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A Study on Investigating 8th Grade Students` Reasoning Skills on Measurement: The Case of Cylinder

Öğrencilerin Ölçme Alanında Akıl Yürütme Becerilerine İlişkin Bir Çalışma: Silindir Örneği

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Abstract

The purpose of this study was to investigate 8th grade students' reasoning skills on measurement. Specifically, this study aimed to find students' reasoning skills on the case of surface area and volume of a cylinder. The data were collected from 271 8th grade elementary school students enrolled in three public and two private schools in Ankara at the end of 2008 spring semester. In order to investigate the elementary students' reasoning skills regarding the area and volume of a cylinder, the Cylinder Exploration Task (CET, Merseth, 2003) was administered as the measuring instrument. The results revealed that 8th grade students had difficulty in solving the problems demanding the conceptual understanding on reasoning and measuring the surface area and volume of cylinder. Discussion of findings and recommendations for future research studies were provided.

Keywords: Reasoning, Measurement, Surface Area, Volume

Öz

Bu çalışmanın amacı 8. sınıf öğrencilerinin ölçme alanındaki akıl yürütme becerilerini, daha spesifik olarak belirtmek gerekirse, öğrencilerin silindirin yüzey alan ve hacimine yönelik akıl yürütme becerilerini incelemektir. Veriler 2008 bahar dönemi sonunda Ankara'daki üç devlet okulu ve iki özel okulda öğrenim görmekte olan toplam 271 öğrenciden elde edilmiştir. Öğrencilerin silindirin alan ve hacmine yönelik akıl yürütme becerilerini tespit etmek amacıyla Silindir İnceleme Envanteri (CET, Merseth, 2003) ölçme aracı olarak kullanılmıştır. Bulgular, 8. sınıf öğrencilerinin kavramsal anlama gerektiren geometri problemlerini çözerken zorluk yaşadıklarını göstermiştir. Bulgular yorumlanmış ve ileriki çalışmalar için öneriler sunulmuştur.

Anahtar Sözcükler: Akıl yürütme, ölçme, yüzey alan, hacim

Introduction

Measurement is an essential concept in mathematics teaching and learning for various reasons. First, it contributes to the development of other mathematical concepts. The study of measurement "…offers an opportunity for learning and applying other mathematics, including number operations, geometric ideas, statistical concepts and notions of function" (National Council of Teachers of Mathematics, 2000, p. 44). Further, understanding and application of measurement concepts emerge in very early ages and in real world contexts (Stephan and Clements, 2003). Finally, it builds connections between mathematics and other disciplines such

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as physical and engineering sciences (Clements, 2003). Due to its importance in mathematics education, measurement concepts are given a great deal of emphasis in the curriculum of a number of countries, including the ones with top performance in international assessments (Chen, B. Reys, and R. Reys, 2009). Similarly, in the Turkish elementary school mathematics program, measurement is considered as a main content domain such as numbers and geometry (MNE, 2006). In the first five years of elementary school, to help develop measurement sense, students are provided with plenty of learning opportunities where they apply concepts of measurement in real life settings. Mainly, grades 1-5 students are expected to build a conceptual understanding of measurement concepts and improve their estimation skills in measurement. When they reach 6th grade, Turkish students begin thinking about how to measure angles, areas, volumes, and liquids. Spatial measurement involves a large portion of measurement in 6-8 grade mathematics. In particular, students are expected to understand area and volume measurement of geometric shapes such as prisms, cubes, and cylinder (MNE, 2006). In teaching measurement, the recently revised elementary school mathematics curriculum aims at enhancing student understanding and providing students with the environments in which they can learn meaningfully (Bulut and Koc, 2006). Despite ambitious goals of the curriculum for teaching measurement and teachers' relentless efforts, in the international assessments such as TIMSS (1999; 2007), Turkish students' performance in geometry and measurement is quite low (EARGED, 2003; TIMSS, 2007). National exams such as the Secondary Schools Entrance Exams (SBS) also do not portray a better picture of student performance (MNE, 2009).

Area and volume measurement

"Area and volume are vital geometrical concepts which underlie many aspects of mathematics" (French, 2004, p. 76). They are also practically and mathematically important concepts both in mathematics and science (Raghavan, Sartoris, and Glaser, 1998). Area refers to a quantitative measure of the amount of two-dimensional surface contained within a boundary (Baturo and Nason, 1996). Hence surface area is the total area of surfaces of a solid shape. Volume is the amount of three-dimensional space in a solid shape that can be quantified in some manner (French, 2004). While it is important to know the formulas and do the computations for measuring the region and capacity, it is also equally important to understand these two concepts (French, 2004). "The research indicated that developing measurement sense is more complex than learning the skills or procedures for determining a measure" (Stephan and Clements, 2003, p. 14). Just recalling the formulas and performing the calculations is not enough for a thorough understanding. However, classroom instruction is mainly focused on memorizing the formulas to solve problems requiring low level of cognitive demand rather than fostering conceptual understanding of surface area and volume measurement (Raghavan, Sartoris, and Glaser, 1998). When students are asked mainly to complete the computations and reach the final answers, they lack engaging in interpreting the results they get (Silver, Shapiro, and Deutsch, 1993). In other words, they do not necessarily know that measuring the area and volume requires counting the number of squares and cubes in the object we are measuring (Battista, 2003).

Area and volume create conceptual difficulties for students (French, 2004). For example, understanding area and volume measurement requires reasoning multiplicatively which is not easy for students (Simon and Blume, 1994). Especially, according to French, surface area and volume are confused by students. Additionally, French notes that it is hard to visualize three-dimensional objects and interpret their two-dimensional representations. In fact, Piaget and his colleagues had already found out that children cannot fully comprehend the relation between two and three-dimensions of the same object until adolescence (Piaget and Inhelder, 1956; Piaget, Inhelder, and Szeminska, 1960). In order to help students not to confuse surface area and volume, students should be given instructional tasks that "aim to counter false intuitions" (French, 2004, p. 76). In this study, the participants were given a geometrical task which requires critical thinking and reasoning to help them check their false intuitions. A firm understanding of student thinking about area and volume measurement is necessary to design the most effective instructional environments and monitor student thinking (Battista, 2003).

It was argued that area and volume measurement requires students to understand "how meaningfully to enumerate arrays of squares and cubes" (Battista, 2004, p. 191). Yet, this claim makes more sense when the focus of the investigation is prisms or solids with rectangular surfaces (Battista, 2003). Working with cylinders or solids with curved surfaces should require a different approach to calculate the surface area and volume. For instance, it is not possible to cover a circular region with small squares without any gaps or overlapping of units. For that matter, the literature does not suggest any common way of comparing the region or internal capacity of a cylinder to another smaller unit, usually a square or cubes. We should note that, it is not within the goals of this study to find smaller units for tiling or filling a solid shape; yet, in this study, 271 8th grade students were given a task involving four interrelated questions to explore their understanding of surface area and volume measurement of a cylinder. We analyzed the students' solutions to discover any pattern in their approach to the task. This research is believed to be a significant step toward improving the knowledge base on student understanding of area and volume measurement.

Conceptual knowledge for reasoning

Students are not successful when they do not have the necessary reasoning skills as reasoning is a base for understanding (Sierpinska, 1994). In order to help students develop reasoning skills, it is vital that they have conceptual understanding rather than just having procedural knowledge (Battista, 2007). Conceptual knowledge has been characterized as knowledge that is rich in relationships (Hiebert and Lefevre, 1986). It can be thought of as a connected web of knowledge, a network in which the linking relationships are as the discrete pieces of information. Hiebert and Lefevre (1986) added that connection process can occur between two pieces of information that have already been stored in memory or between an existing piece of knowledge and one that is newly learned. Conceptual knowledge also grows in creation of relationships between existing knowledge and new information just entering the system. On the other hand, procedural knowledge consists of rules, algorithms, or procedures used to solve mathematical tasks. These are step-by-step procedures that are carried out in a predetermined linear sequence (van de Walle, 2007). Procedural knowledge of mathematics have essential role both in learning and in doing mathematics (Rittle-Johnson and Alibali, 1999). In addition, those algorithmic procedures help to do routine mathematical tasks efficiently (van de Walle, 2007). However, Hiebert (1990) mentioned that even the most skillful uses of procedures will not help developing conceptual knowledge. The connection of procedures and conceptual ideas is much more important than the usefulness of procedure itself (Hiebert and Carpenter, 1992).

Hiebert and Lefevre (1986) stated that during the earlier years of childhood or when children enter the school their conceptual knowledge and procedural knowledge are closely related. When students move through elementary and middle school, conceptual knowledge and procedural knowledge develop separately and the focus of instruction remain procedural (Hiebert and Lefevre, 1986). Van de Walle (2007) added that procedural rules should not be taught in the absence of concepts; however, most teachers only focus on the procedural characteristics of mathematics. This should explain why many students from elementary school through university perform successfully on procedural tasks; but, lack the conceptual understandings (Hiebert and Lefevre, 1986).

Studies on student thinking suggest that the explanations and beliefs that students have for a concept might reveal students' misconceptions (Williams, 1991). Such explanations can only be identified through giving students opportunities to explain their thinking (Confrey, 1990) and reason mathematically. With tasks, which only require procedural knowledge, students might get right answers, but they may have serious misconceptions (Ball, Lubienski, and Mewborn, 2002). Through the tasks emphasizing conceptual understanding rather than memorization and letting students see the relationships among concepts, it might be possible to help them understand the logic behind the mathematical concepts. As Battista (2001) puts it, instead of poor instructional strategies such as memorization, students should receive chances to develop meaningful concepts and opportunities to analyze problems and reason. The studies in the literature examining students' reasoning and understanding related to measurement of geometric shapes (Battista and Clements, 1996; Chang, 1992; Ng, 1998) through investigating elementary students' problem solving strategies and the difficulties they experienced while solving area and volume problems also suggested that in order to facilitate meaningful understanding, it is important to understand students' thinking process (Battista, 2003). In other studies which investigated the conservation of surface area and volume of two cylinders (Tirosh and Stavy, 1999; Stavy, Babai, Tsamir, Tirosh, Lin, and Mcrobbie, 2006), the researchers analyzed students' responses to and difficulties on the problems requiring reasoning and tried to explain students' reasoning through explaining the role of intuitive rules. The researchers suggested that it is necessary to be aware of inappropriate responses of students in order to have knowledge on students' reasoning, and arrange the instructional environment accordingly.

From this perspective, in this study, Turkish 8th grade students' reasoning skills on measurement is investigated through a task that requires students to reason and explain their thinking. In light with the literature, the aim of this research is to investigate 8th grade students' measurement reasoning concerning surface area and volume of a cylinder. In other words, we investigated whether 8th grade students could reason the meaning of measurement concepts behind the symbolic manipulation of formulas. Our purpose was to answer the following research question:

What is the level of 8th grade students' reasoning skills on measurement regarding the surface area and volume of a cylinder?

Method

Participants

Data were collected at the end of the spring semester of 2007-2008 academic year from 8th grade elementary school students enrolled in 3 public and 2 private schools in Ankara, Turkey. More specifically, 145 (53.5 %) of the participants were students in public schools and 126 (46.5 %) were students in private schools. Participants' mean age was around 15. In terms of gender, 124 (45.8 %) female students and 147 (54.2 %) male students participated in the study. All 8th grade students from five schools were asked to volunteer to fill out the questionnaire, and in total, 271 8th grade students participated in the study.

Instrument

In order to investigate 8th grade students' reasoning skills on measurement regarding the relationship between surface area and volume of cylinder, Cylinder Exploration Task (CET, Merseth, 2003) was administered. In CET, students were given two papers (21 cm x 30 cm) and they were directed to roll one paper along the long way (long cylinder) and the second paper along the short way (short cylinder) to make a cylinder. Then, students were directed to explore the relationship between the volume and surface area of these two cylinders. In other words, students were supposed to calculate the surface area and volume of a cylinder and then make generalizations concerning the relationships. More specifically, in the first question, students were asked to calculate the lateral surface area of a short and long cylinder. In order to solve this question students were either expected to remember the formula for calculating surface area of a cylinder or relate the surface area with the area of a rectangle. In the second question, students were asked to compare the volume of a short and long cylinder without using the formula. In the third question, students were expected to use the formula to calculate the volume of a short and long cylinder. Lastly, in the fourth question, students were expected to make a generalization regarding the relationship between lateral surface area and volume of a cylinder and other solid figures. The CET was translated and adapted for Turkish students. For content validity concerns, the original and translated versions were given to two professors from the education department. The questionnaires were revised until 95% agreement reached between the professors. The Cylinder Exploration Questionnaire (Table 1) is given below.

Table 1.
Cylinder Exploration Questionnaire

Cylinder Exploration: Use your 21cm and 30cm paper to visualize the following exercises:

Roll one paper the long way to make a cylinder and tape it. Roll a second paper the short way to make a cylinder and tape it.

1. Calculate the lateral surface areas.

2. Which of the given statements is true? Write a paragraph to explain why.

a. Long cylinder has great volume

b. Short cylinder has great volume

c. The volumes are the same

3. Calculate the volumes of each cylinder.

4. Write a statement which relates the lateral surface area and the volume of a cylinder. Could you make generalizations concerning the relationship between lateral surface area and volume of other solid figures?

The Cylinder Exploration Questionnaire was given to the students during their regular class hour. Forty minutes were given students to complete the given questionnaire without any guidance.

Data analysis

To analyze the data, content analysis technique was employed. Two coders evaluated the papers in order to analyze students' verbal expressions by using a rubric, and ninety nine percent of agreement was found among the raters.

Results

In this study, we aimed to examine 8th grade students' reasoning skills on measurement related to the surface area and volume of a cylinder.

In the first question of the CET, students were asked to calculate the lateral surface areas of short and long cylinders. Results revealed that 183 (67.5%) students calculated the areas correctly. However, 88 (32.5%) students could not calculate the surface areas correctly. Those students mentioned that they could not find the right answers since they did not remember the formula for calculating the surface area.

In the second question, 8th grade students were asked to decide whether the long or short cylinder had greater volume or they had the same volume. Students were also asked to write a short paragraph to explain their answers. Results revealed that 29 (10.7%) students indicated that long cylinder had greater volume. These students explained that since the long cylinder has greater height, it should have greater volume. For instance, Participant 24 responded to this question as follows:

Participant 24: Long cylinder has greater volume because its height is longer than the short cylinder.

In addition, 164 (60.5%) students mentioned that the volumes of two cylinders are same and these students' explanations could be grouped into two categorizations. One of them is related to raw materials. Students stated that the position or shape of the figure is not important in determining the volume. Students mentioned that since the material used (paper) did not change in both situations the volume of cylinders did not change either. The following two excerpts illustrate such cases:

Participant 123: The volume of two cylinders is equal to each other. Since both cylinders were made from same paper. In other words, the paper being used did not change in either case.

Participant 89: The volume of two cylinders was same because we used the same paper and we did not cut any pieces from the paper.

The other categorization of answers was related to the dimensions of the paper. Students mentioned that since the dimension of papers being used did not change, the volumes would not change either. In other words, as illustrated in the following excerpts, they indicated that the volumes were the same because the width and the length of the paper did not change in both situations:

Participant 12: The volume of short and long cylinder was the same. Since the length of the long cylinder is equal to the width of the short cylinder and the width of the long cylinder is equal to the length of the short cylinder.

Participant 76: They are both the same because the dimensions of the papers did not change in both situations. We can prove this by measuring the length and width of both papers.

In addition, results showed that 20 (7.4%) students did not answer this question. On the other hand, only 58 (21.4%) of the students gave the correct answer and responded that the short cylinder had greater volume. These students' answers were also categorized under three subheadings: correct answer with correct explanation, correct answer but insufficient explanation, and correct answer with no explanation. Results showed that 32 (11.8%) students stated that as the short cylinder had greater radius, it has greater surface area and greater volume. Examples for the correct explanations are provided below:

Participant 4: Short cylinder had greater radius because its base radius is bigger.

Participant 28: Base area of the short cylinder is bigger, so its volume is bigger. In calculating the volume, the radius is so important since we use the square of radius while calculating the volume of a cylinder.

An example for the correct answers with insufficient explanation is given below:

Participant 37: Short cylinder has greater volume because it is wider and covers more space.

On the other hand, 11 (4.06%) students mentioned that they, by intuition, believed that the small cylinder has greater volume. These students did not give any further explanations on their answers. Additionally, 15 (5.54%) students noted that the small cylinder has greater volume but they did not give any explanation on their answers.

In the third question, the participants were asked to calculate the volume of the short and long cylinders. Forty one (15.1%) students correctly wrote the formula and calculated the volume of cylinders. On the other hand, 158 (58.3%) students found incorrect answers. Among those students, some of them used incorrect formula and some of them made errors in calculations. For instance, Participant 81 responded as follows:

Participant 81: The formula for the volume of a cylinder is $2.\prod.r.h + 2.\ \prod.\ r^2$.

Additionally, 72 (26.6%) of them did not write any answers to the given question. In other words, they left the question blank.

In the last question, after calculating the volume, the participants were asked to explain the relationship between surface area and volume of a cylinder. Students were asked to write a statement that relates surface area and volume and also make generalizations to other solid figures. Thirty (11.1%) students could write the relationship where they stated that two cylinders with the same surface area could have different volumes since the volume depends on radius and height. An example of such responses is as follows:

Participant 52: The lateral surface areas of two cylinders are equal, but the volumes are different. So, we can say that as the radius increases the volume increases also.

On the other hand, 115 (42.4%) students either gave incorrect answers like both cylinders have equal volumes or they wrote unrelated explanations. 126 (46.5%) students did not answer the question. Results also revealed that 180 (66.4%) students did not comment on anything about the generalization of the relationship between area and volume. In addition, 91 (33.6%) students gave incorrect or insufficient explanations like 'we could not make any generalizations' or 'surface area and volume were directly proportional'. More examples to insufficient and incorrect explanations are provided below:

Participant 197: Two solids with the same lateral surface area may not have same volumes.

Participant 55: We can generalize it to prisms because lateral surface areas of prisms increase as their volumes increase.

Participant 168: We can generalize it because the volume and surface area are equal for all geometric shapes.

In other words, none of the students could make a correct explanation on the generalization of the relationship between area and volume.

Discussion

In this research, our aim was to investigate the 8th grade students' reasoning skills on measurement related to the surface area and volume of a cylinder. Specifically, in the first question, students were asked to calculate the lateral surface areas of short and long cylinders. Results revealed that most of the students correctly solved the question. This question was an easy and direct question that requires procedural knowledge. Results revealed that students were able to calculate the lateral surface areas of short and long cylinders via using formulas. That is, students could easily perform the operations by using formulas (Hiebert and Lefevre, 1986).

In the second question, students were asked to decide whether the long or short cylinder had greater volume or they had the same volume. In addition, in this question students were supposed to reason their answer. Contrary to the first one, students had difficulty in this question since it required reflective thinking behind symbolic manipulation of formulas. Most of the students mentioned that long cylinder has greater volume or both cylinders have same volume. In other words, students have difficulty in reasoning the relationship between the dimensions and volume of a cylinder. Parallel to those ideas results revealed that students have difficulty in solving this question that requires conceptual understanding (Hiebert and Lefevre, 1986).

Similar to the second question, in the third question, students were asked to calculate the volumes of each cylinder. State differently, as in the first one, this question needs direct manipulation of numbers into the volume formula. However, contrary to the first question students had difficulty in solving this question. In other words, this is one of the interesting findings of this study since students had difficulty in solving this question where students could easily use the formula. These results might be attributed to the possibility that students could not remember the formula. Thus, we concluded that if students had conceptual understanding about the volume of solids, they could easily generalize the formula for the cylinder. Thus, even students could not recall the formula for the volume of a cylinder; they could deduce it by using the relationship for the solids. That is, they could use the formula for volume of prisms to find the formula for the volume of a cylinder. In other words, as in prisms, they could multiply the area of the base by the height to compute the volume of the cylinder. However, 8th grade students' lack of understanding of the relationship among the geometrical concepts inhibited their thinking in deduction of formulas by using those relationships.

In the last question, students were asked to make generalizations concerning the relationship between lateral surface area and volume of other geometrical structures. Results revealed that the percentages of students who gave correct answers to that question were very low. Thus, in this question, similar to the second one, results revealed that 8th grade students' conceptual understanding was very low. In other words, students' lack of understanding in the relationship between surface area and volume of a cylinder inhibit their reasoning on generalization of this relationship to the other geometrical figures. To sum up, our data revealed that in terms of reasoning skills on measurement, 8th graders had difficulty in solving the problems demanding conceptual understanding

Conclusion

In conclusion, this study suggests that 8th grade students had problems on reasoning about the relationship between surface area and volume of cylinder. In other words, upper elementary school students had difficulty solving questions that required conceptual understanding. They struggled with deciding whether the long or short cylinder had greater volume or they had the same volume as well as explaining and reasoning on the relationship between surface area and volume of cylinder. It is recommended in current reform documents in mathematics education that teachers should help students develop both conceptual and procedural understanding (Lubinski, Fox, and Thomason, 1998). The connection of procedures and conceptual ideas is much more important than the usefulness of formula itself (Hiebert and Carpenter, 1992). Thus, it is recommended that teachers should create environments for students in which they can communicate with each other, discuss on mathematical concepts, and have opportunity to reason on mathematical ideas. It is believed that with the help of effective instructional strategies, teachers could easily connect the mathematical procedures to the conceptual ideas, and meaningful learning will take place in mathematics classrooms.

Finally, we also need acknowledge the limitations of our research. Although the Cylinder Exploration Task is a strong tool to discover students' level of reasoning skills on measurement, it covers only the concepts related to the surface area and volume of a cylinder. We believe that other assessment tools could give more extensive and stronger results; yet, due to time limitations, we could only use one task to assess reasoning skills on measurement. Additionally, we are aware of the fact that critical interviewing of the students would provide us with in-depth understanding of their level of reasoning. Critical interviews would be helpful to explore the details of their thinking and to ask follow-up questions for further clarification. Hence, further research could be planned to interview the participants one-on-one to explore their understanding of measurement constructs and procedures.

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