the studies that such mistakes that teachers have, affect student learning negatively (Chick, Pham, & Baker, 2006; Yazıcı & Kültür, 2017). As a matter of fact, it is known that among the factors that lead to the development of misconceptions in students, the preferred pedagogical approaches, teaching models and materials (Simon, Tzur, Heinz, & Kinzel, 2004; Tanner & Jones, 2003). In addition, in order to analyze the students 'answers regarding any concept, it is necessary to decide whether the students' answers are correct or incorrect. The next step is to identify students' misconceptions and sources of error (Boz, 2004). In other words, teachers should also adopt an expert approach against students' making mistakes (Bingölbali & Özmantar, 2014). In some studies, conducted in this manner, it has been observed that students also have learning difficulties regarding the concepts of "common trait, infinite set, empty set, element" and "subset" (Demir, 2012; Gür, 2009; Özdeş & Kesici, 2015). In these studies, it was determined that the students frequently used the expression "the elements are too many to count" for the concept of "infinite set". In addition, it was determined that students have learning difficulties about number sets, especially due to not knowing the subset-inclusion relationship. According to Baki and Mandacı Şahin (2004), the set concept is abstract in mathematics and students will have difficulties in learning abstractions (Senemoğlu, 2000). Therefore, by using concrete expressions or examples as memorable in the minds of students, and learning difficulties can be reduced or eliminated (Baykul, 2016).

Teacher's Subject Matter Knowledge

The concept of content knowledge, which is necessary for effective teaching, can be thought of as subject and concept knowledge specific to the field and curriculum that the teacher teaches (Aslan-Tutak & Köklü, 2016). According to Shulman (1986), content knowledge is defined as "the amount and organization of knowledge per se in the mind of the teacher" (p. 9). Content knowledge is the competencies that include subject-specific definitions, features, symbols, algorithms and selection of examples in the teaching process (Davis, 2003; Grossman, Wilson, & Shulman, 1989; Shulman, 1987, 2004). Therefore, it is expected that teachers who do not have sufficient content knowledge about the subject they will teach will also have a low contribution to the learning process (Ball, Thames, & Phelps, 2008). Because teacher's content knowledge includes problem solving methods and strategies, as well as competence in key concepts and rules specific to the subject (Toluk Uçar, 2011). With this, many instructional activities such as asking instructive and thought-provoking questions to students, evaluating student learning, choosing learning activities depend on the subject matter knowledge of teachers (Ball & McDiarmid, 1990). In this regard, teachers who do not have enough knowledge about the subject misstate the concepts related to the subject and cannot use enough appropriate teaching techniques for the subject (Suh, 2005). It is known that the field-specific pedagogical knowledge that the teacher should have depends on the content knowledge (McDiarmid, Ball, & Anderson, 1989). Therefore, it can be stated that subject knowledge is one of the indispensable requirements for guiding the teacher in the teaching process (Karal Eyüboğlu, 2011). As a result, it can be said that teachers' inadequacy in their field knowledge will negatively affect the process of using their field-specific pedagogical knowledge (Even, 1993; Küçükahmet, 2008; Suh, 2005; Yazıcı & Kültür, 2017). As a matter of fact, as a result of teachers' inadequacy in mathematics knowledge (Borko & Putnam, 1996; Richardson, 1996), the fact that the instructional explanations in-class teaching process are mostly based on memorization rather than understanding (Arslan Kılcan, 2006; Henningsen & Stein, 1997) supports this result.

Mathematical Knowledge for Teaching Model and Its Components

Shulman (1986) revealed that there is a relationship between the teaching-specific content knowledge concept and teaching practices. This situation has guided many researches on teacher knowledge (Ball, Hill, & Bass, 2005; Ball et al., 2008; Cochran, DeRuiter, & King, 1993; Gess-Newsome, 1999; Hurrell, 2013; Kind, 2009). As a result of these researches, different knowledge models that classify the knowledge about teaching that teacher should have, showed up such as Rowland's Knowledge Quartet, Fennema and Franke, Model of Teacher Knowledge by Grossman, and Tamir Model (Carpenter, Fennema, & Franke, 1996; Grossman, 1990; Rowland, Huckstep, & Thwaites, 2005; Tamir, 1988). In this context, one of the models developed exclusively for the mathematics education based on Shulman's "Pedagogical Content Knowledge" (PCK) is the MKT model (Ball et al., 2008). Ball et al. (2008) tried to determine the scope of mathematics knowledge required for these applications by focusing on the classroom practices and tasks of teachers in the mathematics teaching process with the

MKT model. Based on the importance of using the mathematics content knowledge in the teaching process, they elaborated the concept of "teaching" and evaluated everything that the teachers did in the mathematics teaching process as a part of the teaching process. They expressed the mathematical activities in the knowledge required in the teaching process as follows: Mathematical activities such as being able to explain definitions and concepts to students, commenting on students' solutions, giving satisfactory answers to why-how questions from students, determining mathematical tasks appropriate for the class and student level, assigning homework to students, evaluating students, communicating with parents about student work. The components of the MKT model are shown in Figure 1 (Ball et al., 2008).

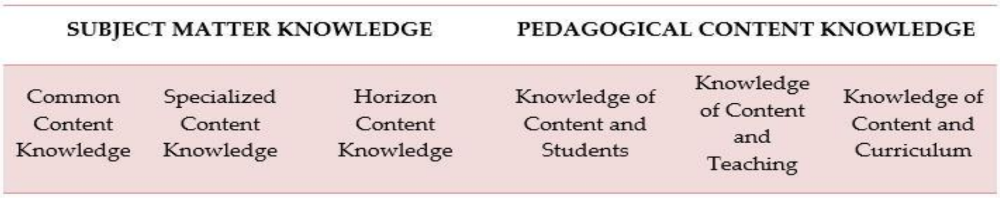

Figure 1. MKT model

What is the Teacher's Specialized Content Knowledge?

Ball et al. (2008) approached the concept of subject matter knowledge as the whole of the necessary information related to the subject that the teacher will teach to make effective teaching. Subject matter knowledge consists of three sub-components as “Common Content Knowledge”, “Specialized Content Knowledge”, and “Horizon Content Knowledge”. Specialized Content Knowledge (SCK) which is one of the sub-components examined in this research could be considered as mathematical knowledge that does not include pedagogical knowledge and can express the concepts such as mathematical statement, operation or concepts in the form of “why” and “how” (Aslan-Tutak & Köklü, 2016; Ball et al., 2008). SCK, which is a requirement for the basic mathematical activities that teachers carry out in-class teaching process, is a component that complements common content knowledge (Hill, Schilling, & Ball, 2004). SCK is the knowledge and skills necessary for "teaching" that no one except mathematics teachers is expected to have. For example, using a hundreds charts by modeling the concept of place value, defining mathematical terms accurately and intelligibly by students, and making solutions more understandable are skills within the scope of SCK (Thames & Ball, 2010). Again, it is a skill that requires SCK for a mathematics teacher to provide satisfactory explanations for students' why questions. For example, it is SCK to clarify the mathematical thinking that allows the rule of divisibility by “4” to be valid if the last two digits of a number are divisible by “4” (Hill et al., 2004). In addition to these, within the scope of SCK, the duties and responsibilities of the teacher are as follows (Ball et al., 2008). Finding examples/representations for certain mathematical situations, arranging in-class activities to be easier or more difficult, asking interesting and non-routine questions, and examining mathematical equivalences and relationships between them. When we examine these duties and responsibilities, it can be said that SCK is a type of knowledge that is not specifically needed for purposes other than teaching.

In mathematics education, teaching the concepts related to sets, which form the basis of many learning areas, is important in revealing the axiomatic structure of mathematics and shaping the logic of proof. Therefore, it is necessary for a mathematics teacher to gain in-depth knowledge and skills in teaching concepts related to sets. It is observed in the studies that both students and teachers have difficulties about basic concepts of sets (Demir, 2012; Gür, 2009; Kolar & Čadež, 2012; Uğurel et al., 2010; Uğurel & Moralı 2010; Yazıcı & Kültür, 2017). In support of this statement, in the researches on mathematics knowledge of teachers, it was found out that teachers have deficiencies in SCK and Knowledge of Content and Teaching (KCT), and the students made the same cognitive mistakes with the teachers (Aslan-Tutak & Köklü, 2016; Kar, 2014; Miheso-O'Connor Khakasa & Berger, 2016; Li, 2011; Speer, King, & Howell, 2015). In addition, it is important that teachers can give satisfactory explanations to students' questions about “The Basic Concepts of Sets”, since it is one of the teaching qualifications examined within the scope of SCK. For this reason, it is thought that examining the mathematical knowledge of mathematics teachers regarding the subject of "The Basic Concepts of Sets" in the teaching process is necessary in terms of teaching mathematics. However, by examining the SCK, which is a knowledge and skill that only mathematics teachers are expected to have, it is considered as a necessity to determine the deficiencies/riches in the knowledge and skills of the teachers regarding the subject of "The Basic Concepts of Sets." Thereby, the main issue of the research was constituted as "What is the status of the specialized knowledge of mathematics teachers about teaching basic concepts of sets?"

When the studies were examined, it was seen that students had misconceptions regarding the concepts of "common trait, community, set, universal set, finite-infinite set, subset" and "equal set" (Cheung et al., 2017; Demir, 2012; Fischbein, 2001; Gür, 2009; Hannula & Pehkonnen, 2006; İpek et al., 2009; Kolar & Čadež, 2012; Linchevski & Vinner, 1988; Monaghan, 2001; Özdeş & Kesici, 2015; Uğurel & Moralı 2010; Zehir et al., 2008). In addition, it was determined that teachers also have misconceptions and deficiencies regarding the concepts expressed (Chick et al., 2006; Uğurel et al., 2010; Yazıcı & Kültür, 2017). Therefore, this research is limited to the concepts of "Set, Universal Set, Infinite Set, Empty Set, Subset"" and "Equal Set".

Method

Model of the Research

The research was carried out with a case study design, one of the qualitative research methods to anatomize the specialized content knowledge of teachers by collecting detailed data. The case study is a research design that enables deeply and systematically analyzes the concept in its reality and allows describing the generated situation (Cohen, Manion, & Morrison, 2007; Creswell, 2012; Yin, 2017). Additionally, the case study is a method that allows the researcher to analyze in detail the restricted situations over time by observing, interviewing, audiovisual recording, documents, and reports (Creswell, 2012; Patton, 2002).

In this research, the status of the specialized content knowledge of the teachers was limited to the concepts of "Sets, Universal Set, Infinite Set, Empty Set, Subset and Equal Set" within the context of the results of the researches.

The Workgroup

The workgroup of the research consists of 18 mathematics teachers at different educational background and professional experience in teaching through the sampling of criterion method from different high schools in a province of the Mediterranean Region in the academic year 2016 to 2017. Three teachers were determined among these teachers on a voluntary basis. The two-week teaching processes of the teachers were followed by video and audio recording. In addition, while determining these three teachers, teachers with ten years or more experience in teaching profession were chosen so that the lesson would not be interrupted due to excitement and similar reasons. In the research, TV1, TV2, and TV3 were used instead of the names of the teachers whose video and audio was recorded. Codes like T1, T2, T3, ..., T15 were used instead of the names of the teachers who had the semi-structured interview only. Information about the teachers in the study group is given in Table 1.

Table 1. Demographic Information about Teachers That Were Interviewed and Observed

Code of the Teacher	Sex	Teaching Experience	Educational Status	Experience in Sets
TV1	Male	15 years	Bachelor's degree	7 year
TV2	Female	12 years	Bachelor's degree	8 years
TV3	Female	15 years	Bachelor's degree	7 years
T1	Male	10 years	Bachelor's degree	4 years
T2	Male	2 years	Bachelor's degree	Never taught
T3	Male	10 years	Bachelor's degree	1 year
T4	Female	8 years	Master's degree	3 years
T5	Female	5 years	Master's degree	2 years
T6	Male	6 years	Doctoral degree	5 years
T7	Female	2 years	Doctoral degree	Never taught
T8	Male	5 years	Bachelor's degree	Never taught
T9	Female	12 years	Doctoral degree	5 years
T10	Female	15 years	Bachelor's degree	1 year
T11	Female	15 years	Bachelor's degree	6 years
T12	Male	20 years	Bachelor's degree	2 years
T13	Male	15 years	Bachelor's degree	5 years
T14	Female	12 years	Master's degree	6 years
T15	Female	6 years	Bachelor's degree	3 years

Data Collection

In the research, the part of Set Knowledge Test for Teaching (SKTT) for the SCK component and "Subject Observation Form (SOF)" prepared by researchers were used as the data collection tool. At this stage, document review was used in the preparation of the content.

SKTT and SCK Dimension

The SKTT was prepared in accordance with the semi-structured interview format, consisting of open-ended questions. In the preparation process of the SKTT, the MKT Model and concept of "Set" were studied with five lecturers and six mathematics teachers who were leading experts. Primarily, the type of information that each component should have was determined with two lecturers by analyzing the components of the MKT Model (one of the experts being among the researchers). Moreover, a question pool was created through having interviews about students' perceptions of the concept of "Set", misconceptions, learning strategies, representations for permanent learning, and material selection with the technique of brainstorming in collaboration with six mathematics teachers and three lecturers. Thereafter, the specialist lecturers in the concept of "Set" built a consensus about the questions. Finally, SKTT was prepared in accordance with the components of the MKT Model. The SKTT that was prepared before the main implementation was implemented to 10 mathematics teachers who have been working in a province of the Mediterranean Region to pilot scheme. Therefore, teachers at educational background were included in the research taking account of professional experiences of the teachers, educational levels, and the school years (sixth grade, ninth grade) that the concept of "Set" was taught. Subsequently, the final state of the SKTT was formed in consequence of the responses of the teachers to the test, their opinions about the test, and their impressions. In this research, data were collected over the SCK dimension of the SKTT.

In the study, the following questions were used to collect data on SCK. In the following questions about the SCK, the duties and responsibilities set out by Ball et al. (2008) in the introduction part of the research were used. The emphasis in SCK for effective teaching is not only the presence of conceptual information. At the same time, this information can be used sufficiently in teaching tasks in the classroom teaching process (Aslan-Tutak & Köklü, 2016; Ball et al., 2008). As a matter of fact, no direct application of any rule or feature is expected in the content of the SCK. On the contrary, the ability to know what the mathematical idea that allows the rule to be made and to answer the "Why" question satisfactorily in line with this information comes to the fore. For this reason, the following questions have been designed in order to answer students' why and how questions, to make mathematical explanations and to select representations / examples for specific purposes. However, the content of the answers expected from teachers is also stated below:

Question 1: The teachers were asked through a student named Bora who said “There is no need to have a common trait between the elements of the set.” to claim that there is no common trait between the elements and therefore they will not form a set. The teachers were expected to mark the “false” option with an explanation that includes “why?” and “how?” They could clarify the claim through the process of combination in sets or the intuitive definition of the concept of “set”.

Question 2: The teachers were asked through a student named Esin who said “a set with a single element would form a universal set.” to claim that a set with a single element would not form a universal set. The teachers were expected to mark the “false” option with an explanation that includes “why?” and “how?”. The teachers could clarify the claim as 'the universal set may vary so elements of the universal set would not have to be more than once' through the definition of “universal set”.

Question 3: The teachers were asked through a student named Ali who said “The number of leaves of all trees on the earth indicates a finite set” to claim that the number of leaves of all trees on the earth indicates an infinite set. The teachers were expected to mark the “false” option with an explanation that includes “why?” and “how?”. The teachers could clarify the claim through the concept of the finite set; the finite set has certain limits and is expressed by a natural number. They could also clarify it with an example like “We can count the number of leaves on our environment as well as the number of leaves on the earth.” Also, it was expected from the teachers to clarify the claim through the statement “We cannot say endless for the things that we described as countless” (Ministry of National Education [MoNE], 2016).

Question 4: The teachers were asked through a student named Elif who said “The empty set is a subset of all sets.” to claim that the empty set is not a subset of all sets. The teachers were expected to mark the “false” option with an explanation that includes “why?” and “how?”. The teachers could clarify the claim through the definition of the concepts of “empty set” and “subset”. They could also clarify it with an example like “There are no elements that are not in any set that has an element in an empty set.”

Question 5: The teachers were asked through a student named Serra who said “If A and B are equal sets, then $A\subset B$ and $B\subset A$.” to claim that the set of negative natural numbers is not equal to the set of negative prime numbers. The teachers were expected to mark the “false” option with an explanation that includes “why?” and “how?”. The teachers could clarify the claim through the theorem that “There is only one empty set” (Nesin, 2008). They could also clarify it through that “Empty set is the subset of all sets.” What teachers should not express here is that “they are incomparable sets due to there are no elements of the empty set”. In Figure 2, a visual of one of the questions prepared in order to analyze the specialized content knowledge of the teachers during the interview is included.

Question 4. A student named Elif claims that "the empty set is not a subset of every set".
For Elif's claim, tick the appropriate option given below and explain the reasoning.

For Elif's claim;		
A. Is correct ( )	B. It is wrong ( )	C. I am not sure ( )
Your reason:	Your reason:	Explain exactly what's leaving you up in the air.

Figure 2. One of the questions prepared for SCK

Subject Observation Form

The subject observation form was prepared in accordance with the components of the MKT Model in order to record the researcher's impressions about the in-class teaching process. In the SOF, explanation sections have been added to the form in order for the researcher to write explanations when necessary. In the creation of the SOF, the work of Ball et al. (2008) named "Content Knowledge for Teaching - What makes it special?" and the book section prepared by Aslan-Tutak and Köklü (2016) for the MKT were used. While preparing the SOF, it has been tried to be created in a way that it will be consistent with the SKTT since the comparison will be made with the SCK component of the SKTT. In this context, SOF has been prepared according to six indicators of SCK. These indicators are as follows:

1. To be able to reveal the reasons for mathematical expressions, operations and concepts,
2. Finding examples / representations to indicate a particular mathematical point,
3. To be able to organize activities in a way that can be both easy and difficult,
4. Asking subject-specific generative mathematical questions,
5. To be able to adapt the mathematical content of the textbooks to the lesson,
6. To be able to quickly evaluate the suggestions or solutions of the students.

SOF's validity and reliability studies were provided by peer-review. For this, the indicators related to the components of the MKT were examined by two field experts and the components that should be added and removed were determined. In the expressions in the SOF, the aspects that the experts found incomplete were completed and the observation form was finalized.

Document Analysis

The document analysis comprises the analysis of written materials containing information about the notion or notions that are intended to be researched. (Yıldırım & Şimşek, 2008). The documents used in the research to support both the data source and other data collection tools consist of ninth grade mathematics textbooks, curriculum, guide books for teachers, studies about "Set" and lecture notes of students. Thus, it was aimed to form a scale at the SCK knowledge level by conducting document analysis. In other words, it was aimed to answer the question: "Which questions are better to establish the SCK levels of teachers related to basic concepts of "Sets?". Firstly, "Set, Universal Set, Infinite Set, Empty Set, Subset" and "Equal Set" concepts are selected as the analysis unit; then studies and misconceptions about these units were determined. Subsequently, the definitions related to the concepts mentioned in the lectures and teacher guide books were examined and subjected to content analysis. Thus, studies related to each concept, mathematics textbooks, and teacher's guide books were examined and keywords were formed. Keywords were determined as "Common Trait" for the concept of set, "Only Element" for the concept of the universal set, "Number of Leaves" for the concept of the infinite set, "Association with the Concept of Subset" for the concept of empty set and equal set. Three specialist researchers, (one of them specialized in MKT, the other two specialized in math education) participated in this phase. Also, the data obtained from the document analysis in the research were used in the analysis phase to support the data collection tools in the comparison of the data. The data obtained during the document analysis were continuously compared with each other and subjected to expert examination to increase credibility.

Data Analysis

In the analysis of qualitative data, descriptive analysis, content analysis, continuous comparison techniques and data analysis methods specific to each data collection tool were used together. Accordingly, it was primarily determined that in which themes the answers written by the teachers to the open-ended questions prepared in line with the SKTT test should be given. Document analysis and content analysis were used for this purpose. As a result of the content analysis, the responses of the teachers who participated in the research were coded and it was established that in which themes the answers should be presented. Thereafter, to determine whether the answers were "correct, false" or explanations were "correct explanation, partly correct explanation, false explanation"; analysis units were established from short words related to the answers or the most common statements in the answers.

According to Yıldırım and Şimşek (2008), during the descriptive analysis, the acquired data were interpreted in accordance with the themes and codes that established by the researcher; and the opinions of interviewees were directly written with quotations. In the analysis of the obtained data from the research, results were presented as themes and codes by using percentage and frequency techniques. Also, in-class teaching processes of the teachers who had video and audio records were transcripted and continuously compared with the SKTT data. According to Coffey and Atkinson (1996), the data analysis process needs to be carried out comprehensively and systematically. However, they also state that it is not possible to make the analysis process a standard process that will remain valid for each research. In the research, the data related to SCK were analyzed to explain reasons for mathematical expressions, operations and concepts contained in SCK content such as "why?" and "how?". Because, answering the "why" and "how" questions about mathematical expressions, operations and concepts is the essence of the knowledge that will determine the distinction between common content knowledge and SCK (Hill & Ball, 2009). The analysis scheme of the acquired data for the SCK component for this research is shown in Table 2.

Table 2. Analysis Process of the Acquired Data for SCK Component

Themes	**Codes**	**Data Analysis Process**
Exposing the reasons of mathematical expressions, operations, and concepts as "Why?" and "How?"	There is no need to have a common feature between the elements of a set.	First Stage
	A set with a single element can form a universal set.	At this stage, the answers to each of the five questions directed to the teachers were analyzed with percentage-frequency techniques as "It is correct", "It is false", and "I am not sure".
	The number of leaves of all trees on the earth indicates a finite set.	Second Stage
	The empty set is the subset of all sets.	At this stage, the reasons written by the teachers to the questions were classified as "False explanation", "Partially Correct Explanation", "Correct Explanation", and "I have no idea" and analyzed without any scoring.
	If Set A and Set B are equal sets, it has to be: $A \subset B$ and $B \subset A$	

If we examine the data analysis process through the sample question in the SKTT;
A student named Ali claims that the number of leaves of all trees on the earth indicates an infinite set. Mark the appropriate option listed below for Ali's claim and explain the reason.
Firstly, the answers of the teachers were analyzed as "It is correct", "It is false", and "I am not sure" in line with the statement "The number of leaves of all trees on the earth indicates a finite set". Thereafter, the teachers were expected to give a reason for the claim, and the reasons were analyzed without any scoring. The textbooks of MoNE (2016) are taken as a reference at this part.

- T1: Since the number of leaves of the trees is limited to the earth, it is a finite set. (Correct Explanation)
- T2: Due to lifetime is insufficient to count the numbers of leaves, it is an infinite set. (False Explanation)

Data that demonstrate the mathematical sentences, operations, and concepts as "why" and "how" were analyzed in two stages. In the first stage, themes related to five questions directed to the teachers were prepared. The teachers' responses were analyzed under these themes. The results of the research were presented by writing titles under the themes prepared. For this, teachers' answers to each of the five questions were classified as "True", "False", and "I am not sure" and these data were analyzed by percentage-frequency techniques. In the second stage, the teachers were asked to write reasons for their answers such as "Why is it true?", "Why is it wrong?" or "Why are you not sure?". The reasons given by the teachers were evaluated without any scoring by classifying as "False explanation", "Partially correct explanation", and "I have no idea". The purpose of that kind of data analysis is to determine whether the justifications of the teachers who gave the correct answers to the questions in the SKTT were correctly written or not. While quoting the teachers, the results of the teachers only who were performed the SKTT were interpreted according to the quotations of only two teachers related to each category. All of the quotations of the teachers whose both SKTT and in-class teaching processes and also video-audio recordings performed (TV1, TV2, and TV3) were included separately to the research. In addition, while the teachers' in-class teaching processes were included, sections about teaching processes with deficiencies related to the concept were written.

Validity and Reliability

Reliability in qualitative research is directly related to the reliability of the observation of the researcher and the detailed presentation of each stage of the research (McMillan & Schumacher, 2010). For that purpose, data triangulation approach that multiple data collection methods and techniques were used, had been adopted within the scope of the research to minimize the possibility of "systematic error" (Yıldırım & Şimşek, 2008). In this research, three types of data collection methods as "interview", "observation" and "document" analysis were used to increase the soundness and reliability of the research. In the research, the teachers were interviewed in line with the SKTT test. In the meantime, natural and unstructured observations were made throughout the unit to evaluate the teachers' lectures by recording. The use of observation technique is aimed to determine the consistency between teachers' answers to the interview and their in-class behaviors.

Intercoder reliability was used to determine the credibility of the research. The acquired data in the research were coded as "I have no idea", "False explanation", "Partially correct explanation", "Correct explanation" and "It is correct", "It is false", "I am not sure" by the two researchers in line with the themes mentioned above. The cases where the same code was used by two researchers were marked as "Agreement" and the cases where different codes were used marked as "Disagreement". Thereafter, the consistency of the coded data was determined by using the formula [Consistency=Agreement / (Agreement + Disagreement)] developed by Miles and Huberman (1994). According to Miles and Huberman (1994), encoder compatibility should be at least 80% for good reliability. Since the coding reliability was found 81.4% in this research conducted by us, the research was accepted as reliable.

Results

During the interviews conducted with teachers in line with the SKTT, five questions were asked to examine the specialized content knowledge related to basic concepts about sets and the teachers were expected to write the reasons in detail as "Why" and "How". The results are written according to the codes in Table 2. In addition, the in-class teaching processes of the teachers followed by the SOF were transcribed, and findings of only one teacher were included after the written answers. In this context;

Question 1: In line with the statement of "There is no need to have a common trait between the elements of the set", questions such as "A student named Bora claimed that there is no common trait between the following elements: '1, a, Ankara, Japan, May, Tuesday', so these elements cannot form a set. Select the appropriate option listed below for Bora's claim and explain the reason" were asked to the teachers. For this question, the results obtained from the teachers' answers are given in Table 3.

Table 3. The Obtained Results from the Answers of the Teachers to Bora's Claim

	Frequency (f)	Percentage Distribution (%)
It is correct.	6	33
It is false.	11	61
I am not sure.	1	6

As it's seen in Table 3, more than half of the teachers stated that the claim was wrong; six of the teachers stated that the claim was true. However, one of the teachers did not answer the claim as correct or false but marked the "I am not sure" option. The teachers were expected to mark the "It is false" option and explain "why and how is it wrong". The justifications of the teachers who marked "It is correct" option for Bora's claim are as follow;

T5: Since there is no common trait between the elements, we cannot form a set with these elements.

T6: A collection of objects with the same features forms a set.

T8: Because there is no significant and distinctive common feature to form a set.

T11: Generally, elements that have several features in common can form a set according to the rooted set definition.

When the reasons were examined, it was established that the source of the misconception of the teachers was the definitions related to the concept of set. In addition to this, the teachers did not provide any reason for "why" and "how". The justifications of the teachers who marked "It is false" option for Bora's claim are as follow;

T2: There is no common trait; however, it is wrong to say it cannot form a set. It is important to understand the "well-defined" part in the definition of set.

T3: There is no condition to have a common trait between the elements of a set.

T4: The elements don't have to have a common trait to form a set. The common trait condition is existed by virtue of that condition makes possible to demonstrate a set.

T13: There is no condition to have a common trait between the elements of a set. Stating the elements clearly is sufficient.

When the reasons were examined, it was established that the teachers realized the claim was wrong. Also, it was found that only T2 and T13 wrote a partially correct explanation of Bora's claim for "why" and "how" it was false. When the explanations of T3 and T3 were examined, it was established that they stated conceptional features rather than reason.

Each of the three teachers who participated in the video and audio recordings of the in-class teaching processes wrote different answers to Bora's claim. For Bora's claim;

TV1: It is false. The elements of a set can consist of different entities. The common trait method cannot be used only in the demonstration of the set.

TV2: It is correct. There is no common trait between the elements. At least at first glance. However, a set can be formed by writing long sentences and finding a common trait.

TV3: I am not sure. There is a common trait between 1, a, and Ankara but I'm not sure about the other three elements. Therefore, either can be formed or cannot be formed.

When the statements were examined, it was established that only TV1 realized Bora's claim was false, however, he cannot manage to write an explanation for "why" and "how" it was false. On the other hand, TV2 and TV3 failed to write both the correct answer and the correct explanation for Bora's claim. A section from the class where TV1 had been teaching the common trait method is transcripted below. It is seen that TV1 does not state any explanation revealing why and how in the in-class tuition either.

TV1: Let's suppose the students of class 9/K is Set A. So what do we get from that? All the students in class 9/K, right?

ST: Yes.

> *TV1: What have we done here? We indicated the set instead of writing down all of the names of the student. So, can I write all sets with a common trait? For instance, how can I form a set with the elements of "Sunday, Monday, January, 1, 2"?*
>
> *ST: We cannot sir, we cannot form a set with these elements.*
>
> *TV1: Why not? Maybe we cannot do that with the common trait method but we can form a set with the list method. Also, these elements form a set. Why? Because a set can consist of different types of elements. The common trait method is just a demonstration.*

As TV2 and TV3 could not write both the correct answer and the correct explanation for the claim, by quoting similar statements of other teachers who wrote wrong explanations, these teachers' in-class teaching processes were not written here.

Question 2: In line with the statement of "A set with a single element can form a universal set": "A student named Esin claims that a set with a single element cannot form a universal set. Mark the appropriate option listed below for Esin's claim and explain the reason." For this question, the acquired results from the teachers' answers are given in Table 4.

Table 4. The Obtained Results from the Answers of the Teachers to Esin's Claim

	Frequency (f)	Percentage Distribution (%)
It is correct.	2	11
It is false.	16	89
I am not sure.	0	0

As it's seen in Table 4, most of the teachers stated that Esin's claim was wrong; only two of the teachers stated that the claim was correct. None of the teachers marked "I am not sure" option and did not write any answer to state it was correct or false either. The teachers were expected to mark the "false" option with an explanation that includes "why" and "how". The justifications of the teachers who marked "It is correct" option are as follow;

> *T8: Due to the universal set is the largest set that we pick elements from it, a set with a single element cannot be a universal set.*
>
> *T10: Universal set is a set that comprises the largest set. Since it has to be large as much comprise all sets, there is no such a universal element with a single element.*

When the reasons were examined, it was found that the teachers wrote incorrect explanations due to the deficiencies in the definition of the concept of "Universal Set". T10 wrote the wrong explanation that the universal set has to be a large set that comprises everything, which doesn't suit with the definition of the universal set. T8's statement "the largest set that we pick elements from it" overlaps a part of the definition of the universal set. However, considering that T8 wrote that a set with a single element cannot be a universal set; it appears T8 thinks that the universal set is a large set with many elements. Therefore, the statements of T8 and T10 about Esin's claim includes incorrect reasons for "why" and "how".

Justifications of the teachers who marked the "It is false" option for the Esin's claim are as follows;

T3: Elements of a universal set do not have to be more than 1. If there is no other set or element involved in the operation, a single element can form a set as well.

T4: The number of elements in the universal set does not have to be larger than 1. Since an empty set is the subset of all sets, it is also a subset of a set with a single element. Thereby, $\varnothing \subset E$.

For example: If E={1}, then $\varnothing \subset E$.

T7: Due to each set is a subset of itself, it is also a subset of all possible sets.

T9: Because, if the set that we are working on is Ø and for example, if we call {a} to the set with a single element, then Ø is a subset of {a}.

T11: A set that can comprise each set we defined can be a universal set. A set with a single element that comprises the empty set can be the universal set of the empty set.

From the reasons written by the teachers, it was established that the teachers realized the claim was false. When the statements were examined, it was established that T3 and T7 wrote partially correct answers to Esin's claim for "why" and "how". When attention is paid to the statement of T7, it's seen that T7 stated the conceptional features rather than reason. However, when the statements were examined, it was established that explanations of T4, T9, and T11 were correct for "why" and "how".

It was found that all three teachers whose video and audio recordings were taken during the in-class teaching process marked the "It is false" option for Esin's claim. For Esin's claim;

TV1: It is false. It can form a universal set. The sets that included in operations can contain only a single element.

TV2: It is false. A set with a single element can form a set. For instance, let's suppose Demir Family has only one child. E={x:x children of Demir family}, A={x:x blond children of Demir family}, B={y:y brunette children of Demir family}

TV3: It is false. A set with a single element can be a universal set owing to the fact that it changes according to the circumstance.

When the statements were examined, it was established that the teachers realized Esin's claim was false; however, they could not manage to write a satisfying answer to the claim for "why" and "how". Also, none of the teachers wrote an explanation by examining together the concepts of "Ø" and "universal set". In the example written by TV2, even though Set A, Set E, and Set B are stated differently; Set E is equal with Set A or Set E is equal with Set B due to the fact that "a family with an only child cannot have both a blond and a brunette child." Thus, it was established that TV2's explanation was wrong for Esin's claim. The statement of TV3 whose teaching process video-audio recorded was transcripted below. (Only the part from the class that he was teaching the universal set.) It is seen that TV1 does not state any explanation revealing why and how in the in-class tuition either.

TV3: If we consider people, trees, animals, and plants as a set, which is generally given as the easiest example, the living creatures set becomes the universal set of these sets. Let's examine these sets; A={0,1,2,3,4,5,6,7,8,9} and Set B is the natural numbers set. Can you form a universal set considering these 2 sets?

ST: We cannot sir… It can be natural numbers set… Sir the real numbers set…

TV3: That's true. The real numbers set can be the universal set of these 2 sets. It can be rational numbers and natural numbers set as well. So can we say that: The universal set is not unique. It depends on the circumstance. In other words, there is no single universal set. In the previous example, "the living creatures" was my universal set. In the present example, the natural numbers set or the real numbers set became my universal set…

It's seen that all of the examples of TV3 are given by selecting sets with the largest elements. Due to it was established from the video record that TV1 and TV2 were pursuing the in-class teaching process similarly, the in-class teaching processes of TV1 and TV2 were not transcripted here.

Question 3: In line with the statement "The number of leaves of all trees on the earth indicates a finite set": "A student named Ali claims that the number of leaves of all trees on the earth indicates an infinite set. Mark the appropriate option for Ali's claim and explain the reason." The acquired results for this question are given in Table 5.

Table 5. The Obtained Results from the Answers of the Teachers to Ali's Claim

	Frequency (f)	Percentage Distribution (%)
It is correct.	3	17
It is false.	13	72
I am not sure.	2	11

As Table 5 shows, most of the teachers stated that Ali's claim was wrong; three of the teachers stated that the claim was true. Nevertheless, two of the teachers marked the "not sure" option regarding the trueness or fallaciousness of the claim. However, teachers were expected to mark the option "It is false" for Ali's claim and explain "How?" and "Why?" it is false. The reasons for the teachers who marked the "It is correct" option for Ali's claim are as follows;

T1: If we stop the time for a moment and start counting the leaves, we can think of a finite number of different leaves. But the leaves are renewed in nature at any moment. Hundreds fall and thousands flourish. So we can't count.

T3: We don't have a chance to enumerate all the elements. Even if the last leaf is counted, new leaves will appear.

T11: It is considered correct according to the definition of the set. It is a finite set. But in fact, human life cannot be sufficient to count the number of leaves.

When the reasons were examined, it was found that false explanations were arising from the deficiencies in the definition of the concept of "Infinite Set". However, the expressions of T1 and T3 contain contradictory statements. Because "the last leaf" in T3's "even if the last leaf is counted" statement indicates that the claim is a finite set. It is also possible to see the same situation in T1's statement "a finite number of different leaves". However, "… we cannot say endless for the things we described as countless" (MoNE, 2016) in the context of the explanation, it was found that the description of T11 is a false explanation that does not coincide with the definition of the infinite set. Therefore, T1, T3 and T11 wrote answers that state false reasons to "Why?" and "How?" statements about Ali's claim. The reasons for some teachers who marked "It is false" for Ali's claim are as follows;

T2: The number of leaves is a countable feature. It may be a very large number but it is not infinite.

T8: Infinite is not a number but a concept. Therefore, the number of trees on earth is still a number and not an infinite set.

T9: Although the number of leaves may seem infinite, it is ultimately finite. Maybe there is no figure to express it, but it still is finite.

T12: The result is that the number of leaves is finite. It is very difficult to count but can be expressed with a natural number.

T14: Since the number of leaves of the trees is limited to the surface of the earth, it is a finite set.

When the reasons were examined, it was found that the teachers realized that the claim was wrong. However, it was determined that T12 and T14 marked the correct explanation to "Why?" and "How?" to Ali's claim. T12 and T14 explained that Ali's claim was wrong in line with the statements that "being expressed by a natural number" and "limited availability" based on the definition of the finite set concept. On the other hand, although T2, T8, and T9 realized that the allegation was false, they based their statements on false grounds.

The teachers who marked "not sure" for Ali's claim and what leave them in suspense are as follows;

T13: I am not sure. The set with elements that cannot be counted is called an infinite set. Tree leaves are too many to count. But for a set to exist, we don't have to know its elements. So it would be wrong to admit that this number doesn't exist because none of us know this number. It is like it is right and wrong at the same time.

T15: Mathematically, it would be an infinite set because we cannot sit and count all the leaves in the world. It depends on our perspective. We refer it as an infinite set to the students. Because, if we could sit and count all the leaves on the earth, we could have a border as much as the world. That is why I am not sure.

When the written statements were examined, it was found that the teachers gave inconsistent answers that the claim was wrong. T13's "none of us know the number" statement and T15's "we have a border as much as the world" statement explains the situation. The reason for inconsistent answers written by the teachers can be thought to be due to the errors in defining the concepts of "finite and infinite sets" and their failure to make definitions by associating these two concepts with each other.

It is established that the teachers whose video-audios are recorded during the in-class tuition marked the "It is false" option for Ali's claim.

TV1: It is false. The trees on the earth and the leaves of these trees have an end. Just because it is on earth, it shows that there is a limit and although it is a very large number it has an end.

TV2: It is false. Tree leaves are numerous. Therefore, it is a finite set.

TV3: It is false. It is finite because we can count. Even if our lives are not enough. Because the world is definite and if we want to count the living and nonliving beings in it we can count.

When the answers were examined, it was found that the teachers realized Ali's claim was wrong and they wrote a satisfactory explanation to "How?" and "Why?" about the claim. Within the definition of "finite set" TV1 and TV3 wrote an explanation to the claim in line with "finite sets have borders and they are definite". On the other hand, it has been found that TV2 makes a statement to the claim that finite sets are countable sets. The statements of TV3 whose teaching process video-audio recorded were transcripted in line with the conceptions of "finite set" and "infinite set. It is seen here that TV3 gives an example of the number of leaves of trees in the course of assertion (SKTT applied later).

TV3: Can you count the number of leaves of all the trees on earth? Is the number of stars in the sky more or the natural numbers we know? I guess you have more or less understanding of what we are doing. Now let's examine the concepts of finite and infinite sets. What do you think is a finite set?

ST: It must have enough elements to count. It means to a certain extent.

TV3: Right. So if the number of elements of a set is equal to a natural number, that is a countable multiplicity and its boundaries are known this set is called a finite set. If A={1,2,3,4,5} then s(A)=5 is a finite set.

In-class teaching processes of TV1 and TV2 haven't been transcripted due to it was established that TV1, TV2, and TV3 had been pursuing the process of in-class tuition.

Question 4: In line with "The empty set is a subset of all sets." to teachers; a student named Elif "Claims that an empty set is not a subset of each set." For Elif's claim, select the appropriate option below and explain the reason." type a question was asked. For this question, the results of the answers written by the teachers are given in Table 6.

Table 6. The Obtained Results from the Answers of the Teachers to Elif's Claim

	Frequency (f)	Percentage Distribution (%)
It is correct.	0	0
It is false.	18	100
I am not sure.	0	0

As it is seen in Table 6, all of the teachers stated that Elif's claim was incorrect. However, the teachers were expected to select the "It is false" option for Elif's claim and explain "How?" and "Why?" the claim was wrong with its reasons. The reason for the teachers who marked "It is false" for Elif's claim is as follows;

T2: Let X be a set. Suppose that the empty set is not a subset of X. So the empty set contains an element that X doesn't have. This contradicts the definition of an empty set. So the empty set is a subset of X.

T4: Since the empty set does not have any element, Ø is considered as a zero-element subset of each set.

T9: Definition of the empty set.

T10: An empty set is a set with no elements. Therefore, it is a subset of every set.

T13: Let's suppose there is a set named A, and the empty set is not a subset of Set A. In other words, an element that is not in Set A has to be included in an empty set, so the element will not be the subset of Set A. However, since there are no elements in the empty set, this does not happen. So the empty set is a subset of each set.

T14: Since even the subset of the empty set will still be itself, the empty set is a subset of each set.

T15: The empty set is a subset of each set. Because $\varnothing = \{x : x \neq x\}$. Let A be a set. Let's accept $\varnothing \not\subseteq A$. So Ø must have at least one element that does not belong to Set A, which contradicts the definition of the empty set. Then $\varnothing \subset A$ statement is always true.

When the reasons are examined, the teachers T4, T9, and T10 express a feature of the concepts of "empty set" and "subset" rather than justification; it was found that they wrote a partly correct explanation to Elif's claim "How?" and "Why?" the claim was wrong. However, when the explanations of T2, T13, T14, and T15 were examined, it was determined that Elif had written the right explanation because of her statements stating "How?" and "Why?" the claim was wrong. Because it was found that teachers explained the claim in with the definitions of empty set and subset concepts.

It was found that teachers whose teaching course processes were followed by video and audio recordings marked the "It is false" option for Elif's claim.

TV1: It is false. Each set has a subset, which is an empty set.

TV2: It is false. Each element in Set A is the subset of the Set B and Set A is the subset of Set B. According to this definition of subset, there are no elements in the empty set that are not in Set B. In other words, the empty set does not have an element that can be called a subset of any set. Therefore, the empty set is a subset of each set.

TV3: It is false. Ø is a subset of each set. Because it has only one subset. (2^0=1)

When the written explanations were examined, it was found the teachers realized that Elif's claim was wrong, but they could not write a satisfactory explanation to "How?" and "Why?" the claim was wrong. It was determined that none of the teachers examined the definitions of the concepts of empty set and subset together. However, it was found that TV1, TV2, and TV3 express a feature of the concepts of empty set and subset rather than justification, therefore they wrote partly correct explanations to "Why?" and "How?" for Elif's claim was wrong.

A section from the class where TV2 had been teaching the concepts of the empty set and the subset is transcripted below;

TV2: A set without an element is called an empty set. The empty set is indicated by one of these symbols Ø or {}. A= {Ø} or A={{ }} sets are not empty sets.

…

TV2: Now A={a,b,c,d}, B={a,b,d}, C={a,b,c,d,e} let's examine the subset relationship between sets with the help of Venn scheme.

…

B⊂A and A⊂C according to the figure next to it. It can also be written in B⊂C…

Features of the Subset

1. Each set is a subset of itself.

2. The empty set is a subset of each set.

3…

Example: Now let's take Set B = {a, b, c}. And let's write its subsets…

How many subsets there are?

It is seen that TV2 does not provide a detailed explanation of concepts of "empty set" and "subset" and does not relate the concepts to each other. However, it has been found that among the properties of the subset, the empty set is a subset of each set.

Question 5: "For sets A and B to be equal sets, should be A⊂B and B⊂A." Through the teachers' statements, a student named Serra claims that "The set of negative natural numbers and the set of negative prime numbers are not equal to each other". For Serra's claim, select the appropriate option listed below and give a reason." question is asked. For this question, acquired answers written by the teachers are given in Table 7.

Table 7. The Obtained Results from the Answers of the Teachers to Serra's Claim

	Frequency (f)	Percentage Distribution (%)
It is correct.	4	22
It is false.	13	72
I am not sure.	1	6

As Table 7 shows, most of the teachers stated that Serra's claim was incorrect; four of the teachers stated that the claim was correct. However, only one of the teachers pointed the "I am not sure" option regarding the rightness or wrongness of the claim. However, teachers expected to mark the option "It is false" for Serra's claim and explain "How?" and "Why?" it is false. The reason for teachers who chose "It is correct" option for Serra's claim are as follows;

T4: Both are the empty sets. Therefore, they do not have elements. Equal sets are sets of the same elements. The given two sets can be equivalent. Because the number of elements is equal. (Both have zero elements.)

T11: If the negative natural numbers set = {}, If the negative prime numbers set = {}, both sets are empty sets. It's like nothing. How can we compare non-exist elements and say they are equal? In other words, equality between existing assets is sought.

T14: Since there are no negative natural numbers set and negative prime numbers set, these two non-exist sets can't be equal.

When the reasons are examined, the explanations of T4, T11, and T14 like "They cannot be compared due to there are no elements of the empty set." In line with the statement if "$A \subset B$ and $B \subset A$, then $A=B$" (MoNE, 2016) in the definition of the equal set and "The empty set is the subset of all sets." could be stated as $\varnothing \not\subseteq \varnothing$", which could be interpreted as any two empty sets are equal. Even from the theorem that "there is only one empty set" (Nesin, 2008), it could be stated that the negative natural numbers set and the negative prime numbers set are equal sets. Hence, T4, T11, and T14 wrote in their explanations regarding Serra's claim that they gave incorrect reasons for" How?" and "Why?" the claim was wrong. Some of the teachers who marked the "It is false" option for Serra's claim are justified as follows:

T2: These two sets are empty and they are equals.

T7: There is no negative natural number, so if we call X is a set then X = Ø. There is also no negative prime number. If we call Y is a set then Y = Ø. So X = Y.

T9: There are prime numbers as there are no natural negative numbers. Therefore, both are empty sets. They are equals.

T12: Sets which consist of the same elements are equals. Here, both sets are equal because they are empty.

T13: They both are empty sets. And they are equals.

When the reasons written by the teachers were examined, it was found that the teachers realized that the claim was wrong. However, it has been found that only teachers T7 and T12 have written the correct explanation of Serra's claim that "Why?" And "How?" are wrong. Based on the definition of equal sets T7 and T12 explained Serra's claim was wrong with its reasons. Therefore, T7's and T12's reasons are classified as correct explanations. On the other hand, T2, T9, and T13 express a feature of the concept of "empty set" and "equal set" rather than justification, therefore, it is determined that Serra's claim is partly correct in explaining "How?" and "Why?". It was established that none of the teachers wrote an explanation to clarify that negative natural numbers and negative prime numbers are subsets of each other. Having selected the option of "I am not sure" for Serra's claim, T3 did not give any reason as to what had left him in suspense.

Two of the three teachers whose video and audio in-class teaching processes are recorded marked the "It is false" option for Serra's claim; it was found that only one of them marked correctly. For Serra's claim;

TV1: It is correct. Both of them are empty sets, but we can't say empty sets are equal.

TV2: It is false. Both sets are empty. They are equals.

TV3: It is false. Because both sets do not have elements. They are empty sets and they are equals.

When the explanations were examined, it was concluded that TV2 and TV3 realized that Serra's claim was wrong but they could not write satisfactory explanations for "Why?" and "How?". It has been found that TV2 and TV3 express a characteristic of "empty set" and "equal set" rather than justification, and therefore they wrote a partly correct explanation for Serra's claim "How?" and "Why?" it is wrong. It was found that TV1 correctly marked for Serra's claim. The statement of TV1 "...we cannot say empty sets are equal to each other" contradicts the theorem that "there is only one empty set" (Nesin, 2008), so the rationale for TV1 is a false explanation.

A section from the class where TV1 had been teaching the concepts of the empty set and the subset is transcripted below. Here, it is seen that TV1 does not provide any explanation for the claim in in-class teaching. In this case, it can be said that TV1 does not have enough SCK for the concepts of "empty set" and "equal set".

TV1: C = {x: x> 0 and x, a negative integer}

Let's interpret the Set C given on the board. "x" will be both a negative integer and greater than zero. What kind of set is this then?

ST: Empty set.

TV1: Then we have defined the empty set. So, the set without the element is called the empty set.

TV1: …how do we show the empty set?

The empty set is denoted by one of the symbols "{}"or "Ø".

What about s (Ø) =? If I ask what it is, you'll know right away.

Tell me.

ST: Zero, sir.

TV1: It is correct. How about this? If A = {Ø}, what is the number of elements of set A?

ST: Zero, sir. It has only one element, sir.

TV1: Guys, it has only one element.

Why is there an empty set in the Set A…

TV1: Now let's take an example of the equal set.

If A = {x: 0 < x <5, x∈Z}, B = {x: x <6, x∈N}, let's write these sets together with the list method. You write first.

ST: Here it is, sir. It is A={0,1,2,3,4,5}, B={1,2,3,4,5,6}

The number of the elements is the same, equal sets, sir.

TV1: Set A, nice you did right. We chose the elements of x from integers in the set A, didn't we? Let us remember what the integers were…

There is something missing in Set B. What were the natural numbers, my friends?

I will write it down. N = {0,1,2,3,4,5...} starts from zero and goes forever. We didn't get zero in set B, so we should write zero. And the key in Set B is "less than" "<". So we won't take the "6" element. So set B = {0,1,2,3,4,5}. Same as Set A.

My friends, it is not enough to have the same number of elements to be equal sets. All of their members must be identical.

When the findings are generally interpreted, teachers write generally correct answers to the questions asked, but the statements they give to "how?" and "why?" it reveals that they are incapable of explaining that this is the way it is. In other words, it is seen that teachers regarding the basic concepts in the sets write explanations based on the characteristics of the concepts rather than their specialized content knowledge. This situation arises especially in the concepts of universal set, infinite set and empty set. However, it was also observed that teachers wrote wrong explanations as a result of wrong definitions of the concepts. As a matter of fact, in order to indicate a reason within the scope of SCK, the conceptual understanding must be realized (Aslan-Tutak & Köklü, 2016). For example, it has been observed that teachers make incorrect explanations for the concept of the universal set that "there should be a big set that encompasses everything". For the infinite set, it was observed that the teachers made definitions contradicting with the statement "…we cannot call things an infinite set that we consider to be unable to end by counting" (MoNE, 2016). It was determined that teachers did not write explanations by associating the empty set concept with the concept of subset. Instead, it was determined that they wrote explanations that empty sets are incomparable sets because they have no elements. As it's seen in Table 8, reasons were written by the teachers for the questions related to justify the mathematical expressions and operations as "why" and "how", were classified as "partially correct explanation" and "false explanation".

Table 8. Dispersion of the Data to Explain the Reasons for Mathematical Expressions, Operations and Concepts as "Why" and "How"

	False Explanation	Partially Correct Explanation	Correct Explanation	I have no idea
There is no need to have a common feature between the elements of a set.	T5, T6, T8, T11, TV1, TV2, TV3	T1, T2, T3, T4, T7, T9, T10, T12, T13, T14, T15		-
A set with a single element can form a universal set.	T8, T10, TV2	T1, T2, T3, T5, T6, T7, T12, T13, T14, T15, TV1, TV3	T4, T9, T11	-
The number of leaves of all trees on the earth indicates a finite set.	T1, T2, T3, T8, T9, T11, T13, T15	T4, T5, T6, T7, T10	T12, T14, TV1, TV2, TV3	-
The empty set is the subset of all sets.		T1, T3, T4, T5, T6, T7, T8, T9, T10, T11, T12 TV1, TV2, TV3	T2, T13, T14, T15	-
If Set A and Set B are equal sets, it has to be: $A \subset B$ and $B \subset A$.	T3, T4, T11, T14, TV1	T1, T2, T5, T6, T8, T9, T10, T13, T15, TV2, TV3	T7, T12	-

Discussion and Conclusion

Common content knowledge (CCK), which is one of the subject matter knowledge components in MKT model, is the mathematical knowledge used by everyone who deals with mathematics in areas where mathematics is used intensively (Aslan-Tutak & Köklü, 2016). It was established in the research that the teachers generally gave superficial or deficient answers by approaching the questions regarding the determination of SCK with general content knowledge. Instead of providing deeply mathematical explanations, the teachers gave answers at a general level of mathematical knowledge that someone engaged in mathematics could answer. In the researches, it is seen that pre-service mathematics teachers do not perform conceptual learning regarding "The Basic Concepts of Sets" and they have difficulty in making the formal definition of the concepts (Doruk & Çiltaş, 2020; Fischbein & Baltsan, 1999; Speer et al., 2015; Zehir et al., 2008). In this study, we can express the erroneous or incomplete information that teachers have about the concepts of "Set, Universal Set, Infinite Set" and "Equal Set" as follows:

- It has been observed that the teachers have erroneous perceptions that the elements that make up the set should have common traits, based on the expression "well defined" in the set definition;
- Teachers did not include the expression "shown in a closed curve or polygon" regarding the representation method with Venn diagram in the classroom teaching process;
- Inability to make a satisfactory explanation that there is no need to have a common trait among the elements of the set (cannot answer based on the combination feature in sets);
- Not to include in definitions that the universal set may change according to the situation studied;
- Trying to explain the concept of infinite sets by considering it alone without associating it with a finite set;
- On the subject of subset, the students cannot satisfactorily explain "why 2^n" is the rule of 2^n, which is used to find the number of subsets of a set with n elements;
- Not making a definition by associating the concept of "equal set" with the concept of "Subset" ($A=B \Leftrightarrow A\subset B$ and $B\subset A$).

When the above results are evaluated in the context of the list of teachers' roles and responsibilities in classroom teaching within the scope of SCK (Ball et al., 2008), SOF data and question expressions, we can say the following:

It was observed that the teachers were not sure whether the elements given as "1, a, Ankara, Japan, May, Tuesday" (Question 1) would form a set or not, as they did not have common traits. It was also observed that teachers made incorrect instructional explanations regarding this situation. This situation was also observed in the teachers followed up with SOF. This situation shows that teachers have misconceptions about whether the set consists of elements with the same features. A similar situation can be seen in the researches carried out by Linchevski and Vinner (1988) and Yazıcı and Kültür (2017). The participants stated that "There has to be a certain feature between the objects to form a set." TV2, whose teaching process was followed by using SOF, stated that elements can form sets even if they do not show common traits at first glance. However, it can be said that TV2 made incorrect explanations in terms of instruction based on the following statements: "…by writing long sentences, one can somehow find the common trait." It can be said that this result is due to TV2's misconception that a collection of well-defined elements can form set only if the objects have common traits. Similarly, Demir's (2012) study shows that due to most of the set examples written by the teacher and students were formed to the common trait, it has similar results in terms of the tendency of the perceptions of teachers and students about the concept of set. In the context of the statement "A set with a single element can form a universal set" (Question 2), the following results were obtained: It was observed that only three teachers were able to make an instructionally correct explanation that the universal set could consist of a single element (a set consisting of only one element) depending on the situation in which it was studied. On the other hand, it was determined that other teachers made incorrect instructional explanations that the universal set should be a very large set covering every set. In the SOF, it was determined that none of the teachers wrote explanations by examining the empty set and the universal set concept together in-class teaching process, supporting this result.

It was seen in the research that the teachers made false explanations for the claim that the number of leaves of all trees on the earth would form an infinite set (Question 3). The source of the false explanations is the misconceptions of the teachers that supposing the elements of the infinite set are too many to count. The statement of MoNE (2016) which is given in the textbook (MoNE, 2015) for the concept of infinite set summarizes the situation and shows the misconceptions of teachers: “We cannot say endless for the things that we described as countless” (p. 19). It can be said that this error may be caused by the wrong explanations written in some textbooks as "sets with too many elements to be counted are called infinite sets" (Karakuyu & Bağcı, 2016, p. 15). It was established in the research carried out by Gür (2009) that students frequently refer to the statement “its elements are too many to count” for the conception of infinite set. Based on the research conducted by Gür (2009) and the findings of this research, this situation could be interpreted as difficulties that teachers have been having during the in-class teaching might also appear for students. When we examine this situation from a different perspective, it has been stated in some studies that the difficulties experienced by teachers in the teaching process may be a reflection of their K-12 education years or may be a result of misleading learning that continues uncorrected in undergraduate education (Aslan-Tutak & Adams, 2015; Browning, Edson, Kimani, & Aslan-Tutak, 2011; Jones, 2000). In other words, it can be thought that the difficulties teachers experience in the context of field knowledge arise from their past student experiences.

Among the basic concepts examined in the research, the other ones that the teachers have mostly incomplete knowledge are “equal set” and “subset”. Here, the question expressions prepared regarding the concepts of empty set, equal set and subset were examined together (Question 4, 5). It is seen that the teachers stated that the equal sets must be the set with the same elements and the same number of elements. However, none of the teachers associated the subset with the equal set ($A=B \Leftrightarrow A \subset B$ and $B \subset A$). This result was determined in the same way in all teachers who followed the teaching process with SOF. The reason for this situation could be that teachers have been using linguistic statements instead of mathematical symbols or the teachers have superficial knowledge about the concepts. Yazıcı and Kültür (2017) also stated that none of the teachers who correctly defined the concept of the equal set did relate the concept of the equal set with the concept of equal set. Thus, it could be considered that teachers don’t take the textbook as a reference.

Within the scope of the research, the in-class activities of TV1, TV2 and TV3 were followed using SOF. It has been determined that the SCK levels of these teachers are partially sufficient. In particular, it has been observed that they are insufficient in making instructional explanations about explaining the reason for mathematical expressions, rules and operations in the teaching process, which is an indicator of SCK. It is thought that the lack of conceptual understanding of teachers due to the fact that they have superficial SCK is effective in this result. As a matter of fact, TV1 gave "a ton of sugar" as an example for a finite set during the in-class teaching process, while she/he gave "human hairs" as an example of an infinite set in the next example. However, TV1 was insufficient to explain the following question of the students instructionally: "Why would a ton of sugar be a finite set, while a hair shaft would be an infinite set?" Afterwards, TV1 tried to shape the teaching process with abstract examples over number sets by changing the examples of “sugar” and “hair”. This is thought to be due to TV1's definition of an infinite set as "the set with an uncountable number of elements". In addition, it was observed that TV1 did not explain the concept of infinite set over the concept of finite set (heuristic definition). Therefore, it can be said that TV1 could not provide satisfactory answers for the concepts of finite and infinite sets in terms of instruction. In this context, it can be said that the type of knowledge and skills that a teacher should have in-class teaching process in order to provide more effective mathematics education and to include more consistent instructional explanations is in-depth SCK.

As a result of the research, it was determined that there are deficiencies in the SCK knowledge of teachers about "The Basic Concepts of Sets". Provasnik, Gonzales, and Miller (2009) found similar results in their study. It was observed that they approach the question items prepared to determine the SCKs with common content knowledge, that is, they only use the definitions related to the concepts. That is, it has been observed that teachers use the definitions of the concepts, associate the concepts with each other and try to reach the concepts within the scope of SCK by reasoning from known concepts. It is known that SCK is not an ordinary mathematics knowledge and it is a type of knowledge that the teacher must have in order to be able to provide effective mathematics education in classroom activities (Ball et al., 2008; Speer et al., 2015). Based on this knowledge, teachers need to compensate for the shortcomings mentioned above. Because, as stated by Ball et al. (2008), it is obvious that a teacher who does not have in-depth knowledge of a subject to teach cannot contribute enough to students' learning.

Suggestions

The teachers should have deep SCK to perform effective mathematics teaching about basic concepts in sets. In this context, it must be ensured that teachers should spend more time on practices that will improve the SCK about the basic concepts mentioned, also, teachers should utilize the scientific studies and activities. Nevertheless, teachers should benefit from both high school mathematics textbooks and scientific books written in mathematics, and informative seminars should be organized to access deep information for SCK.

It is thought that prospective about SCK may not have sufficient and deep knowledge in sets, therefore, teachers can overcome the deficiencies through lessons at the level of bachelor degree. In this context, it can be said that the difficulties identified in "sets" can be overcome with the "teaching" courses that have been implemented in education faculties since 2018. For this, the content of teaching courses can be designed based on practice, and the knowledge that teachers should have in the context of SCK can be gained.

The SCK levels of teachers can be determined by extending the scope of this research and studying other mathematics education subjects. It is recommended to ensure that the results of such research should be delivered to teachers. Thus, the deficiencies in SCK might be compensated.

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