Problem Solving in Realistic, Arithmetic/algebraic and Geometric Context

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Abstract

In the last decades, voluminous research has been dedicated to the modeling process and students' understanding of word problems (verbally set problems with realistic context). These problems were considered as a natural framework for the development of the meaning of mathematical relations and for linking mathematical knowledge and everyday situations. In this study we examine three different contexts of verbally set problems: realistic, arithmetic/algebraic and geometric. The research sample consists of 62 fourth-grade elementary school students (10 – 11 years old). The results show that there is a significant relationship between students' achievement in problem solving in the three different contexts as well as a relationship between the choices of strategies in different contexts. It is shown that students solve problems without the use of visual-schematic representations. Surprisingly, not even in geometric context did student use visual representations. Therefore, a joint activity of students and teachers in constructing visual-schematic representations should be an important aspect not only of solving problems with realistic context, but also of solving geometry problems and problems posed in the mathematical language.

Introduction

In school, mathematics problems are posed in verbal, graphical or symbolical form or in the combination of these representations. Verschaffel, Greer, and De Corte (2000) describe two categories of verbally set problems: word problems and verbally stated numerical problems. Word problems are problem situations that enable students to apply mathematics in everyday life, so we will refer to them as problems in realistic context. On the other hand, verbally stated numerical problems are problems formulated as a mathematical sentence (e.g. Calculate the first addend if the second is 23 and the sum is 50). We will refer to them as problems in arithmetic/algebraic context. Besides word problems and verbally stated numerical problems, there are problems that include concepts from geometry and measurement, to which we will refer as problems in geometric context.

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Potentials and limitations that a context gives to a verbally set problem have become a topical issue. Lately, the focus has been mainly on the problems with realistic context. Several studies indicated that many students have low achievement on tasks with realistic context (Schwarzkopf, 2007; Verschaffel et al., 2000). The reasons for the difficulties in problem solving are seen in (1) understanding and recognizing the problem (Van der Schoot, Bakker Arkema, Horsley, & Van Lieshout, 2009); (2) making the difference between relevant and irrelevant information (Verschaffel et al., 2000); and (3) identification of mathematical procedures needed for problem solving (Verschaffel et al., 2000). On the other hand, there is a great didactic potential in problems with realistic context. The context could be used for making a connection with the solving strategies (Van den Heuvel-Panhuizen, 2005). When the context includes a situation that is easy to imagine, students could present the problem visually and make it easier for solving. They could also find a solution with the use of informal strategies or they can use their real-life experience in problem solving.

Unlike tasks posed in realistic context, tasks posed in arithmetic/algebraic context (mathematical sentences), do not have a realistic situation that helps students to choose the strategy for solving and presenting the problem visually. However, students do not have the above mentioned difficulties with understanding the situation of a problem and making the difference between relevant and irrelevant information when solving problems in arithmetic/algebraic context. The importance of problems in arithmetic/algebraic context is seen in the development of mathematical communication and language, which are needed for algebraic thinking (Sfard, 1995; Van Ameron, 2003).

Finally, there are many results which show that visual representation of a problem facilitates problem solving (Boonen, Van der Schoot, Van Wesel, De Vries, & Jolles, 2013; Boonen, Van Wesel, Jolles, & Van der Schoot, 2014; Hegarty & Kozhevnikov, 1999; Montague & Applegate, 2000; Van Garderen, 2006). Researchers concluded that the use and creation of visual shematic and pictorial representations are factors that determine students' success in problem solving. Therefore, if a problem is posed in geometrical context, the context itself suggests appropriate drawing (visual representation). This could be an argument to assume that students would be more successful at solving problems in geometrical context than in realistic or arithmetic/algebraic context.

The process of word problem solving is frequently described as a modeling process. The phases of the modeling process of essential importance are understanding the situational model (recognizing relations between the relevant elements of the text) and creating the mathematical model with visual schematic representations. In this study we examine the extent of students' use of visual models when solving problems in realistic, arithmetic/algebraic and geometric context and we investigate if students use the same visual models in the modeling process (when they solve problems in realistic context), as in arithmetic/algebraic and geometric context.

Even though realistic, arithmetic/algebraic and geometric contexts were separately investigated, modern literature does not show (or investigate) whether students have more success in solving problems in one of the contexts comparing to the others and whether some of the contexts favor a particular solving strategy or the use of visual representations.

The choice of context of a verbally set problem is an important question for the teaching practice. If students have more success in some of the contexts, that is the context in which students most successfully recognize the mathematical structure of a problem. On the other hand, if there is a strong relationship between students' achievement in all the three contexts, we could not say that a realistic situation (in realistic context) or an explicitly offered visual model (in geometric context) are factors that determine students' success in problem solving. If students do not use visual schematic representations (geometrical pictures) in problem solving in the geometric context, it could not be expected of them to use these in the arithmetic context either or as a phase in the modeling process in the realistic context. Finally, if some of the contexts is more appropriate for the use of some of the strategies (arithmetic or algebraic), the context could be chosen to guide the students to solve the problem arithmetically or algebraically, depending on the aim of teaching.

Realistic, Symbolic and Geometric Context of a Problem Through History

The historical development of algebraic thinking can be used as a guideline in the teaching practice when choosing the representation of an algebraic concept (or procedure) and its level of abstractness. Abstract algebraic concepts, such as solving problems with equations, are gradually developed through cognitive processes that last for years. Hence, we can expect the similarities between the phylogenesis and ontogenesis of algebraic thinking and the difficulties that students have on different levels of learning to be close to the difficulties that generations of mathematicians have experienced (Sfard, 1995).

Algebraic ideas were first expressed through examples (procedurally), figures, words, and finally through symbols. Word problems with realistic context represent a clear relationship between arithmetic, which is procedural, and algebra, which is based on symbolical language (Sfard, 1995; Van Ameron, 2003). This view came from investigation of the historical development of mathematical symbolism. Babylonian, Egyptian and Chinese algebra first dealt with problems from everyday situations (exchange of goods, money, etc.). This turns out to be a wide context for developing ways of reasoning and methods of problem solving. The existence of students' natural interest and capacity for these problems enables their use in the teaching practice and makes them suitable for developing students' informal strategies. One of the significant phases in the historical development of algebraic thinking is ancient Greek geometric algebra (Sfard, 1995). Ancient Greeks used geometric objects as visual representations, because there was no better means for reification of complex calculations. Hence, from the view of historical development, the most natural context for problem solving should be realistic, then geometric and in the end algebraic. A lot of time and experience with abstractions is needed for students' readiness for modern algebra, or in the context of our study, for solving verbally set problems with equations.

Previous research did not compare students' achievement in solving problems with the same mathematical structure in these three contexts (realistic, geometric and arithmetic/algebraic). The study which compared achievement in solving word problems, symbolically posed problems and verbally stated numerical problems showed that teachers' and researchers' beliefs about the difficulty of these kinds of problems differ from the obtained results (Nathan & Koedinger, 2000). The teachers and the researchers assumed that verbally set problems were more difficult for students than symbolically posed problems. On the contrary, symbolical problems were the most difficult to students. We can say that this result is in accordance with the historical development. However, this study did not consider students' achievement in solving problems posed in geometric context.

Word Problems and the Modeling Process

One of the definitions of word problems is that they represent a verbal description of a problem situation in which the answer could be found by performing mathematical operations on numerical data provided in the text of the problem (Verschaffel, Depaepe, & Van Dooren, 2014). This is the base for traditional problem solving – the process that is considered as a mere application of operational rules. It assumes noticing the relations between the structure of the problem situation and the structure of the symbolical mathematical expression (English, 2009). In this process, students are basing their analysis and calculations in problem solving on the superficial association between the quantitative elements in the text and mathematical operations, which is referred to as a key-word strategy (De Corte, Greer, & Verschaffel, 2000), and identified as a frequent obstacle in problem solving (Verschaffel et al., 2000).

It is assumed that approaches, such as the key-word approach, are supported by teachers' practice, in which they tend to emphasize only the mathematical structure of a problem and not the contextual aspect (Depaepe, De Corte, & Verschaffel, 2010). In other words, teachers' focus on outcomes is widespread (Gravemeijer, 1997). By solving word problems this way, students only exercise their computational skills by imitating the procedure of problem solving given in the textbooks and do not use conceptual understanding and correct mathematical reasoning (Boesen, Lithner, & Palm, 2010; Jonsson, Norqvist, Liljekvist, & Lithner, 2014; Lithner, 2008). However, these approaches do not lead to

successful problem solving when numerous elements and relations between them are needed for the construction of the efficient mental model (Thevenot, 2010; Van der Schoot et al., 2009). Several studies which investigated the ways of overcoming these difficulties emphasized the importance of the modeling process in word problem solving. In this process, students should: (1) construct the internal representation of the problem (Depaepe et al., 2010; Moore & Carlson, 2012; Voyer, 2011) and (2) choose a problem solving strategy (De Corte & Verschaffel, 1991; Van den Heuvel-Panhuizen, 2005). Students' choice of strategy is related to the internal representation which is based on students' interpretation of the problem.

The modeling process assumes an interaction between the real world and the mathematical world (Schwarzkopf, 2007). It is a complex process with many phases: (1) constructing the internal model of a problem situation which is related to understanding the elements and relations, (2) transforming the situation model to a mathematical model, (3) working with the mathematical model with the aim of getting the result, (4) interpreting the results of calculations (5) evaluation of the results from the aspect of calculations (6) communicating the results (Blum & Leiss, 2007). This multiphase model is not entirely sequential; it assumes returning to some of the phases of the model many times.

For solving complex word problems, previously mentioned modeling phases are inevitable. More generally, there are two components of skills needed for successful solving of verbally set problems (word problems, verbally stated mathematical problems and geometrical problems): 1) relational processing, i.e. noticing the relation between relevant elements in the text and 2) construction of the visual-schematic representations (Boonen et al., 2013). These situations require deep relational consideration and the construction of visual-schematic representations. Visualization of the problem situation enables understanding of relations in the problem, which leads to successful problem solving.

Visual-schematic Representations
Besides understanding verbally set problem and constructing relations between elements in the text, numerous authors are emphasizing the importance of the construction of visual-schematic representations (Boonen et al., 2013, 2014; Hegarty & Kozhevnikov, 1999; Montague & Applegate, 2000; Van Garderen, 2006; Van Garderen & Montague, 2003). The studies report that the nature of created visual representations determines students' efficiency in problem solving. However, students do not use visual-schematic representations in the modeling process (Verschaffel et al., 2000; Verschaffel, Greer, Van Dooren, & Mukhopadhyay, 2009; Verschaffel, Van Dooren, Greer, & Mukhopadhyay, 2010; Şahin & Eraslan, 2016).

There are two types of visual representations recognized in literature: pictorial and visual-schematic representations (Hegarty & Kozhevnikov, 1999; Presmeg, 1997). Students who create pictorial representations tend to focus on the visual appearance of given elements. However, a number of studies showed that creation of detailed visual representations is in negative relationship with the achievement in problem solving (Ahmad, Tarmizi, & Nawawi, 2010; Hegarty & Kozhevnikov, 1999; Kozhevnikov, Hegarty, & Mayer, 2002; Presmeg, 1997; Van Garderen, 2006; Van Garderen & Montague, 2003). On the other hand, students who create visual-schematic representations integrate relevant elements of the text in a coherent representation (Ahmad et al., 2010; Van Garderen, 2006). This explains why creating the visual-schematic representations is in positive correlation with achievement in problem solving (Hegarty & Kozhevnikov, 1999; Van Garderen, 2006; Van Garderen & Montague, 2003).

There are two approaches that encourage modeling and schematization: 1) to offer students "predefined representations", which the students could create again in their own manner (Diezmann, 2002), or 2) to use students' models as starting points (Gravemeijer, 2002; Van Dijk, Van Oers, & Terwel, 2003). The answer to the debate which approach is better could be found in the research that investigates cognitive flexibility in solving strategies (Heinze, Star, & Verschaffel, 2009). Students' ability to generate relatively new processes in problem solving depends on their participation in the activity of constructing representations and on the characteristics of the final representation which is the result of guided co-construction (Ainsworth, 2006; Keijzer & Terwel, 2003; Terwel, Van Oers, Van Dijk, & Van

den Eeden, 2009). Heinze et al. (2009) highlight that flexible and adaptive use of strategies and representations is part of the cognitive variability which enables fast and precise problem solving. The development of these abilities is not simply based on the growth of experience. We could suppose that their acquisition is based on complex cognitive processes. Flexible/adaptive use of strategies and representations is considered an important aspect of mathematics teaching. There is widespread consensus that students should be able to solve mathematical tasks not only quickly and precisely, but in an adaptive way, too, i.e. they should be able to apply strategies and representations adaptively, considering the problem and context characteristics (Elia, Van den Heuvel-Panhuizen, & Kolovou, 2009; Verschaffel, Greer, & De Corte, 2007).

Aim and Rationale of the Study

In this study, we are investigating how the context of a problem affects students' achievement, solving strategy and construction of visual models in problem solving. We have chosen the mathematical structure of a problem and presented it in three different contexts: realistic, arithmetic/algebraic and geometric, without changing the mathematical structure of the problem.

Contexts are chosen in the way that can suggest different strategies of problem solving and different mental images that result with different visual representations. We expect that students will solve problems in realistic context with arithmetic strategies and with frequent use of pictorial representations, problems in arithmetic/algebraic context with algebraic strategies, and problems in geometric context with the use of visual-schematic representations.

Hence in this paper, we pose the following research questions:

1. Is there a significant difference in students' achievement in solving problems in varied contexts (realistic, arithmetic/algebraic or geometric)?

2. Is there a significant relationship between students' achievement in problem solving in all the three contexts?

3. Does the context of a problem affect students' choice of the solving strategy (arithmetic or algebraic), and visual schematic representation? We will also analyze students' strategy choice when visual representation is used in problem solving and if that strategy led students to the correct result.

Method

The research sample consisted of 62 fourth grade students (10-11 years old) from three different schools in Belgrade (Serbia). Schools were randomly selected from the list of schools available to the researchers' institution for educational research. Schools are located in different parts of the city with similar socio-economic structure.

Before the experimental procedure we had conducted an unstructured interview with teachers about their teaching practice. All teachers worked based on the official mathematics curriculum and used mathematical tasks from the same textbooks. The national curriculum in Serbia prescribes that students have experience with all the contexts investigated in this study (realistic, arithmetic/algebraic and geometric). Algebraic symbolism and equation solving are introduced early, from the second grade (8 - 9 years old) and used for solving verbally set problems. We could conclude that students had less experience in geometric context than in realistic and arithmetic/algebraic ones because geometry makes about one fifth of the total number of classes. On the other hand, most tasks in geometry are related to measurement (e.g. perimeter, area) and transformed to algebraic tasks; hence it is considered that this number of classes is sufficient for context understanding.

The research is based on the testing technique. We constructed three different tests with six verbally set problems for the purpose of the study (Table 1). In each test, the corresponding tasks have the same mathematical structure, but they are represented in different contexts: real context, algebraic/arithmetic context (mathematical sentences) and geometric context. As we argued, students

had experience with all the contexts and had knowledge about the mathematical procedures that tasks in the tests required. The main idea behind designing such tasks is that students can solve them with procedures they know and that tasks can be solved in an easier way with the line segment model. The usual tasks from students' textbook have a less complicated mathematical structure, as did the first three in the test (Table 1), while the 4th, 5th and 6th tasks in the test are more complicated than standard tasks from the textbook. As calculation errors were not important for this study, we used numbers that are easy to calculate with.

Table 1. Verbally set problems in different contexts used for testing

Task No	Realistic context	Arithmetic/algebraic context	Geometric context
1.	Chocolate and juice cost 130 dinars. Chocolate and two juices cost 180 dinars. What is the price of chocolate and what is the price of juice?	Sum of two addends is 130. Sum of the first addend and double second addend is 180. Which are the numbers?	Sum of the length of line segments AB and CD is 130 mm. Sum of line segment AB and double line segment CD is 180 mm. What are the lengths of AB and CD?
2.	Sum of Nadja's and Lena's years is by 20 years greater than Nadja's years and by 15 years greater than Lena's years. How old is Nadja and how old is Lena?	Sum of two numbers is by 20 greater than the first addend and by 15 greater than the second addend. Which are the numbers?	Sum of lengths of two line segments is by 20 mm greater than length of the first segment and by 15 mm greater than length of the second segment. What is the length of each line segment?
3.	Sonja has several stickers, Nadja one sticker more than Sonja and Mila one more than Nadja. All together they have 48 stickers. How many stickers does each girl have?	Sum of three consecutive numbers is 48. Which are the numbers?	Line segment AB is given. Line segment CD is by 1 cm longer than line segment AB, and line segment EF is by 1 cm longer than line segment CD. Total length of all three line segments is 48 cm. Calculate the length of each line segment.
4.	Sister has 4 times greater amount of money than brother. When sister spends 60 dinars, then sister and brother have an equal amount of money. How much money does the sister have and how much money does the brother have?	One number is 4 times greater than the other. When the greater of those two numbers is decreased by 60, it will be equal to the second number. Determine the numbers.	Line segment CD is 4 times longer than line segment AB. If line segment CD is 60 mm shorter, it will be equal to the length of line segment AB. Determine the length of all line segments.
5.	Maria has 10 stickers more than Jovana. Jovana doubled her number of stickers and then gave Maria 20. They concluded that they have an equal number of stickers. How many stickers did each of the girls have at the beginning?	One number is by 10 greater than the other. When the other is doubled and decreased by 20, and the first is increased by 20, they will be equal. Which are the numbers?	Line segment AB is by 10 mm longer than line segment CD. When length of CD is doubled and shortened by 20 mm, and AB is extended by 20 mm they have equal lengths. Determine the length of line segments AB and CD.

Table 1. Continued

Task No	Realistic context	Arithmetic/algebraic context	Geometric context
6.	In a bigger container there is 3 times more milk than in a smaller container. When 9 *l* of milk is added to the bigger and 8 *l* to the smaller container, there will be 2 times more milk in the bigger than in the smaller container. How much milk was in each container at the beginning?	The first number is three times greater than the second. When the first is increased by 9 and the second by 8, the first will be two times bigger than the second number. Which are the numbers from the beginning of the task?	Line segment AB is three times longer than line segment CD. When AB is extended by 9 cm, and CD by 8 cm, AB will be two times longer than CD. What is the length of AB and CD at the beginning of the task?

Students solved tests individually. They were given unlimited time to solve the tasks in the test, but they all finished within one hour. The tests were one week apart. The potential limitation of this method could be found in the possibility that some of the students recognize the mathematical structure of the task from the first test when they work on the second or the third test. This could result with better achievement on the second/third test than on the first one. In order to minimize this limitation, students from different schools worked on the tests in different order – the first school worked on realistic context, then on arithmetic/algebraic and in the end on geometric, the second school worked on arithmetic/algebraic context, then on geometric and in the end on realistic and the third school on geometric, realistic and in the end on arithmetic/algebraic context.

Data Analysis

To answer the question of whether there is a significant difference or relationship in students' achievement when solving problems in realistic, arithmetic/algebraic and geometric contexts, we are recording if a student solved the problem correctly in every context. As in Verschaffel's and De Corte's study (1990) about strategies of word problems solving, we did not take into consideration calculation errors. We considered the result correct if a student had recognized the mathematical structure of a problem and chose a suitable mathematical model for its solving.

We analyzed every task on the tests (Tasks No 1 to 6, Table 1) in the three contexts. For every task, we made a categorical variable *Context of correct solution i* ($i = 1, \ldots, 6$) in which we recorded if the task is correctly solved in realistic, arithmetic/algebraic or geometric context. Variables *Context of correct solution i* have three values r – correct solution in realistic context, a - correct solution in arithmetic/algebraic context, and g – correct solution in geometric context. For the analysis of the difference in number of students' correct solutions in every context, we used Chi square test of goodness of fit on variables *Context of correct solution i.*

Important indicators of students' achievement are the number of students who solved the task correctly in realistic context (n_r), number of students who solved the task correctly in arithmetic/algebraic context (n_a) and number of students who solved the task correctly in geometric context (n_g). We also calculated *average achievement* – the percentage of the correct answers in each task, by dividing the sum of correct answers in all contexts ($n_r + n_a + n_g$) with 186, which is the total number of values on the task (62 participants on 3 tests).

In order to analyze the relationship between students' achievement in all contexts we created three dichotomous variables: *Achievement in R, Achievement in A and Achievement in G*, which referred to students' achievement in realistic, arithmetic/algebraic and geometric contexts. Each variable has two possible values: student solved the problem correctly or student did not solve the problem correctly. To analyze if there is a relationship between the three variables, we used Chi-square test of independence with Bonferroni correction for multiple comparisons ($p < 0.017$) and phi coefficient as a measure of association.

We are also investigating if the context of a problem affects students' choice of the solving strategy (arithmetic or algebraic) and the visual schematic representation. Hence, we are analyzing if a student used arithmetic or algebraic strategy in solving, if he/she used a visual representation and what type of representation he/she used. This way we can answer if there is a relationship between the creation of models/visual representations on the one hand and the achievement and choice of strategy on the other.

For the analysis of students' strategies, we made three dichotomous variables *Strategy in R*, *Strategy in A* and *Strategy in G* with possible values *arithmetic* or *algebraic.* First, we compared these variables with Fisher-Freeman-Halton test with Bonferroni correction for multiple comparisons (p < 0.017), not taking into consideration correctness of the result, only the strategy that students used in their attempt to solve the problem. Fisher-Freeman-Halton test is used as alternative for Chi square test, since variables did not meet all assumptions needed for Chi square test. In order to investigate the relationship between students' used strategy and correctness of the result, we compared variable *Strategy in R* with *Achievement in R, Strategy in A* with *Achievement in A* and *Strategy in G* with *Achievement in G* using Fisher-Freeman-Halton test. Since there are no multiple comparisons in this analysis, we used the significance level p = 0.05.

For the analysis of the use of visual representations in problem solving we recorded the number and type (visual-schematic or pictorial) of visual representations in each context. We have analyzed the strategy that a student used in problem solving when he/she used the visual representation, and if the strategy led him/her to the correct result.

To answer the question if there are differences in number of used visual representations in different contexts, we made variable *Context of visuals* with three posible values: *r* – use of visual representation in realistic context, *a* – use of visual representation in arithmetic/algebraic context, and *g* – use of visual representation in geometric context and performed Chi square test of goodness of fit.

In the end, we made three dichotomous variables *Visuals in R*, *Visuals in A* and *Visuals in G* that denote if a student used visual representation on at least one task in realistic, arithmetic/algebraic and geometric context. Each of the variables has two possible categorical values – student used the visual model or did not use the visual model in at least one task. We performed Chi square test of independence with Bonferroni correction (p < 0.017) to compare the difference in the number of visual representations that students used in each of the contexts, and phi coefficient as a measure of association.

Results

In the beginning, we are presenting the results of the analysis of the categorical variables *Context of correct solution i,* by which we are investigating differences between the number of correct solutions in every context. Chi square test of goodness of fit showed that there was no significant difference in the number of correct answers in three contexts in each of the six tasks, i.e. each of the p values is greater than 0.05 (Table 2). Besides the results of Chi square test, Table 2 represents the number of students who solved the task correctly in each context and average achievement - average percentage of the correct answers.

Table 2. The results of Chi-square test of goodness of fit ($n = n_r + n_a + n_g$) for analyzing difference in number of correct answers in different contexts which are n_r, n_a and n_g and average achievement

Task No.	Variable	$\chi^2(2, n)$	p	(n_r, n_a, n_g)	Average achievement
1.	*Context of correct solution 1*	0.105	0.949	(59, 56, 56)	91.94%
2.	*Context of correct solution 2*	0.292	0.864	(51, 46, 47)	77.42%
3.	*Context of correct solution 3*	0.275	0.872	(52, 43, 48)	76.88%
4.	*Context of correct solution 4*	0.110	0.947	(23, 25, 25)	39.25%
5.	*Context of correct solution 5*	2.167	0.338	(8, 13, 15)	19.35%
6.	*Context of correct solution 6*	0.182	0.913	(11, 10, 12)	17.74%

Chi square test of independence and phi coefficient performed on variables *Achievement in R, Achievement in A and Achievement in G* showed that there was a moderate to strong relationship between achievements in different contexts in almost every task (p < 0.017, φ > 0.3 for moderate, φ > 0.5 for strong, Table 3).

Table 3. The results of the Chi square test of independence for investigating the relationship in achievement in different contexts

Task No	*Achievement in R Achievement in A*			*Achievement in R Achievement in G*			*Achievement in A Achievement in G*		
	$\chi^2(1, 62)$	*p*	φ	$\chi^2(1, 62)$	*p*	φ	$\chi^2(1, 62)$	*p*	φ
1.	-	0.267	-	-	0.267	-	-	0.472	-
2.	29.602	0.000	0.691	32.454	0.000	0.723	30.352	0.000	0.700
3.	12.098	0.004	0.442	15.336	0.001	0.497	18.394	0.000	0.544
4.	27.170	0.000	0.662	21.870	0.000	0.594	33.211	0.000	0.732
5.	16.182	0.000	0.511	12.928	0.000	0.457	32.743	0.000	0.727
6.	42.656	0.000	0.829	33.427	0.000	0.734	38.123	0.000	0.784

The lack of the relationship in the first task (p > 0.017) is the result of high success in its solving. More than 90% of students solved the task correctly in all three contexts (see Average achievement in Table 2). The incompatibility is in the rest of the sample, which is less than 10%.

Fisher-Freeman-Halton test performed on variables *Strategy in R, Strategy in A* and *Strategy in G* showed that there is a significant relationship (p < 0.017) between students' choice of strategy (arithmetic or algebraic) in different contexts, except in the comparison of *Strategy in R* and *Strategy in A* in the first task, on which *p* value is greater than 0.017 (Table 4). Further, Phi coefficient in Table 4, showed that there is a strong relationship in each task (φ > 0.5) except in the first one in which the association is moderate (φ > 0.3).

Table 4. The results of the Fisher-Freeman-Halton test for investigating the relationship between strategy use in different contexts and values of phi coefficient (n-number of students who tried to solve the task in both contexts)

Task No.	*Strategy in R Strategy in A*			*Strategy in R Strategy in G*			*Strategy in A Strategy in G*		
	n	*p*	φ	n	*p*	φ	n	*p*	φ
1.	62	0.030	-	60	0.006	0.485	60	0.004	0.443
2.	54	0.000	0.573	53	0.001	0.527	51	0.000	0.732
3.	58	0.000	0.781	54	0.000	0.761	55	0.000	0.734
4.	49	0.000	0.745	44	0.000	0.795	43	0.000	0.741
5.	34	0.003	0.525	35	0.000	0.767	36	0.000	0.708
6.	25	0.000	0.761	22	0.002	0.726	22	0.000	0.913

To answer the question about the relationship between the solving strategy and success in problem solving (students' achievement), we also used Fisher-Freeman-Halton test and phi coefficient. We did not get significant results in the first four tasks (p > 0.05), therefore we are reporting results on 5th and 6th tasks that are statistically significant (Table 5).

Table 5. The results of Fisher-Freeman-Halton test for investigating the relationship between strategy choice and correctness of the solution (n- number of students who solved the task) on 5th and 6th tasks

Task No.	Realistic			Arithmetic/algebraic			Geometric		
	n	*p*	φ	n	*p*	φ	n	*p*	φ
5.	42	0.007	0.438	43	0.056	0.299	41	0.040	0.327
6.	28	0.019	0.486	28	0.005	0.559	28	0.063	0.351

Results in Table 5 show a weak to moderate relationship between the solving strategy and correctness of the solution in the 5th and 6th tasks (p < 0.05, 0.3 < φ < 0.5). In algebraic/arithmetic context in 5th and in geometric context in 6th task probabilities are close to significance level (p=0.056 and p=0.063 respectively), hence we consider this result as significant. For further analysis of this result, we had to analyze the percentages of students who solved the problem correctly or incorrectly by using some of the strategies in these two tasks (Table 6). The results in Table 6 show that the group of students who chose arithmetic strategy had bigger percentage of incorrect results than the group of students who used algebraic strategy. In the 5th task, 95.8% of students who attempted to solve the problem using arithmetic strategy solved the problem incorrectly in realistic context, 79.3% in arithmetic/algebraic and 76% in geometric context. In the 6th task, the percentage of incorrect answers is similar.

Table 6. Percentage of strategy choice and correct/incorrect results in the 5th and the 6th task

Task No	Solving strategy	Realistic		Arithmetic/algebraic		Geometric	
		Correct	Incorrect	Correct	Incorrect	Correct	Incorrect
5.	Arithmetic	4.2%	95.8%	20.7%	79.3%	24.0%	76.0%
	Algebraic	38.9%	61.1%	50%	50%	56.3%	43.7%
6.	Arithmetic	18.8%	81.3%	12.5%	87.5%	26.7%	73.3%
	Algebraic	66.7%	18.3%	66.7%	33.3%	61.5%	38.5%

The results in Table 7 show the number of students who used visual representations in problem solving. Twenty students used the model in realistic context, 17 in arithmetic/algebraic and 25 in geometric context. Chi-square test of goodness of fit performed on variable *Context of visuals* (which contains the context of visual representation) showed that the differences in these numbers are not statistically significant $\chi^2(2, 62) = 1.38$, $p = 0.454$.

Table 7. Number of students who used models in different contexts and the correctness of the tasks

Strategy		Realistic	Arithmetic/algebraic	Geometric	Total
Arithmetic	Correct	15	12	16	43
	Incorrect	3	2	3	8
Algebraic	Correct	2	3	5	10
	Incorrect	-	-	1	1
Total number of models		20	17	25	62

Data in the Table 7 also show that students mostly used arithmetic strategies when they constructed visual models (51 from 62 models). Also, when students used visual models in problem solving, the solutions were correct in a large number of cases (43 from 51 correct arithmetic and 10 from 11 correct algebraic word problems with the model). We have also recorded the type of representation that students used. A small number of students (10 in realistic and 1 in algebraic context) created detailed illustrations and figures, and all of them were in realistic context.

With the aim to investigate if there is a relationship between the constructions of visual models in different contexts, we singled out the answers of students who used models in at least one task in the context. We created variables *Visuals in R*, *Visuals in A* and *Visuals in G*. Thirteen students used models in some of the contexts, and they made 78 visual representations in solving the tasks in the test. Chi-square test of independence showed that there is a strong relationship between the models used in different contexts (p < 0.017, φ ≥ 0.5, Table 8).

Table 8. The results of the Chi square test for investigating the relationship in the use of visual representations in different contexts

Visuals in R Visuals in A			*Visuals in R Visuals in G*			*Visuals in A Visuals in G*		
$\chi^2(1, 78)$	*p*	*φ*	$\chi^2(1, 78)$	*p*	*φ*	$\chi^2(1, 78)$	*p*	*φ*
44.67	0.000	0.757	17.785	0.000	0.478	25.255	0.000	0.569

Discussion

Previous research and historical development recognized realistic and geometric contexts, as contexts that facilitate solving of verbally set problems (Nathan & Koedinger, 2000; Sfard, 1995). By contrast, our results show that students are equally successful in all the three investigated contexts (realistic, arithmetic/algebraic and geometric). Reformulation of the problems (contextual change) did not result with the change in students' achievement, i.e. there was no significant difference in the number of correct answers in three different contexts in each of the six tasks (Table 2). This is also supported with the fact that there is moderate to strong relationship between the students' achievement in problem solving in all the three contexts (Table 3). In other words, if a student solved the task correctly in one context, he/she solved it correctly in the other contexts, too. The question is: did students who solved the problems correctly understand the structure of the problem and recognize it in different contexts, or did they use superficial methods in problem solving (such as key-word approach), as in Verschaffel's et al. (2000) research, a high percentage of students (between 77% and 92%, Table 2) solved the first three tasks correctly. As we pointed out, problems in students' textbooks have similar complexity with the first three tasks. They have a structure that facilitates understanding, making the difference between relevant and irrelevant information and identifying mathematical procedures needed for problem solving (Van der Schoot et al., 2009; Verschaffel et al., 2000). Based on these arguments, we can say that high achievement in the first three tasks is partially the result of superficial methods that students used in problem solving and this confirms the previous finding that problems with this kind of structure and complexity are easy to solve (Boesen et al., 2010; Jonsson et al., 2014; Lithner, 2008).

Surprisingly, our results mostly showed a strong relationship between the choices of strategy in different contexts (Table 4). In other words, students used the same solving strategies in realistic, arithmetic/algebraic and geometric context. They did not show the ability to flexibly apply different strategies and representations considering the contextual characteristics (Elia et al., 2009; Heinze et al., 2009; Verschaffel et al., 2007). We will illustrate this result with the work of one student on the first task (Table 1) in all contexts (Figure 1).

Figure 1. The solution of the first task (Table 1) created by one student (realistic, arithmetic/algebraic and geometric context)

In the theoretical part of this paper we argued that students who do not solve word problems correctly often use the key-word approach (De Corte et al., 2000; Verschaffel et al., 2000). We will illustrate the approach with example given in Figure 2.
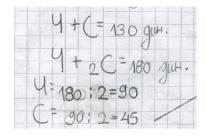

Figure 2. The incorrect solution of the first task (Table 1) in realistic context

The student, whose work is presented in Figure 2, correctly wrote the relations: "chocolate and juice are 130 dinars" and "chocolate and two juices are 180 dinars". However, the student focused on

the element "two juices", and divided the amount of money for chocolate and two juices by two. The students' misunderstanding of situational model is also manifested through the fact that he/she connected the obtained result and price of juice and divided it by 2 again in order to get the price of juice. Therefore, the student based the problem solving on the superficial association of some of the quantitative elements.

In most of the problems, students' success in solving the problems is not in a relationship with the choice of the solving strategy (Table 5). The exceptions are the 5th and 6th tasks (Table 1) in which the students' success in solving them was low (19% and 18% respectively). Besides the 4th task, the 5th and 6th tasks have relations that are not instantly visible, and they have more elements needed for the construction of efficient mental model (Thevenot, 2010; Van der Schoot et al., 2009). As shown in Table 6, arithmetic strategies in solving these problems led to incorrect results. If a student chose arithmetic strategy in solving the 5th or 6th task, his/her result was wrong in more than 74% of cases (Table 6). In the analysis of the students' solutions we singled out two key misleads: applying the superficial strategy based on the key-word and representing the relations with algebraic symbolism without recognizing the adequate meaning of the relations (De Corte & Verschaffel, 1991; Depaepe et al., 2010; Moore & Carlson, 2012; Van den Heuvel-Panhuizen, 2005; Voyer, 2011). In the last approach, even if the students did write the relations using algebraic symbolism correctly, they did not succeed in using them to solve the problem. This is illustrated in Figure 3 (left), where the student wrote the relations between numbers correctly using algebraic symbolism, but instead of the solution, he/she wrote: "Sorry, I don't know to solve the problem". By analyzing the phases of modeling process (Blum & Leiss, 2007), we can conclude that students have developed an adequate situational model. However, the mathematical model without visual representations that students developed was not adequate and did not enable students to continue the mathematical calculation. In the presented situation, students could not choose an adequate solving strategy because they did not construct an appropriate mental representation of the problem such as a visual schematic representation (De Corte & Verschaffel, 1991; Depaepe et al., 2010; Moore & Carlson, 2012; Voyer, 2011). In the theoretical background it is argued that students tend to skip some of the phases of the modeling process, such as construction of an internal model of problem situation and transformation of a situational model to a mathematical model (Blum & Leiss, 2007; Schwarzkopf, 2007). The importance of the construction of an internal model is presented in Figure 3 (right), which shows that the student could solve problem algebraically, but he/she did not construct the right internal model of the problem situation.
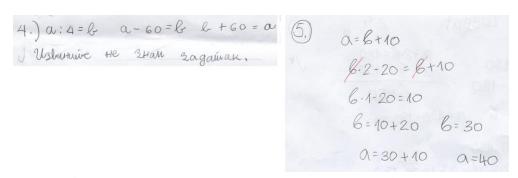

Figure 3. The use of algebraic strategies in attempts to solve 4th and 5th task in realistic context (Table 1)

Surprisingly, some of the students were successful in algebraic strategies that are not in the syllabus for the first four years of mathematics education. As we described in the Method section, according to the official mathematics syllabus in Serbia, by the end of the fourth grade, students are familiar with the operations in the natural number system and simple forms of equations. However, some of the students wrote and solved systems of equations that are not included in the syllabus for the first four grades. The students used the same procedures in geometric context. We will illustrate the solution of the first task in arithmetic/algebraic and geometric contexts, when students managed to explicate the relations in the problem and use the above mentioned procedures successfully (Figure 4).
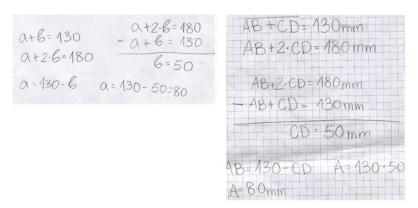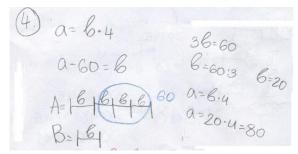
Figure 4. Algebraic strategies in solving the 4th task (Table 1)

Previous studies emphasize the importance of creating visual models in the modeling process (Hegarty & Kozhevnikov, 1999; Presmeg, 1997), especially visual schematic representations (Boonen et al., 2013, 2014; Depaepe et al., 2010; Mayer, 1985; Moore & Carlson, 2012; Van Garderen, 2006; Voyer, 2011). Our results confirm these findings. Students who created visual models were successful in problem solving (Table 7). There were 53 correct solutions out of 62 solutions in which students used a visual representation. We also confirmed the view that creation of models facilitates the recognition of relations and choice of solving strategies (Hegarty & Kozhevnikov, 1999; Van Garderen, 2006; Van Garderen & Montague, 2003). Even if students mostly used arithmetic strategies after they created models, they were also successful when they used algebraic strategies of solving (Table 7). We will illustrate this with one solution of the 4th task (Figure 5).
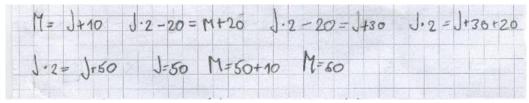

Figure 5. The use of a visual model and algebraic strategy in problem solving (4th task, Table 1, arithmetic/algebraic context)

Our results also show that there are no differences in the numbers of students who used visual models in different contexts and that there is a strong relationship between their use in different contexts (Table 8). This is surprising since geometric context should be the context which facilitates visual representations. It is interesting that students who used models in problem solving created abstract visual representations and used a line segment to represent the relations in all contexts. This led to successful problem solving, which is in accordance with previous studies (Ahmad et al., 2010; Blum & Leiss, 2007; Hegarty & Kozhevnikov, 1999; Presmeg, 1997; Van Garderen, 2006; Verschaffel et al., 2000). A small number of students (10 in realistic and 1 in algebraic context) created detailed illustrations and figures in realistic context, i.e. students' answers included a small number of pictorial representations which did not lead to the correct solution of the problem. This is in accordance with Boonen et al. (2014).

In the theoretical part of this study we referred to the question about the starting point in problem solving, which can be a "self-constructed model" (Gravemeijer, 2002; Van Dijk et al., 2003) or a "previously built representation" (Diezmann, 2002). Based on the results of this research, in which students did not make self-constructed models, we share the view with the researchers who maintain that construction of a model is a process that requires mutual activity of students and the teacher (Ainsworth, 2006; Keijzer & Terwel, 2003).

As we argued, students in our research were successful in applying algebraic and arithmetic strategies in problem solving on simple tasks. However, only a small number of students solved complex tasks by engaging formal methods of solving. The example of a successful formal method on a complex task (5th task, Table 1) is presented in Figure 6.

Figure 6. Correct algebraic strategy in solving the 5th task (Table 1)

The results confirmed the stance that formal strategies of solving are available to small number of students in the first years of schooling (Boesen et al., 2010; Carpenter, Moser, & Bebout, 1988; Jonsson et al., 2014; Lithner, 2008; Verschaffel et al., 2000). Students had difficulties in making the connection between conventional symbolism and informal approaches developed when they face a problem they need to solve. Students in our research did not try to make algebraic symbolism simpler by creating different visual models that can help them in problem solving.

Conclusion

In this paper, we have analyzed students' achievement, solving strategies and visual schematic representations when students are solving verbally set problems in different contexts with the same mathematical structure. Besides word problems that have realistic context, we used verbally set problems in arithmetic/algebraic and geometric contexts where mathematical relations are expressed with mathematical language. There is voluminous research that highlights the importance of the understating of the situational model in the process of word problem solving. In contexts that do not require understanding of situational models (arithmetic/algebraic or geometric) we expected two different outcomes: students' achievement will be higher because they do not have to understand elements and relations in a realistic situation; or students' achievement will be lower because there is no realistic situation to help them use informal strategies of solving and everyday experience. However, we did not get statistically significant differences in students' achievement in these three contexts. We could partially attribute this result to students' use of superficial strategies in problem solving, such as key-word approach. In our research, students could correctly solve the tasks in which relations were instantly visible, but superficial strategies did not lead them to the correct solutions in more complex tasks.

On the other hand, the absence of difference in achievement in different contexts could be found in the absence of difference in the used strategies and visual models. A realistic situation in a verbally set problem did not result with more frequent use of informal arithmetical strategies in problem solving. To represent problems with more complex mathematical structure, students used algebraic symbolism. However, the achievement in these tasks was low, hence we may conclude that most of the students were not able to use formal algebraic strategies, or to efficiently adapt some of the arithmetic strategies.

Similarly, there was no difference in the use of visual models in different contexts. Based on this result, we consider that students at this level of education are not able to use visual representations as a phase in the modeling process when they are solving word problems (problems with realistic context). If students do not use visual schematic representations (geometric pictures) when solving problems in the geometric context, we cannot expect them to use visual schematic representations in the arithmetic/algebraic context either, or as a phase in the modeling process. Therefore, more emphasis should be placed on the process of construction of visual representations in geometric context. We consider that it could result with an increase of students' achievement in problem solving in other contexts.

After four years of mathematical education and experience with problem solving with formal methods, students ignore visualization as an accessible means (and phase) of problem solving. Based on this result, we can conclude that flexibility and adaptability of strategy and model choice in the problem solving process could be developed only by their systematic teaching. Different contexts that could foster various mental representations are not going to induce their use without explicit teaching. This is in accordance with the view of researchers who consider that problem solving with the use of models should be part of mathematical syllabus in the first years of education (Aztekin & Taşpınar Şener, 2015; Elia et al., 2009; Şahin & Eraslan, 2016; Van den Heuvel-Panhuizen, 2003; Van Dijk et al., 2003) and that more detailed research is needed regarding methodical approaches to modeling on the elementary level (Aztekin & Taşpınar Şener, 2015). The use of representations that are presented to students, or co-constructed representations (Ainsworth, 2006; Diezmann, 2002; Keijzer & Terwel, 2003; Van Garderen, 2006), should become an explicit goal of mathematics teaching even if students know how to solve problems symbolically. Students need to have their ability for modeling already developed when they are facing more complex problems. The construction of models itself should be a task for the teaching practice and not only a means of problem solving. Teachers should change their attitudes towards mathematical problems by shifting emphasis from merely explaining the mathematical structure of a problem with formal mathematical language to understanding the situational model and representing it by means of visual schematic representations.

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