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Investigation of Mathematics Teachers' Usage Frequency of Learner Generated Examples in Classroom and Its Reasons

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Abstract

In the classroom, examples are typically presented by the teacher and students try to improve their understanding of the relevant concept by examining the given examples. In addition to use in this way, example generation is defined as a problem-solving activity in which individuals can develop different strategies. In recent studies, it has been considered a pedagogical method as well as a research tool. The purpose of this study is to investigate high school mathematics teachers' usage frequency of learner generated examples (LGEs) and to reveal the reasons behind them. The sample consisted of 196 high school teachers, with different year of mathematics teaching experiences (ranging between 1 and 36 years). Data were collected using an instrument consisting of strategies that were brought together as a list by Watson and Mason (2005) and used by researcher in order to reveal teachers' LGE usage frequency. The relationships between the frequency of use of LGE, the year of mathematics teaching experience of the teachers, and the type of high school they work, were investigated by using regression analysis. Additionally, 16 semi-structured interviews were conducted with voluntary teachers in order to obtain in-depth knowledge about their LGE usage frequency. The highest mean score for usage frequency belongs to teachers with 21 years and above mathematics teaching experience, whereas the lowest score belongs to teachers with 6-10 years mathematics teaching experience. Science high school teachers have the highest mean score for LGE usage frequency, whereas vocational high school teachers have the lowest score. However, only 'year of mathematics teaching experience' has a significant, unique contribution to the prediction of LGE usage frequency. Analysis of the qualitative data revealed that constraints related to students, educational policies, parents, topics, classroom environments, and teachers' belief and attitudes have considerable effect on teachers' example usage frequency. Also, their knowledge of content and student, along with knowledge of content and teaching, affect LGE usage.

Keywords

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Mathematics Teachers
Learner Generated Examples
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Introduction

Every step taken by teachers in teaching environment occur in order to design a better environment for student learning. However, to design a "learning environment" compatible with the changing conditions of the world, a teacher should consider several components. Examples are a significant part of this process and play a crucial role in conceptual understanding (e.g. Dahlberg & Housman, 1997). They are also an efficient communication tool between teacher and student (Goldenberg & Mason, 2008; Peled & Zaslavsky, as cited in Bills et al., 2006). For all these reasons, examples play an outstanding role in the learning process.

Examples play an important role in student's comprehension processes not only in mathematics but also in other disciplines. For scientific disciplines working with concepts such as mathematics, it can be said examples are more powerful tools than definitions and theorems. According to Watson and Mason (2005) "mathematics is learned by becoming familiar with examples that manifest and illustrate mathematical ideas and by constructing generalizations from examples" (p. 2).

There are many types of examples in mathematical contexts. The most well-known and widely used ones are examples of a mathematical object, counterexamples, and non-examples (Goldenberg & Mason, 2008). However, with different perspectives, every example of a particular type can be included in other types of example sets. For example, '0.9' is an example of a decimal number, a non-example for numbers whose squares are larger than itself, and a counterexample for the argument 'squaring makes larger' (Goldenberg & Mason, 2008). Examples are used for different purposes during the instruction. While teachers prefer easy examples to provide a better understanding of the theoretical knowledge described in the introduction of a new topic, more complex examples are used to expand the context. Rowland (2008) explains the first type as the inductive dimension of example use. These are examples of a rule or a procedure. The second type, which consists of similar examples considered 'exercises' is used in the later phases of the lesson to provide a basis for mathematically complex ones. Without taking into account this distinction, examples chosen by teachers can be designed as a result of careful planning before the lesson or as a result of the interaction with the students during the lesson. Mason and Spence (1999) call the second one 'knowing to act in the moment'. It is obvious that second type is crucial for students learning. Therefore, the use of examples in the classroom may facilitate students' learning and may prevent learning if not used correctly (Zodik & Zaslavsky, 2008). Many mathematics teacher-training programs do not attach sufficient importance to this issue and do not provide preservice teachers with a systematic preparation on how to choose, design, or use instructional examples (Zodik & Zaslavsky, 2008). Thus, preservice teachers' use of examples effectively in classrooms is often left to their own personal experience (Kennedy, 2002; Leinhardt, 1990). Although experienced teachers as years have developed processes that they are not aware, there seems to be a difficulty for relatively novice teachers to choose the right examples to contribute students' conceptual learning. Rowland (2008) used a middle school teacher's mathematical example to explain this case. The teacher who is in her twenties tells the students that the "x-axis comes first" as a reminder, before demonstrating a point on the coordinate plane, and then exemplifying it with the point (1,1). It is clear that the example is ineffective for the rule "x-axis comes first" that the teacher wants to emphasize. This is one of the common mistakes a novice teacher could make. Rowland, Thwaites, and Huckstep (2003) describe this mistake as 'to make the role of the variables ambiguous' and mention two others flaws as "the numbers used to describe a procedure are more appropriate and more recognizable for another procedure and make random choices when more careful selection is needed".

Another dimension of the use of examples in mathematics lessons is examples generated by learners. This process is different from the ones in which students work on examples given by their teachers. The purpose of examples given by the teacher changes when they are used or produced by the students.

Example Generation

In the classroom, the teacher typically provides examples for students and the students try to improve their understanding about the relevant concept by examining the given examples. In addition to use in this way, example generation can be defined as a problem-solving task in which individuals develop different strategies (Zaslavsky & Peled, 1996). It can be also considered as an open-ended task in the sense of Sullivan, Clarke, and Clarke (2013). According to them "a task has open goals when it has more than one (preferably many more than one) possible responses, and we call such tasks open-ended" (p.57). To solve an open-ended task, students should keep in mind the meaning of the concept and think about possible ways to reach the solution, rather than following a rule. These processes might trigger a better understanding of the concept and develop an efficient way of thinking. We use the term task as Sullivan, Clarke, and Clarke (2013, p.13) defined in their study, because example generation activities trigger student work, presented them as problems that construct a context and a starting point for their learning like tasks. Many researchers (Dahlberg & Housman, 1997; Watson & Mason, 2005; Zaslavsky, 1995) have also emphasized that example generation is also a pedagogical method or a research tool.

Even though example generation has not been benefited from its present potential pedagogical power, it encourages active participation in mathematics (Watson & Mason, 2002; Zaslavsky & Zodik, 2014). Because this strategy emerged from the perspective that mathematics is a constructive activity, and that students can learn mathematics in the richest way when they create new objects, relationships, questions, problems, and meanings (Watson & Mason, 2005, p. ix). To investigate the contribution of example generation to mathematical understanding, Iannone, Inglis, Mejia-Ramos, Simpson, and Weber (2011) compared the success of two groups of students on proof construction: those who generated their own examples and those who studied on worked examples. No significant difference was found, and they speculated that the tasks examined might not have been appropriate for demonstrating the power of example generation. They pointed out that in order to use example generation more effectively, better-designed tasks should be used. However, the expression "better-designed" implies a gap in mathematics education literature on this subject, which leaves an open door for future research on the field.

Example generation tasks also have additional advantages in comparison to solely working on provided examples. First of all, it requires different cognitive skills (higher order thinking skills) compared to working on examples from a workbook or a teacher (Moore, 1994). Some researchers (e.g. Alcock & Simpson, 2005; Dahlberg & Housman, 1997; Meehan, 2007; Watson & Mason, 2005) also believed that one way of overcoming the difficulties experienced in the proof process is to encourage students to produce their own examples. In Dahlberg and Housman's (1997) study, the example generation task they created for the participants allowed the students to integrate many other examples (including different function types) into their concept images, in order to solve the question and explain their solution. Furthermore, the group working with example generation tasks was more successful than the other groups (the groups, which used memorizing, analyzing or reformulating strategies) in presenting the correctness of hypotheses and presenting clarifications. Thus, the researchers have stated that it might be more useful to ask students to generate their own examples and verify them during the teaching of new subjects. Regarding to this result, Zaslavsky and Zodik (2014) asked participants (mathematics teachers) in their study to generate examples continuously from certain concepts (*Make up another or more like or unlike this*) and asked remaining participants to verify whether the given example provide the characteristics of the desired concept. At the end of the study, it was observed that the example spaces of the participants – which is the network of example types belonging to the individual regarding to the concepts - develops, the examples that have been placed incorrectly in this network have emerged and generating examples and verifying them is a sign of understanding of the learners. Besides, it is stated that this kind of teaching requires immediate decision-making skills, but it can also be a catalyst for future teaching. O'Neil (2018) who investigated the factors affecting the use of structured examples and learner generated examples in their study, concluded that teachers used structured examples more than learner generated examples. As the reasons for this result, it was found that teacher felt less control over the learner generated examples, it would be very difficult to give immediate responds to the questions that would raise when using learner generated examples, and the students were not used to such examples. In some other researches (e.g. Sağlam & Dost, 2016), students found difficult to perform such mathematical tasks, however it has been observed that when these tasks are given to the students, they concentrate on the properties of the concept rather than performing routine operations (Hazzan & Zazkis, 1997; Sağlam & Dost, 2016).

Watson and Mason (2005) have set out a list of strategies[1] based on classroom activities related to example generation which can be used by teachers in classrooms and can provide a way to design learner generated example (LGE) tasks:

- **Make up an example:** This task type allows teachers to understand students' thinking and understanding (e.g., "Give me an example of a number between 3 and 4" (p. 151)).
- **Make up an example with some constraints:** In this task, students should take into consideration some principles to find an example. By adding some constraints to looked up example, it is more likely that the students will focus on the principle, which helps them to find the wanted instance, rather than random example selection. (e.g., "Make up a unitary fraction that has 6 unitary fractions bigger than it" (p.151)).
- **Add constraints sequentially:** This task helps the learner to reach some generality (e.g., "Create a quadrilateral. Make one with no edges parallel to edge of paper. Make it have one reflex angle. Can you make it have 2 reflex angles?" (p. 152)).
- **Make up another or more like or unlike this:** This task helps students to realize the other dimensions of examples they worked on.
- **Make counterexamples and nonexamples:** (e.g., "Find a prime number that cannot be expressed as 4k±1 for any positive integer *k*" (p. 154)).
- **Confound expectations:** This kind of example compels learners to move away from their strong concept images (e.g., "Give a number for which the square is not larger than itself" (p. 153)).
- **Characterize all objects that satisfy specified constraints***:* These examples belong to a set of examples that are the results of some constraints (e.g., "Find triples of numbers that can be the 3 sides of a triangle. What can be said about them?" (p. 154)).
- **Reverse:** By reversing the strategy, a closed task turns into an open-ended one (e.g., "The answer to a division problem is 5 with a remainder 2: What could the question be?" (p. 154))

[1] Watson and Mason (2005) named items in this list as "strategies". But these strategies are different from Antonini's work. Antonini (2006) used name of example generating strategies (trial and error, transformation, analysis) as methods for solving example generating task. But the strategies in this list are ways for constructing learner generated example tasks.

- **Explore distinctions:** In this task, students can explore the limitations of definitions (e.g. In isosceles triangles, a perpendicular from the apex divides the base into two equal parts. In what other triangles does this case happen?)
- **Bury the bone:** In this task, the final stage of the solution is the start point. This triggers a better understanding of rules or the invention of new ways (e.g., "The solution to a linear equation is p=6; what could the equation be? Make it as complicated as you can" (p.155)).
- **Use features of methods or objects as starting points:** Different from upper strategy, features of procedures or methods themselves are used for reconstructing a process (e.g., "Which shapes, when cut along a straight line, produce pieces whose shapes are all similar to the original?" (p. 155))
- **Find:** Asking different variation of 'find' questions (e.g., "Find examples of ...", "Find the example that ...", "Find all examples that ..." (p. 156)).
- **Use wild-card generation:** In this strategy, starting examples are not familiar or obvious ones for further work (e.g., "Drop a ruler onto a rotating coordinate grid to get a straight line" (p. 156)). In this example, the coefficients of the equation formed for the straight line are not likely to occur from the integers as the students are accustomed.

These strategies were observed in real classroom settings and have various pedagogical benefits on learning a concept such as realizing the differences, exploring the boundaries of a mathematical definition, understanding learners' comprehension etc. Teachers used these strategies to encourage conjecturing, to construct mathematical objects, for mathematical discovery, to explore mathematical concepts, to learn about mathematical structures, to explore dimensions of variation, to provide wider perspective, and so on (Watson & Mason, 2005). Watson and Mason (2005) indicate that these strategies do not guarantee mathematical learning. They need to be used consciously and deliberately to aid learning, like other strategies. However, the aforementioned benefits of these strategies on learning mathematics render them important. Therefore, some research questions are raised regarding how often teachers utilize these strategies or which factors influence their usage frequency. In a study (Zodik and Zaslavsky, 2008) on the factors affecting the example choice of teachers in general sense, it was found that the example space of the teachers has influence on example choice, and especially the counter examples were found to be spontaneous examples types that was given as a response on students' spontaneous questions. Studies related to learner generated examples are mostly about on the teaching of a subject/concept (Aydın, 2014; Dinkelman & Cavey, 2015); effectiveness as a teaching, research and evaluation method (Bentley & Stylianides, 2017; Dahlberg & Housman, 1997; Iannone et al., 2011; Zazkis & Leikin, 2007, 2008; Zaslavsky & Zodik, 2014); contribution to the development of some skills (example production, generalization, example space) (Park & Kim, 2017; Watson & Shipman, 2008; Zazkis & Marmur, 2018). However, there are few studies on how often and in what ways teachers use these examples. To reveal how often learner generated examples are used in classrooms and what factors affects their usage frequency may be important for students' learning and to update teacher education programs with recent point of views because there are lots of researches which shows these examples have positive influence on students' comprehension as well as teachers' pedagogical development.

Study

The purpose of the study is to investigate high school mathematics teachers' usage frequency of LGE tasks, and to reveal the reasons behind them. The relationship between the usage frequency of LGE tasks, the year of mathematics teaching experience of the teacher, and the type of high school they work, was also investigated. The research questions of the study are:

1. To what extent do high school mathematics teachers use learner generated examples in the classroom? Does this use create a pattern within themselves?
2. What motivates them to use learner generated examples in the classroom?

3. What is the relationship between the usage frequencies of LGE tasks, year of mathematics teaching experience, and the type of high school they work in?
4. Which domain of mathematical knowledge for teaching (MKT) is taken into consideration when providing justification for usage frequency of LGE tasks?
5. Which learner generated examples do teachers use mostly, which ones rarely? Why?.

Theoretical Framework

Mathematical Knowledge for Teaching and Teachers' Task Use

Being the product of teaching, learning can occur when students work on carefully and purposefully selected tasks. The knowledge of the teachers about their students and the knowledge they possess are among the main factors influencing the choice of appropriate tasks due to effective teaching (Ball, Thames, & Phelps, 2008). In particular, it is reported that three types of teachers' knowledge are effective in generating examples: knowledge of mathematics, knowledge about student learning and pedagogical content knowledge (Harel, 2008; Zodik & Zaslavsky, 2008). In this sense, MKT is a factor that affects teachers' choices and use of appropriate tasks in classes.

According to recent studies, MKT has many dimensions (Fennema & Franke, 1992; Ball et al., 2008). One of them is content knowledge. Content knowledge is an important part of teaching as independent from the field worked in. However, recent studies propose that content knowledge is not sufficient alone to guarantee learner comprehension. Shulman (1987) put forward seven different categories of knowledge an educator needs for effective teaching: general pedagogical knowledge, knowledge of learners' characteristics, knowledge of educational context, knowledge of educational purposes and values, content knowledge, curriculum knowledge, and pedagogical content knowledge (PCK). The most influential one is PCK because it is the blending of content and pedagogy (Shulman, 1987) and it contains materials, applications, examples, and other content specific representations to make the content more comprehensible for students (Petrou & Goulding, 2011). In the context of mathematics, Fennema and Franke (1992) proposed another model for MKT based on Shulman's work. They argued that content knowledge, knowledge of pedagogy, and knowledge of students' cognition and teachers' beliefs are the components of MKT. In another study on MKT, Ball et al. (2008) also presented a model based on Shulman's work. They defined MTK as "mathematical knowledge needed to carry out the work of teaching mathematics" and separated it into two main domains: Subject Matter Knowledge (SMK) and PCK. SMK contains the three sub-domains common content knowledge (CCK = general mathematical knowledge and skills), horizon content knowledge (HCK = how a concept is related to previously learned concepts or concepts which will be learned in the subsequent years or concepts outside the curriculum (Jakobsen, Thames, Ribeiro, & Delaney, 2012), and specialized content knowledge (SCK = specialized knowledge needed for effective teaching); PCK's subdomains include knowledge of content and students (KCS = knowledge of how students learn particular content), knowledge of content and teaching (KCT = knowledge on how to teach a particular concept), and knowledge of content and curriculum (KCC= knowledge of designing programs in order to sequence the topics, select appropriate materials, etc.). In this study, the MKT model presented by Ball et al. (2008) forms a part of the theoretical framework for examining the teachers' usage frequency of learner generated examples in the classroom. The use of the MTK as the theoretical framework is based on the assumption that teachers' mathematical knowledge and pedagogical knowledge will influence their choice of examples, which is a teaching activity.

There are other factors that influence task use in classroom. See Figure 1 for the model proposed by Sullivan, Clarke, and Clarke (2013).

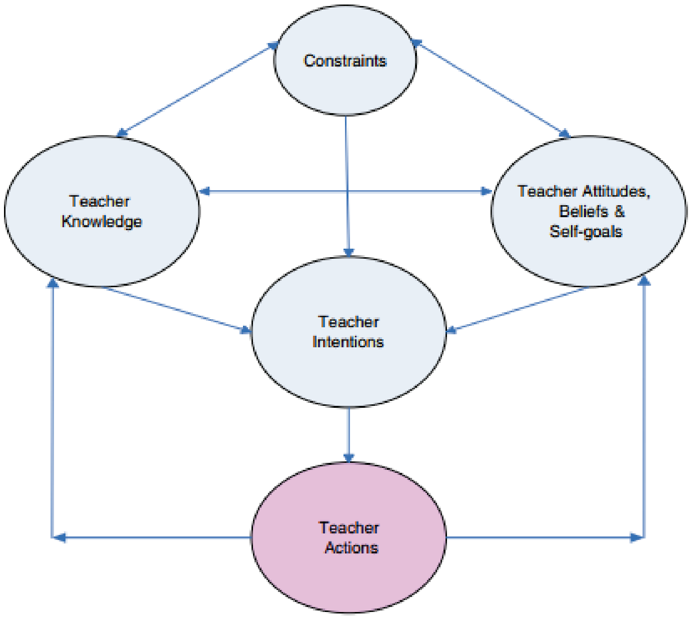

Figure 1. A Model of Factors Influencing Task Use (Sullivan, Clarke, & Clarke, 2013, p. 3)

Sullivan Clarke, and Clarke (2013) adapted this model from Clark and Peterson's (1986) model. In the model, the relationship of four main variables (teacher knowledge, teacher intentions, constraints, teacher's attitudes, beliefs and self-goals) with each other and their effect on teacher behaviors are shown. According to Clark and Peterson (1986), teacher can develop a number of beliefs and attitudes as a result of interaction with the students in the classroom and teachers' behavior may be restricted by a number of environmental factors or external influences (such as curricula, school management). Sullivan, Clarke, and Clarke (2013) stated that the first three variables (constraints, teachers' beliefs and teachers' knowledge) in the model they adapted were mutually influencing each other and these three variables together influenced the teacher's intention. This model and MKT are used as a theoretical framework for analyzing the qualitative data of the study. In this model, MTK was examined under the heading of teacher's knowledge. The use of this model as a theoretical framework is based on the assumption that learner generated examples are also tasks and that the choice of instructional tasks of teachers will affect the use of learner generated examples.

Method

In the study, convergent parallel mixed method design (Creswell, 2014, p. 15) was used. The purpose of using mixed method design is to determine which type of learner generated example was used by teachers mostly (quantitative part) and to reveal the reasons behind that (qualitative part).

Sample/Participants

The research sample included 196 mathematics teachers in the city of Ankara, who had various years of mathematics teaching experience (ranging between 1 and 36 years) and worked in different types of public high schools, including science high school (SHS), vocational high school (VHS), Anatolian high school (AHS), and religious vocational high school (RVHS). Experience has been accepted in this study as years working as a mathematics teacher. Teachers participating in the study were selected by using appropriate sampling. Table 1 shows the distribution of teachers by school type and year of mathematics teaching experience. The year of mathematics teaching experience is categorized as 1-5 years, 6-10 years, etc., in order to view the distribution clearly. This categorization was not used in the analysis. The distribution of the participants according to school type parallels the overall distribution of schools throughout the city. Since the most common school type is AHS, the number of teachers who participated in the study who work in AHS is higher than the others.

Table 1. Distribution of Participants by School Types And Year of Mathematics Teaching Experience

	Year of Mathematics Teaching Experience					
School Type	**1-5**	**6-10**	**11-15**	**16-20**	**21+**	**Total**
Science high school	1	0	0	1	7	9
Anatolian high school	5	6	22	30	32	95
Vocational high school	5	7	10	9	12	43
Religious vocational high school	5	8	10	19	7	49
Total	16	21	42	59	58	196

Data Collection Tools and Analysis of the Data

Data for the quantitative part of the study were collected using an instrument consisting of strategies that were brought together as a list by Watson and Mason (2005) and used by researcher in order to reveal teachers' LGE usage frequency. The instrument (see appendix) includes LGE tasks. All items include a short explanation and examples of the task from different topics. In this respect, the instrument was developed in order to reveal the level of teachers' use of different types of LGE tasks that can be used in teaching. The diversity of examples in each item is derived from the different topics of the high school mathematics curriculum in each grade level. It was assumed that using examples from different topics would allow teachers to understand the tasks better, as there are few mathematics teachers who teach all grade levels. Data would be invalid and unreliable if the example of LGE tasks were of topics that s/he has never taught. In order to ensure the language validity of the items in the instrument, the items were controlled by a language expert and an English speaking field expert. In addition, expert opinions were taken to determine whether the given examples reflect the LGE tasks.

Regression analysis was used to examine the relationship among LGE task usage frequency, the year of mathematics teaching experience of the teachers, and the type of high school they worked in. The internal consistency of instrument was determined using Cronbach's alpha. The reliability of the items was 0.89. According to De Vellis (2003), reliability coefficient of 0.7 and above is ideal.

In addition, principal component analysis was used to determine if there was any pattern in the data set. Tabachnick and Fidell (2007, p. 635) recommend the use of principal component analysis to make an empirical summary of the data set. This analysis was conducted to determine whether there was a pattern among the frequency of using the LGE.

The qualitative part of the study consisted of data that came from 16 semi-structured interviews with the participants. Pseudonyms were used for participants in the analysis. The purpose of the interviews was to have a better understanding of the reasons behind their use of LGE tasks. Voluntary participants were interviewed for 15-20 minutes. The teachers who attended the research filled out the instrument first and then were asked whether they wanted to participate in an interview about instrument. Interviews were carried out by researcher with voluntary participants. Interview questions were asked two mathematics teachers before the interviews in order to examine the questions in terms of clarity and being goal directed and then finalized. Teachers working in different school types were interviewed. So purposeful sampling (with maximum diversity) was used. The following interview questions were asked:

1. When you consider the instrument, which LGE tasks do you use the most/least in the classrooms you teach? Why?
2. Can you compare examples generated by students with those provided by teachers in terms of
   a) conceptual understanding/retention of learning, and
   b) advantages and disadvantages?
3. Do you experience any problems in your lessons when you use examples? If so, what kind of problems do you experience?

4. During your undergraduate education, were you trained about how to use examples in the classroom?
   a) If you were trained, what was the content? Could you tell us how this training contributed to your teaching methods?
   b) If not, what kind of training would you like to have? What kind of contributions would such training provide you, in the sense of the difficulties you experience when using examples in the classroom?

The first question of the interview was asked in order to reveal the teachers' usage frequency of LGE and the reasons behind that; the second question was asked to reveal how teachers evaluate the LGE based on their PCK, and indirectly to reveal the reasons why they use/do not use LGE, and the last two questions were to determine teachers' experiences about example use. In addition to the interviews, the researcher's note about LGE during the implementation of instrument and teachers' view on items on the instrument were analyzed as a part of qualitative data.

In accordance with the convergent parallel mixed method design qualitative and quantitative data were analyzed separately and the results were interpreted together. The data obtained from the instrument were examined using frequencies, percentages, and statistical analysis. The data obtained from interviews was transcribed and analyzed using descriptive and content analysis.

For the internal reliability of qualitative data, another field expert re-coded 25 % of the data to determine the inter-coder reliability. The data that are re-coded for inter-encoder reliability should not be less than 10 % of the total data set (Neuendorf, 2002). Cohen's Kappa coefficient was used to measure inter-coder reliability. The Cohen's Kappa coefficient was calculated and found to be 0.77. The agreement that is 0.70 or over is a sign of reliability (Miles & Huberman, 1994).

Results

The findings section of the study includes analysis of the qualitative and quantitative data. The quantitative data analysis results are presented as follows.

Analysis of Quantitative Data

Table 2 shows the teachers' usage frequency of LGE tasks in terms of year of mathematics teaching experience. Teachers with more than 20 year of mathematics teaching experience had the highest mean score for frequency of use, whereas teachers with 6-10 years of mathematics teaching experience scored the lowest. Teachers with 1-5 years of mathematics teaching experience got scores similar to those with 6-10 years of mathematics teaching experience. When teachers' usage frequency of LGE tasks is compared based on school type, teachers from SHS have the highest score (3.25), AHS teachers have the second highest mean score (3.22), and VHS teachers have the lowest score (2.99).

Table 2. Usage Frequency Mean Scores in Terms of Year of Mathematics Teaching Experience and School Type

Year of mathematics teaching experience	N	SHS		AHS		VHS		RVHS		Total		Min	Max
		$\overline{X}$	StdS	$\overline{X}$	StdS	$\overline{X}$	StdS	$\overline{X}$	StdS	$\overline{X}$	StdS		
1-5 years	16	3.00	-	2.92	0.49	2.94	0.46	2.92	0.47	2.93	0.42	2	4
6-10 years	21	-	-	3.09	0.71	2.76	0.67	2.82	0.49	2.88	0.6	2	4
11-15 years	42	-	-	3.25	0.64	3.12	0.83	3.09	0.7	3.18	0.69	1	5
16-20 years	59	3.62	-	3.14	0.58	2.62	0.73	3.23	0.65	3.10	0.65	1	5
21+ years	58	3.23	0.8	3.34	0.6	3.31	0.69	3.07	0.92	3.29	0.67	1	5
Total	196	3.25	0.71	3.22	0.6	2.99	0.72	3.08	0.66	3.13	0.65		

SHS = Science High Schools, VHS = Vocational High Schools
AHS = Anatolian High Schools, RVHS = Religious Vocational High Schools

Table 3 shows the mean scores for usage frequency of LGE tasks in terms of instrument items. Most of the teachers prefer to use 'Make up an example' (Task 1=T1) and 'exploring distinctions' tasks (T9). The least preferred tasks are 'wild-card generation' (T13), 'using the features of methods or objects as starting points' (T11), and 'burying the bone' (T10).

Table 3. Mean Score for Task Usage Frequency with Regard Items

	T1	T2	T3	T4	T5	T6	T7	T8	T9	T10	T11	T12	T13	Tt
Mean score	3.50	3.14	3.24	3.23	3.19	3.00	3.08	3.00	3.63	2.93	2.95	3.19	2.58	3.1

Table 4 shows the distribution of answers for usage frequency of LGE tasks in percentages in terms of options. As seen in the table, "always" and "never" are the least chosen options for task use.

Table 4. Usage Percentages with Regard to Item Options

	Percentages												
Options	**T1**	**T2**	**T3**	**T4**	**T5**	**T6**	**T7**	**T8**	**T9**	**T10**	**T11**	**T12**	**T13**
1	3.1	3.1	4.1	3.6	9.2	7.7	6.1	10.2	3.1	9.7	10.2	6.1	19.4
2	9.2	18.4	19.4	14.3	17.9	19.4	19.9	20.9	8.2	24.0	22.4	18.9	29.1
3	35.7	45.4	31.6	41.3	27.0	42.9	37.2	32.7	24.0	37.8	34.7	31.1	30.1
4	35.7	28.1	37.8	36.2	36.2	24.0	33.2	29.1	52.0	20.9	27.6	36.7	16.8
5	16.3	5.1	7.1	4.6	9.7	6.1	3.6	7.1	12.8	7.7	5.1	7.1	4.6

1: Never 5: Always

To highlight the patterns in the data set, 13 items in the instrument were subjected to principal component analysis by using SPSS Version 23. Tabachnick and Fidel (2007) recommend the principal component analysis approach in order to obtain an empirical summary of the data set (p. 635). In this sense, purpose of the study is compatible with using a principal component analysis approach. Prior to performing principal component analysis, the suitability of the data for factor analysis was assessed. The Keiser Meiyer Olkin (KMO) value was 0.90, which is excellent according to Sharma (1996, p. 116) and Bartlett's test of statistical significance. Oblimin rotation was used. Principal component analysis revealed the presence of two components with eigenvalues exceeding 1, explaining 42.6% and 10.7% of the variance respectively.

Table 5. Pattern Matrix[a]

	Components	
	1	**2**
T2	.817	
T1	.812	
T5	.741	
T3	.738	
T4	.607	
T6	.507	
T12	.441	.345
T10		.853
T11		.813
T13		.741
T8		.659
T7	.360	.447
T9	.358	.368

As seen in the pattern matrix, factor loads for items 7, 9, and 12 are high in both components. After extracting items 7 and 9 (Table 6), both components show a number of strong loadings and all variable loads are substantial on only two components (after extraction KMO value is 0.88 and Bartlett's test is statistically significant). These two components, with eigenvalues exceeding 1, explain 43.3 % and 12.7% of the variance respectively.

Table 6. Pattern Matrix[a]

	Component	
	1	**2**
T2	.812	
T1	.805	
T5	.744	
T3	.734	
T4	.618	
T6	.515	
T12	.451	.339
T10		.845
T11		.807
T13		.735
T8		.668

According to the result of analysis T1, T2, T3, T4, T5, T6 and T12 are in the first component while T8, T10, T11 and T13 are in the second component.

To determine if year of mathematics teaching experience and school type are predictors of usage frequency of LGE tasks, multiple regression analysis was performed. For this purpose, school type was re-coded by using dummy variable coding (SHS= 1, all others = 0; VHS = 1, all others = 0; RVHS = 1, all others = 0). Before that, regression assumptions (normality, linearity, homogeneity, noncollinearity, fixed variance, and independence of residuals) were checked. The residuals fell into a random pattern; the histogram and normal distribution curves for the predicted values showed a normal distribution; and in a normal P-P plot, no major deviation from normality was observed. The correlation among independent variables range between -0.179 and 0.340, tolerance values (Table 7) between 0,927 and 0.858, and VIF values between 1.165 and 1.079.

Table 7. Coefficients[a]

	Unstandardized Coefficients		Standardized Coefficients			95,0% Confidence Interval for B		Correlations			Collinearity Statistics	
Model	**B**	**Std. Error**	**Beta**	**t**	**Sig.**	**Lower Bound**	**Upper Bound**	**Zero-order**	**Partial**	**Part**	**Tolerance**	**VIF**
1 (Constant)	37.931	1.761		21.543	.000	34.458	41.404					
Prof. Exp.	.216	.085	.187	2.547	.012	.049	.384	.204	.181	.179	.922	1.084
RVHS	-1.207	1.495	-.061	-.807	.420	-4.155	1.742	-.048	-.058	-.057	.858	1.165
VHS	-2.451	1.558	-.119	-1.573	.117	-5.524	.623	-.119	-.113	-.111	.865	1.157
SHS	-.974	2.976	-.024	-.327	.744	-6.844	4.895	.038	-.024	-.023	.927	1.079

a. Dependent Variable: LGE usage frequency
$R=0{,}233$, $R^2=0{,}054$
$F_{(4, 191)}=2{,}738$, $p=0{,}030$

The results of the regression analysis for predicting the level of teachers' use of LGE according to school types and years of mathematics teaching experience variables are given in Table 7. According to Table 7, there were positive and low correlation between the years of mathematics teaching experience and LGE usage frequency; SHS school type and LGE usage frequency ($r=0.20$ and $r=0.038$); negative and low correlation between RHVS school type and LGE usage frequency and VHS school type and LGE usage frequency ($r=-0.048$ and $r=-0.119$). However, when the other variables were controlled, only the correlation between the SHS school type and LGE usage frequency changed negatively ($r=-0.024$). School types and year of mathematics teaching experience variables give a low and significant relationship with LGE usage frequency ($R=0.233$, $R^2=0.054$, $p=0.03$). According to the standardized regression coefficient (β), the relative importance of the independent variables on the LGE usage frequency is the years of mathematics teaching experience and the school types VHS, RVHS and SHS. According to the results, the regression equation is:

LGE usage frequency = (37.9)+(0.21)x(Prof. Exp)–(1.2)x(RVHS)-(2.45)x(VHS)-(0.97)x(SHS).

However only 'the year of mathematics teaching experience' made a significant, unique contribution to the prediction of LGE usage frequency. One-year increase in the year of mathematics teaching experience results in 0.21 points increase on LGE usage frequency. This model explains the 5% of the variance in teachers' LGE usage frequency.

Analysis of Qualitative Data

At the end of the semi-structured interviews with 16 participants of the study, the reasons for frequency of use of LGE tasks were examined within the context of factors affecting teachers' usage of LGE tasks and MKT. The obtained data were first analyzed by taking into account the teachers' MKT.

Effects of MTK on the usage of LGE

When the data were examined within the scope of SMK, which is the first dimension of MKT, it was determined that the teachers considered themselves proficient in Common Content Knowledge (CCK). None of the interviewed teachers saw themselves as having insufficient mathematical knowledge, and indicated that they were not experiencing any difficulty using LGE in their classes.

Regarding Specialized Content Knowledge (SCK), which is the second dimension, the interviewed teachers view themselves as competent in providing appropriate examples and in forming strategies for LGE. In addition to the teachers who recognize LGE's connection with the proof process, as is indicated in the literature, some teachers have given concrete examples of their use of LGE tasks.

> SHS-1: Our students are good in terms of their level of knowledge. We cannot give them conventional question types as if they were conventional students. Using some of our prior knowledge [mathematical knowledge at university], we choose examples from a more advanced category. Our students from the 9^{th}, 10^{th}, and 11^{th} grades, in particular, are highly curious as to where the question comes from. We end up having to show them when we prepare for their classes… I mean where one thing comes from, the proof of it… For example, we use proof methods very effectively starting from 9^{th} grade. In our exams, we try to include at least one proof.

> AHS-1: While finding the roots of a quadratic equation, I would, for example, ask a student to give us a number. He says 2. Then I ask him to give me another. He says 3. An example I use frequently is "Let's write down a couple of quadratic equations, the sum of whose roots is 3.".

Teachers indicated that although they face no difficulty in using strategies for LGE tasks, or in preparing examples for their classes, it depends on the topic but they do not have sufficient knowledge for finding examples that relate mathematics' topics to daily life.

R: Are there any problems you face when constructing your own examples?
AHS-2: It depends on the topic. When teaching sets, I can find contemporary examples, as well as when teaching functions, but let me think of a topic you cannot easily find examples …. (thinking) … It depends on the topic.

R: Ok, if you could have training on using examples in class, what kind of training would you like to have?
VHS-1: Of course, I would like to have such training. First, it should be student-oriented. Examples should be from daily life. That would be my priority.

R: Think about your undergraduate education, have you had any training on how to use examples in class?
AHS-3: No, I have not because I received my BA from the school of science not from the school of education. I wish I had such a training. We probably could not have had that because it was the school of science. Now I think about how I can find examples in my own field, how I can get students' attention. I find myself lacking in those areas.

SHS-1: I only had a month-long internship, and I became a teacher after lecturing once within that one-month period. I tried to learn more about the teaching profession when I became a teacher. We had knowledge but we did not know how to use that knowledge. We are full of knowledge but unfortunately we do not know where we can use that knowledge …

Horizon Content Knowledge (HCK), which is the last sub dimension of SMK, does not come up in any area of the teachers' use of examples or strategies for LGE task.

Compared to SMK, PCK, which is the second dimension of MTK, has the greater effect on the use of strategies for LGE tasks. Knowledge of Content and Students (KCS), which is a subcategory of PCK, is one of the most frequently emphasized dimensions by the participants. KCS, which mostly comes out as the reason of not using, is also regarded as one of the reasons for using depending on the type of strategy and the advantages that example generation provides.

AHS-4: When a student cannot make sense of the question, s/he cannot solve it. Thus, we frequently ask them to generate examples. We use this technique in many of our classes. However, when I ask them to generate examples with constraints, I am aware that some classes are more advanced both mathematically and mentally, and they are readier for this. Thus, I can ask them to generate examples, but some of our classes really have gaps in their knowledge of mathematics, and filling those gaps takes time so it causes us to lose important time, and that is why I do not prefer to do it. I mean, I myself can give one example after another and I can ask them to give me the same or similar examples. In other words, I get feedback after 5-6 examples, which I cannot get in the first instance. Thus, we cannot go into the constraints in some of our classes. It does not work in every class.

VHS-2: It has huge advantages in terms of conceptual comprehension because when we first start it is the basic first type (first task). Now on our exams or when trying to learn things, students always want to learn them (first task). What we call basic type is related to comprehension. Students always say, "… are you going to ask from the first questions or from the later ones"? Alternatively, they ask in later stages, "Which of them are more important for us"? Therefore, I try to use these examples (first task) conceptually and this gives us an advantage.

R: Okay, let's have a look at the example generation strategies on the next page. You have circled "never" for most of the example generation strategies.

RVHS-1: For example, the compound of the function was given. It was asked retrospectively. Our students already find it difficult to find the compound of functions. Here it is the exact opposite. We ask for the return. This is where students fail the most.

AHS-3: The one I use the most frequently is 1 and then 9.

R: Why do you use these the most frequently?

AHS-3: I feel like they would find it easier to answer these.

VHS-2: Wild-card generation. Here, students prefer to deal with whole numbers and natural numbers when doing operations. They do not like irrational numbers, fractional numbers, or root numbers. Thus, they can even ask this in the exams: if the result is a rational number they can utter sentences like is the result of the question wrong.

All interviewees think that students' generation of their own examples is more effective in terms of retention of learning and conceptual comprehension.

AHS-5: Their own examples, the situations they have come up with, are surely more retentive, because they are making things concrete in their own minds.

AHS-6: Kids may not enjoy the theory of the topic. When a student gives an example, he or she actually says, "I have understood this, I can generate examples," and this of course means that the example he/she gives is more effective in terms of retention.

AHS-4: When they give the examples themselves, other students also understand the topic better because their examples are more related to the things they are interested in or talk about among themselves. They influence one another. It seems as if the example I give is not appropriate for them but when other students give examples, they are easier to understand because they are from daily life and match their age. Then the other students also understand the examples better.

Another sub dimension of PCK is knowledge of content and teaching (KCT). Teachers indicated that they determine the strategies they use based on the characteristics of the topic and the teaching method they follow. As not all strategies for LGE task are suitable for every topic, they can choose their strategies according to their teaching.

R: Then you use all of them a lot.

SHS-1: Right.

R: You have used "frequently" only for the last 3 strategies. Why are they used less, in your opinion?

SHS-1: Now, you know it is difficult to find examples for everything. You know in mathematics it is difficult to show the opposite of everything.

R: Do you ask the students to generate examples at the beginning of the topic?

SHS-2: I do it at the very beginning of the topic as an activity.

R: You mean you do not use them much in the following steps of the topic?

SHS-2: No, I do not use it much later. I use it at the beginning of the topic for them to explore and discover.

AHS-4: For instance, I ask them to give me an example of a number whose square is smaller than itself. When I get their examples, I immediately start to refute them. This is more effective for the kids. To see why their idea cannot work … They thought that way but they realize why that way of thinking was wrong and I think this leaves a more effective stamp on their minds. And I use the sixth example generation strategy more frequently (Looks at the 7th strategy: Number triples…). For example, I do not use this as such. I give them the rules and then ask what numbers fit into this rule. I ask them to

> form the numbers. I think it talks about triangle inequality here. I cannot work without giving the rule.

When the teacher's views that can be examined under the perspective of knowledge of curriculum and content (KCC), the last sub dimension of PCK, are taken into consideration, there were two teachers who gave concrete examples for the use of example generation strategies.

> VHS-4: For example, let's say we are going to introduce a topic. Because they have learned it before they are already ready. For instance, $\Delta=0$. But, where are we going to carry this? We are going to carry this to the equality of roots. When I say it is $\Delta=0$ in the parabola, I actually say this: tell me other meanings of it as well. The parabola is tangent to the x-axis. Tell me another meaning. There are two roots, either equal or coincident. What else … I increase retention in learning as such.

> VHS-3: We also give examples ourselves when we lecture students about a topic. Then we ask them to make their own examples, by saying, for instance, "Now you generate an example." The child will think about divisibility by 8 after divisibility by 4.

Effects of Constraints and Teachers' Beliefs on Usage of LGE

Constraints stand out as the most important factor affecting teachers' use of strategies for example generation. Constraints affect the use of strategies depending on the student, the teacher, school policies, the family, education policies, the topics covered in class, and the class environment.

The most frequent constraint among those depending on students was the insufficient preparedness of students. Teachers indicate that they cannot spare time for such strategies (strategies that require more teaching time) because they spend their time trying to cover gaps that happened in the previous levels. In addition to this, it is thought that most of the strategies here require advanced thinking skills. Because many of the students in their classes are not at this level. So, teachers do not want their students to face such strategies. They fear that otherwise they would lose those students who have low motivation and attitude towards mathematics.

> RVHS-2: Now, if the student has prior knowledge of the topic, he or she can give me an example before I do, but if the student does not add anything to what I teach in the classroom then he or she cannot, and that's what happens. There is no preparedness. I mean, I cannot lecture on the topic and then ask for an example from the student. I give an example and then ask them.

> AHS-2: If I had students who have a higher level of preparedness then I would solve the problems through different examples.

> AHS-4: For example, there are such classrooms where you can actually see the kids; you see that they too can learn. When you solve five of the same examples, then you can skip the next one with that student. But, when you have a mixed class, the good student starts to get bored and may try to sabotage the class. This has a negative influence on the general atmosphere of the class as well as on you. I mean the teacher does have emotions, too.

> VHS-3: At the end of the day, mathematics is a chain. A child who has not understood these in the 7^{th} and 8^{th} grades cannot do it in the 9^{th}, 10^{th}, 11^{th}, and 12^{th} grades.

A constraint that depends on the teacher was said to be the insufficient self-improvement of teachers, expressed as being related to the existing education system.

> AHS-6: We are not used to [such questions]. In a test you have one single answer. And here you have many. I mean, we have taken a certain type of education. We also should improve ourselves … Nevertheless, this is also related to the system itself.

The evaluation system at schools and the time factor related to that was another issue. Evaluations based on centralized exams force teachers to cover the same topics at the same time as other teachers, and also to show students all question types they may face on the university entrance exam. Otherwise, teachers would be pressured by their peers, the parents, and the students. This prevents sparing time for these strategies.

> AHS-7: Because children hear "the other class did this," "the other class did that," the parents, the students, and the staff are anxious. The anxiety is common for all. That's why we do not follow the curriculum but the "Assessment Selection and Placement Center (ASPC)" when we lecture.

> AHS-4: Maybe there is a way to work on it, but it is difficult to teach students the outcomes in a certain time frame. It puts us behind the schedule. We have a common exam. Then we refrain from this method so that we could keep up. I just use the example of confound expectations.

Apart from these, one of the interviewees indicated that the number of students in a class might be a hindrance in using these strategies and that these strategies are more appropriate for smaller (in terms of number of students) classes.

Constraints that can be examined in terms of education policies are the university entrance exam, the intense curriculum, and the incompatibility between the university entrance exam and the curriculum. Indeed, the university entrance exam stands out as the most influencing factor.

> AHS-5: We lecture on asymptotes and all other topics but then say, "This will not be in the university entrance exam," and move onto how they could solve such a question on the test. There is an increasing one here, and there is a decreasing one there… The students do not have such an exam. They do not face these. The evaluation of ASPC is also the same. There is no evaluation for open-ended questions.

> AHS-1: We actually try to use these strategies. But the high school curriculum is too loaded! I mean, it could be better if the curriculum was not this full, and we could try to reveal the creative power of the student. This is important but there is no time.

The last factor that affects the use of example generation strategies is the beliefs that teachers hold and the goals they have formed as teachers. Teachers' beliefs, such as their students' being used to ready-made information and formulas, and their not having the necessary habits to use such strategies, as well as the teachers' principles may positively or negatively affect their use of strategies.

> AHS-3: Unfortunately, our students are all used to ready-made things. They prefer that all the time.

> RVHS-3: They do not come to high schools having acquired the habit of critical thinking. They all want ready-made forms.

> R: Let me ask it this way. Are these strategies advanced for your students in this school, for example?
> AHS-4: I think there can be 3 or 4 students who can handle these. If I apply them, 3 or 4 students may get it, but because I am for the general population of the class I do not prefer them, in order not to seem to be favoring those 3 or 4 students and lecturing only to them.

AHS-4: In secondary school, they still maintain their study habits but when they come to high school, I don't know how to put it, but I do not want to scare them. Maybe I also have fears. If I ask a question they cannot answer or if I make them feel like they cannot succeed, I would lose, maybe that's what I fear. If I had tried, I may have had different results.

Factors Affecting the Use of LGE Tasks

The factors affecting teachers' use of LGE tasks are summarized in Figure 2.

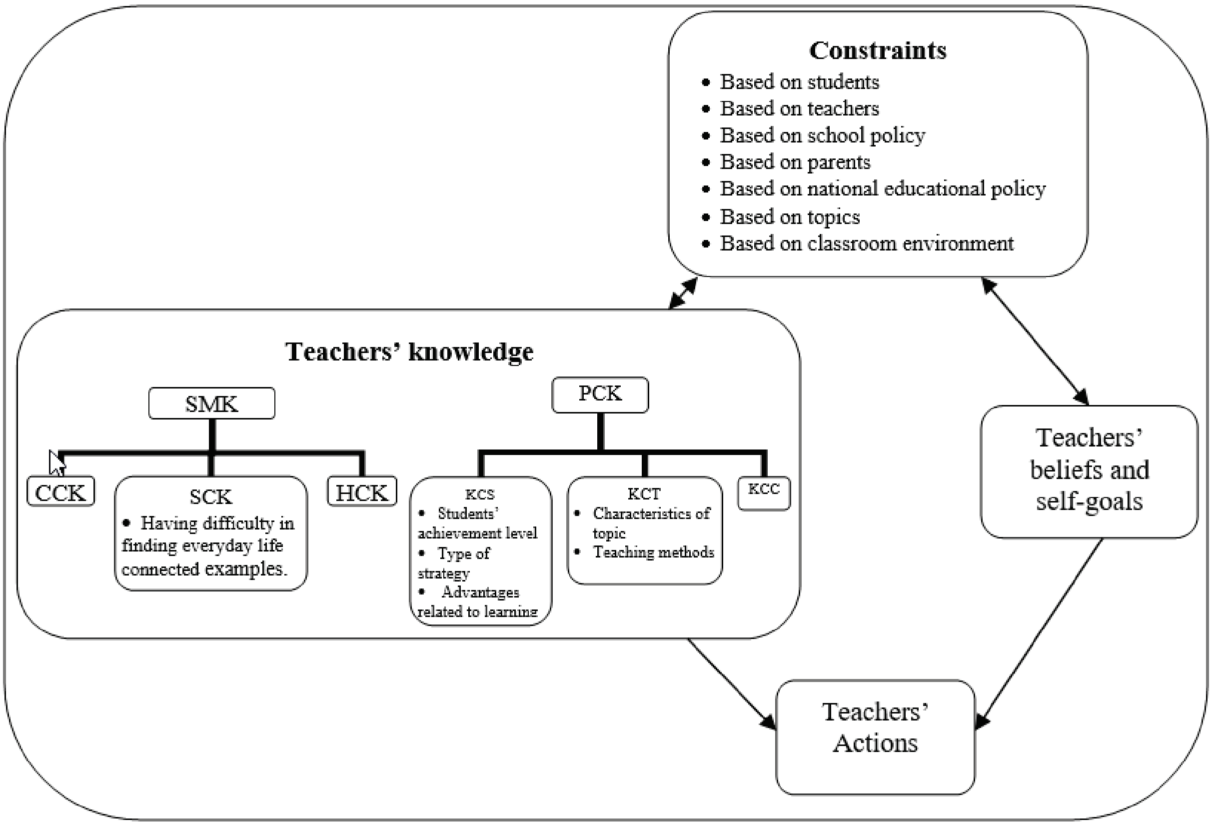

Figure 2. Factors That Affect Strategy Usage for LGE Tasks

In this figure which is constructed by taking into consideration the model in Sullivan, Clarke, and Clarke's (2013, p. 3) study, the headings are elaborated. As already mentioned, under the heading of teacher's knowledge mathematical knowledge for teaching have been examined. The knowledge, teachers have on teaching mathematics especially those in the PCK dimension, stands out as a limitation. For this reason, teacher knowledge and constraints interact with each other. For example, the most obvious relationship can be drawn between KCS and the constraints depending on students. Low student achievement is seen as a constraint for LGE use. The type of LGE used varies according to the educational policies of the country and the structure of the subject in lecture restricts the use of LGEs in some cases. Similarly, teachers' teaching objectives and beliefs about students arise as factors that influence the use of LGE tasks. All of these factors have the potential to influence what the teacher does in the classroom.

Discussion and Conclusion

Results on Quantitative Findings

The main purpose of the study was to investigate high school mathematics teachers' usage frequency of strategies for LGE tasks, and to reveal reasons behind them. Results indicated that teachers' with over 21 years of mathematics teaching experience have a higher usage frequency than teachers with less experience. The main reason behind that may be that teachers did not receive any specific instruction during their teacher training programs on the use of LGE tasks, and acquired the knowledge of valuable tasks in terms of student learning over time with self-experience. Zodik and Zaslavsky (2008) also stated that preservice teachers did not receive any systematic training on the example use. Thus, novice teachers benefit only from their own experience (Kennedy, 2002; Leinhardt, 1990).

The most preferred strategies are 'make up an example' (T1) and 'exploring distinctions' (T9). A high usage frequency for T1 is an expected result. During the interviews, most of the teachers indicated that they use this strategy particularly after the presentation of a new concept. It does not require higher order thinking skills, and the responses to these types of questions do not waste class time on irrelevant topics. Furthermore, teachers (especially the novice ones) may feel more comfortable giving feedback on the answers of these questions. O'Neil (2018) stated that one of the reasons for teachers not to use LGEs is anxiety about not being able to answer spontaneous questions raised within the classroom. The other preferred strategy, exploring the distinctions, is an important skill for mathematics learning. These examples provide a better understanding of the mathematical structure for learners by enabling them to explore the boundaries of definitions, as well as a better understanding of the linguistic differences (Watson & Mason, 2005).

The least preferred strategies are 'wild-card generation' (T13), 'use the features of methods or objects as starting point' (T11) and 'bury the bone' (T10). T11 and T10 are structurally based on reversing the process or the method. In interviews, some of the teachers pointed out that some of their students have difficulty directly applying the method. Therefore, they think they would be unsuccessful if they ask students to reverse a process or method. Likewise, T13 is another type of task that students are not familiar with. Teachers are always choosing problems, examples or exam questions that have whole numbers as answers. So the students may think that their answers are wrong if they reach a rational or an irrational number as the answer. Therefore, this is a habit that is fed by teachers. However, such examples may allow the students to expand their example space. Without using this type of tasks teachers ignore the pedagogical benefits of these tasks. Since the choice and treatment of examples has the potential to make student learning difficult or easy, the teachers have a challenging responsibility and needs to consider many features of the examples (Zaslavsky & Zodik, 2007). But there may be other reasons or constraints behind these preferences, such as time.

Principal component analysis (PCA) was performed to reveal the relationship among variables and analyze the structure of these variables. Two components have emerged as the result of PCA. T1, T2, T3, T4, T5, T6 and T12 fell into first component and T8, T10, T11 and T13 into the second. According to this result, it is seen that there is a pattern between the usage frequency of LGEs. The distribution of strategies into each component is consistent with the usage frequency of teachers. The first component contains the most preferred strategies, in terms of usage frequency, and the second component contains the less preferred ones. This distribution may also be consistent with the complexity of strategies or the time required to solve questions under a strategy, since the majority of teachers' preferences depend on the time their students' need to solve a question or the students' readiness. In addition, as O'Neil's (2018) study suggests, the fact that the examples in the second component have a more complex structure and that there will be less controls on these examples and that it can be more difficult to give immediate responses to these examples may have produced this result. Again, as O'Neil reported in the study, the fact that students are not used to such examples may be a general reason for not using them. T7 (characterize all objects that satisfy specified constraints) and T9 (explore distinctions), on the other hand, are two strategies that are extracted from PCA and have important roles in learning mathematics. T7 focuses on some constraints and the purpose of this strategy is about raising awareness of a class of objects that satisfy that constraints (Watson & Mason, 2005), and students can see connections between different (or seemingly different) mathematical constructs. As previously mentioned, T9 is a strategy that helps learners explore the limits of a definition. So, the strategies of T7 and T9 may create a contradiction in teachers' usage because of their clear teaching and learning advantages and their complexity for learners. This contradiction can explain the close factor loads in both components.

Regression analysis was performed in order to determine the contribution of school type and years of mathematics teaching experience in the use of LGEs. According to regression analysis, year of mathematics teaching experience is a predictor of usage frequency of LGE tasks, while school type does not contribute to the prediction of LGE usage frequency. The regression model revealed that a one-year increase in year of mathematics teaching experience results in a 0.21 point increase on LGE usage frequency. This result supports the idea that teachers' experience with examples develops over the years in the profession. If teacher candidates start with more knowledge and experience on the use of examples in mathematics classes, the pedagogical power of examples may increase. However, the regression model explains only 5 % of the variance in LGE usage frequency. So, there may be other variables that have bigger role in explaining this variance.

Results on Qualitative Findings

According to interview results, teachers' use of example generation strategies is influenced predominantly by the components of PCK and constraints they expose. SMK plays small role for the use of LGE task. There may be different reasons for that result. As one of the participants mentioned, most of these strategies have an open-ended task structure and using them reveals multiple answers, as opposed to what teachers and students are accustomed to. These strategies may bring to light some of the students' misconceptions, which teachers should notice and correct. The components of SMK may undertake a greater role in the use of the LGE tasks if they are used more frequently because they are used moderately. The participants in our study tend to use more simple strategies in the classroom that yield only one correct answer. Therefore, the teachers did not face questions having multiple solutions that may place great demands on SMK and thus, they may not realize the importance of their SMK in their usage frequency. Also, example generation tasks may appear different based on the individual that respond the question (teacher, student, preservice teacher etc.) (Zazkis & Leikin, 2007). Apart from that teachers primarily have difficulty in finding examples that are relatable to daily life. The emergence of such answers may arise from teachers' desire to increase their students' attitudes and motivation towards mathematics.

Within PCK, KCS and KCT are the leading factors influencing strategy use. KCS has three-sub categories under this heading: Students' achievement level, type of strategy and advantages related to learning. The participants of study who took part in the interviews think that all students' achievement level in the classroom is not appropriate for using these strategies, since the classes are not homogeneous in terms of student achievement, and they are trying to reach a level that they can address to all students. Some of these strategies require higher order thinking. Thus, some types of the strategies (most probably the strategies in the first component of PCA) are suitable for all students while others are not. Along with student achievement level and strategy type, some strategies have teaching/learning advantages. According to participants, using strategy T1 at the beginning of a topic is very advantageous in terms of conceptual understanding. For KCT, characteristics of some topics or teaching methods do not allow teachers to use some of the strategies. For instance, one of the participants (AFL-1) mentioned that it is not easy to find a counter example for every concept or the teaching method they use is not always an appropriate strategy for an LGE task.

Constraints are another major factor that affects LGE task use. Constraints based on students, school policy, and national educational policy are the most influential ones among these factors. Primarily, students' readiness, intensive curriculum, and university entrance exams affect strategy use. Specifically, the university entrance exam triggers all other factors since all classroom activities (exams, examples, homework, etc.) are shaped around preparation for this exam. Teachers are trying to prepare their students for this university entrance exam by presenting them with all question types that they may encounter on the exam. However, LGE tasks are not found in this exam so teachers do not want to waste time on these tasks. Students, their parents, school administration, and other teachers in schools apply pressure on teachers to place emphasis on preparation for the university entrance examination, starting in early years of high school. Therefore, class time is being reserved for these activities. The extra class time is allocated to complementary learning activities for students. Since teachers also come from this educational system and provide training under similar conditions, these tasks are not familiar to them, so, it becomes a constraint for them.

Lastly teacher beliefs and self-goals affect their use of LGE tasks. Based on their experience, they think that the student profile in their classrooms is not compatible with these kinds of tasks. They feel they may lose student's motivation or positive attitudes towards mathematics if they use LGEs, which students are not familiar with or which are difficult for students. When students have limited field-specific knowledge, they are limited in their use and generation of new examples (Moore, 1994).

Implications

The results of the study raise some forward-looking research proposals. It is thought that the creation of practical courses such as the use of LGE can be very useful in transforming theory into practice in teacher education programs. At the same time, researches on the effectiveness of these courses in teacher education can be done. In such a study, the gradual generation of good examples in the context of a mathematical topic will also help to uncover the pedagogical benefits of such tasks. In this way, the LGEs that teachers use less may become more useful. Likewise, in the classroom, teaching experiments can be conducted to determine how teachers can use example generation activities appropriately. Such studies will also provide concrete data for the lessons to be used in teacher education. For each type of LGE used in the study, examples can be created on certain subjects and presented in lesson plans to guide the teachers. According to the results of the study, year of mathematics teaching experience and school type explain 5% of the variability in the use of LGE. This result suggests that there may be different variables affecting the use of LGE. Thus, the effect of different variables on the use of LGE such as the type of bachelor degree, whether the teachers have master or doctoral education can be investigated.

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Appendix 1

Dear Participant,

The purpose of this study is to determine mathematics teachers' use of learner generated example in their classrooms. Thank you very much for taking part in this study. You can reach me from ysaglam@hacettepe.edu.tr about the result of the study.

Yasemin Sağlam Kaya

Year of mathematics teaching experience: ………………

What type of school are you currently working in?

( ) Science High School	**( ) Religious vocational high school**
( ) Anatolian High School	**( ) Vocational and technical high school**

How often do you use the following learner generated examples in your classes? The examples given in each item is provided only for concretizing the strategy. While answering the questions, please consider the strategies you used in a similar way.		Never				Always
1	**Make up an example:** Asking students to generate an example "Give me a set example", "Give me a function example", "Give me a proposition example", "Give me a continuous function example"					
2	**Make up an example with some constraints:** Asking students to generate an example with some constraints in the form of "Give me an example of a data set with 5 elements whose mean is 8", "Give me an example of a parabola whose graph is tangent to the x-axis", "Give me a geometric sequence example with a common ratio of 2", " Give me a function example that has limit at a point 'a' but is not continuous"					

3	**Add constraints sequentially:** Asking students to generate examples sequentially by adding some constraints in each step "Give me an example of a function. Give me an example of one to one function. Give me an example of one-to-one and onto function", "Draw a quadrilateral. Draw a quadrilateral with two equal sides. Draw a quadrilateral with two parallel edges and two equal edges. Draw a quadrilateral whose two sides are equal, two sides are parallel and opposite angels are equal.", "Give an example of five numbers divided by 4. Give an example of five numbers divided by 4 and 6. Give me an example of five numbers divided by 4, 6 and 8", "Give me a function example that has only right limit at a point 'a'. Give me a function example that has right and left limit at a point 'a'. Give me a continuous function example at a point 'a'"					
4	**Make up another or more like/unlike this:** Asking students to generate examples which extend previously explored instances such as "Give me an example of linear equation with exactly one answer. Give me another example of a linear equation which have different features", "Give me an example of a linear equation whose graph intersect x-axis. Give me another linear equation whose graph intersects x-axis and which has different features", "Give me an arithmetic sequence example. Give me another arithmetic sequence example with different properties", "Give an example of a rotational permutation. Give me another rotational permutation example with different properties "					
5	**Make counter-examples or non-examples:** Asking students to generate an example in the form of "Give me a counter-example for the situation that every relation is not a function", "Give me a non-example for the numbers whose square root is smaller than itself", " Functions do not have to be commutative under compound operation, give a counter-example", "Give an example of a non-polynomial function", "Find a counter-example for the statement that every continuous function may not have a derivative"					
6	**Confound expectations:** Asking students to generate an example which challenges their expectations such as "Find two numbers whose product is less than each of the numbers", "Find two functions that are commutative under compound operation", "Give examples of two functions that have the same derivative"					

7	**Characterize all objects that satisfy specified constraints:** Asking students to generate examples in the form of "Find three numbers that could be the measures of the lengths of sides of a triangle. What is the characteristic of these numbers?", "Make up second order equations with equal roots. What are the common characteristics of these equations?", "Specify the parabolas that intersect the x-axis at two points. What is the common feature of the equations of these parabolas"					
8	**Reverse:** Asking students a question that reverses the givens such as "The product of diagonals of a quadrilateral which are perpendicular is 36. What could be the question?", "The answer to a division problem is 5 with a remainder 2: What could the question be? ", "Determine maximum and minimum points of a function whose derivative graph is given"					
9	**Explore distinction:** Asking students to generate examples that distinguish particulars of a definition "In isosceles triangles, a perpendicular from the apex divides the base into two equal parts. In which triangles does this case also happen?" "In a parallelogram, the opposite edges are parallel. In which quadrilateral does this case also happen?", "When the graph of the sine function is examined it can be seen that it repeats itself at certain intervals. Which functions also have this property?", "The volume of a rectangular prism is equal to the product of the base and height. In which solids volumes can be found in a similar fashion?"					
10	**Bury the bone:** Starting with what is normally the final stage of a procedure; asking students to reverse a method and hiding the answer in increasingly complex ways like "If the solution of a linear equation is 6, what could be the equation?", " If the compound of two functions is $2x + 1$, what could these two functions be?", "If the equation of a tangent of a circle drawn at a point is $2x + 5y-2 = 0$, what is the equation of the circle?"					
11	**Use the features of methods or objects as starting point:** In this strategy, we don't use the answer but the byproduct of a procedure or a method itself as a basis for reconstructing a process such as "Find a fraction for which adding any positive number to the numerator and adding double that number to the denominator increases the value of the fraction. Characterize all such fractions"					

12	**Find:** Asking students to generate examples in the form of "Find the..." "Find examples of two triangles which are similar according to SAS (side angle side) rule.", "Find an example of a probability that includes a dependent event", " Find a geometric sequence example with the same common ratio", "Find two skew lines in the space",					
13	**Wild-card generation:** This technique gives starting examples for further work that might not have been obvious previously. Values are likely to be 'nasty' numbers; they do not allow students to use the methods they always used. "Close your eyes and determine 3 points on the coordinate plane. Calculate the area of the triangle which was constituted by these three points (Determine another three points if it does not constitute a triangle).", "Asking students to draw two regular shapes with the same size but on different locations on a coordinate plane and find out which reflection, rotation, and translation is needed to transform on shape to the other", "Work in groups of two. Select a vector without each person in the group telling the other. Later, calculate the angle measure between these two vectors", "Close your eyes and set three points in the coordinate system. Write a parabola equation that pass through these three points.",					