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# Mathematical Opportunities: Noticing and Acting

Oğuzhan Doğan [1], Hülya Kılıç [2]

**Abstract**

Recent studies on the improvement of quality of teaching and learning revealed that teachers should act as a facilitator, coach or guide to foster students' knowledge and skills. However, teachers' professional knowledge and skills is one of the determinants of how effectively they can act in accordance with such roles in a classroom setting. To support the development of both pre-service and in-service teachers' professional skills is thought as a valuable attempt to improve the effectiveness of teaching in schools. Therefore, under a university-school collaboration program, we provided an opportunity for pre-service mathematics teachers to work with students throughout a year and we investigated their pedagogical content knowledge and noticing skills. We aimed to determine whether or not pre-service teachers attended to mathematical opportunities occurred while interacting with students and we analyzed how they responded to those opportunities. In this paper we discussed their responding actions through two sample instances. Such an analysis is likely to contribute to the relevant literature in terms of identifying the nature of pre-service teachers' in-the-moment noticing as well as evaluating their responding actions for mathematical opportunities.

**Keywords**

Mathematical opportunities
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[1] Yeditepe University, Faculty of Education, Department of Mathematics and Science Education, Turkey, oguzhand@gmail.com
[2] Yeditepe University, Faculty of Education, Department of Mathematics and Science Education, Turkey, kilichul@yahoo.com

## Introduction

An ongoing transformation of how knowledge and learning are perceived and defined is an impetus for changes in how teacher competencies were evaluated. If knowledge is not perceived as an end that teacher possess and transfer to their students but as a process that students and teachers construct with the help of communication and experiences, then each moment in teacher-student interaction will become an indispensable point of this knowledge construction process. Similarly, in mathematics education, when mathematics learning is not perceived as a static manner that composed only of memorization of certain mathematical formulas and operations but a dynamic manner that focus on students' comprehension of real-life situations with using numbers, shapes and relations between them, then interaction, communication and discussion in a mathematics classroom will gain as much importance as definitions and exercises. A teaching approach, which emphasizes the development of students' thinking skills by placing students' thinking into the core of mathematics classroom and placing teachers as a facilitator of this classroom, is highlighted both in recent national and international mathematics curricular materials (e.g., Ministry of National Education [MoNE], 2017; National Council of Teachers of Mathematics [NCTM], 2014). The importance of students' mathematical reasoning, problem solving, communication and connection skills and teaching activities to improve these skills is spoken more loudly in these curricular materials.

Since a mathematics classroom is one of main environments for students' construction of their mathematical knowledge, the quality of experiences in classroom can be considered as a main facilitator of students' mathematics learning. The quality of students' experiences in mathematics classroom can only be ensured by teacher's awareness of what their students do and say through instruction and teacher's proper intervention with the help of this awareness (van Zoest, Stockero, Leatham, Peterson, Atanga, & Ochieng, 2017). In this context, teachers' ability of detecting a pedagogically rich classroom opportunity and teachers' ability of using this opportunity to enhance students' understanding is defined as teachers' noticing skills (van Es & Sherin, 2002). These pedagogically rich classroom opportunities can be students' answers of a specific problem, problem solution steps, or explanation of their reasoning. Teachers' noticing skills has become an important part of teacher competence, especially in the last 15 years, and has begun to make room for itself as a separate lane in educational research (Jacobs, Lamb, & Philipp, 2010; Mason, 2011; Stockero, Rupnow, & Pascoe, 2017). Although research on noticing skills has focused on many different subcomponents, the common issue is how teachers perceive the complex events in the classroom (Jacobs et al., 2010; Mason, 2011; van Es & Sherin, 2008). The main point here is that teachers should be aware that many events have taken place in classroom at a time and they should filter the moments that have revealed an opportunity to enhance students' learning.

Along with the importance it has, measuring or improving teachers' noticing skills might be a challenging process (van Es & Sherin, 2008). In order to overcome this challenge, one of the questions that needs to be answered in detail is that what classroom moments should be noticed. According to Leatham, Peterson, Stockero and van Zoest (2015), the moments that a student presents their mathematical thinking provides an important pedagogical opportunity for eliciting and supporting that student's understanding. They named such moments as Mathematically Significant Pedagogical Opportunities to Build on Student Thinking (MOSTs) and developed a framework for MOSTs instances. Their description of MOSTs might be an efficient answer for what should be noticed in a classroom setting.

Because role of student thinking in learning mathematics is emphasized in recent teaching approaches, we aimed to investigate the nature and development of the pre-service teachers' noticing skills such that how they attended and responded to the mathematical opportunities (MOSTs) occurred during their interactions with students. As a result of this examination, a classification was developed to describe pre-service teachers' (in-the-moment) attending and responding actions.

***Theoretical Framework***

As noted above, teachers' ability of drawing attention on the classroom moments that have chance to provide an educational opportunity to enhance students' understanding is called as noticing skills (Van Es & Sherin, 2002). While "noticing" is a common component of all teaching processes, noticing efficiently is often considered as an important and complex teacher skill.

In several studies that aim to identify and improve teachers' noticing skills, teachers and prospective teachers are often requested to watch and analyze various lesson videos either their own or others' and asked to answer questions about these videos (Ainley & Luntley, 2007; Colestock & Sherin, 2009; Erdik, 2014; Rosaen, Lundeberg, Cooper, Fritzen, & Terpstra, 2008; Teuscher, Leatham, & Peterson, 2017). Researchers attempt to describe teachers' noticing skills in terms of what they say or write about these lessons and they construct coding schemas in various ways. For example, based on teachers' writing about the course videos they watched, Van Es and Sherin (2002) described teachers' noticing skills with four levels from 'baseline' to 'extended'. If teachers were only writing their observations, they were categorized in Level 1. If they provided some comments and incomplete interpretations on the situation they observed, then they were categorized in Level 2. If they provided their reasoning behind their interpretations, then they were categorized in Level 3 and if they provided pedagogical suggestions following to their interpretations, then they were categorized in Level 4. In another study, Van Es and Sherin (2008) categorized teachers' noticing skills in the following four dimensions: actor (student, teacher, other), topic (mathematical thinking, pedagogy, climate, and classroom management), stance (describe, evaluate, and interpret) and specificity (general, specific). Van Es and Sherin (2008) reviewed and coded each of teachers' interpretation according to these dimensions.

The initial findings of the researches on teachers' noticing skills revealed that experienced teachers' noticing skills are better than the inexperienced teachers (Baş, 2013; Jacobs et al., 2010; Miller, 2011; Star & Strickland, 2008) but this skill can be improved by professional development programs Barnhart & van Es, 2015; Jacobs et al., 2010; Sanchez-Matamoros, Fernandez, & Llinares, 2015). Therefore, it can be concluded that one of the important factors that make research on noticing skills valuable is that having a room for improvement. Although skill development generally requires a long process, there are studies that show noticing skills can be improved even within about 15 weeks (Star & Strickland, 2008).

Van Es and Sherin (2008) proposed three aspects of noticing as "(a) identifying what is important in a teaching situation; (b) using what one knows about the context to reason about a situation; and (c) making connections between specific events and broader principles of teaching and learning" (p.245). For the first aspect, they claimed that a question which a student asks to his/her friends because s/he does not understand is a "remarkable" moment. Since the teacher would have the opportunity to assess students' understanding or misunderstanding from the conversation between two students. For the second aspect, van Es and Sherin (2008) gave an example of asking "why", "how" type of questions to the students. Because such questions could be an indicator of teachers' attempt to evaluate whether the student has understood the subject completely or not. The third aspect of noticing is the ability of making connections between specific events and the broader principles they represent. Teachers could find ways to improve their noticing skills by changing what they have noticed (shifting focus from teacher behaviors to student's perception) and how they have reasoned about it (shifting from just reporting to synthesizing). However, van Es and Sherin (2008) claimed that the development of teachers' noticing skills was not always the same and straightforward for all dimensions of noticing, but up-and-downs might be observed for each dimension at different time intervals.

Because researchers interpreted how noticing can be measured and how it can be developed differently (Erickson, 2011; Mason, 2011), some researchers have narrowed down the scope of noticing skills to a certain degree and focused on noticing of students' mathematical thinking (Jacobs et al., 2010; McDuffie et al., 2014). For example, Jacobs et al. (2010) defined noticing of students' mathematical thinking in terms of three interrelated skills: 1) attending to students' strategies, 2) interpreting students' insights and understanding, and 3) deciding how to respond based on their interpretations. Jacobs et al. (2010) argued that the teacher must pay attention to the verbal and written strategies that students use to reveal their mathematical thinking. Then, by passing through his/her own knowledge and experience, s/he must interpret the strategy of the students. And eventually, based on his/her interpretation, s/he should give a proper response in order to improve students' mathematical thinking.

Based on this description of noticing students' mathematical thinking (Jacobs et al., 2010), two interrelated questions arise for the first dimension (attending): should teacher pay attention to every classroom event? If not, what are the classroom moments that teachers should attend? Stockero and Van Zoest's (2013) concept of "pivotal teaching moment" (PTM) might be a fruitful answer to this question. PTM are the classroom moments in which the flow of course was disrupted and gave teacher an opportunity to change or improve the nature of students' mathematical understanding. These PTMs are generally emerged when students give contradictory answers, when students' answer has a strong connection with a more general mathematical context, or when their answer is one of the common misconceptions of that relevant content.

Leatham et al. (2015) have developed this PTM framework and proposed Mathematically Significant Pedagogical Opportunity to Build on Student Thinking (MOST) framework to describe teaching moments that have chance to develop students' mathematical understanding. In order to be a MOST case, a classroom moment should i) be based on students' mathematical thinking, ii) be mathematically important, iii) uncover a pedagogical opportunity. Firstly, it should be possible to make evidence-based interpretation about students' mathematical thinking and this students' thinking should be related with a mathematical concept. Secondly, the mathematical concept that students' thinking highlight must have an important place among mathematics learning objectives of the relevant class level. Lastly, there should be a cognitive need and enough time to discuss that concept. If these three basic criteria are met then there is a "mathematical opportunity" (Leatham et al., 2015). This MOST framework is briefly presented in Figure 1. Thus, it can be argued that Leatham et al. (2015) attempted to elaborate a more detailed and observable framework for the first two stages of (attending and interpreting) Jacobs et al.'s (2010) noticing skills framework.

Although the number of studies on how teachers respond to these pedagogical opportunities is limited, it is possible to find some seminal research in related literature. For example, Jacobs, Lamp, Philipp and Schappelle (2011) assessed the teachers' ability of deciding how to respond (as the 3rd component of noticing ability) within 3 levels: robust evidence, limited and lack of evidence. In their written reflections, if they noted that they would summarize the previous performance of students, explain students' mistakes, ask why and how questions, and present possible examples to help students to understand the given problem, then their responding was categorized as *robust evidence*. If they stated that they could lead students to solve the problem by giving concrete materials or examples but did not specify what questions they would ask, then their responding was categorized as *limited evidence*. Lastly, if they made general comments or expressed their own thoughts, such as 'this problem is difficult for students', 'they can't solve this', 'they should use materials' then their responding was categorized as *lack of evidence*.

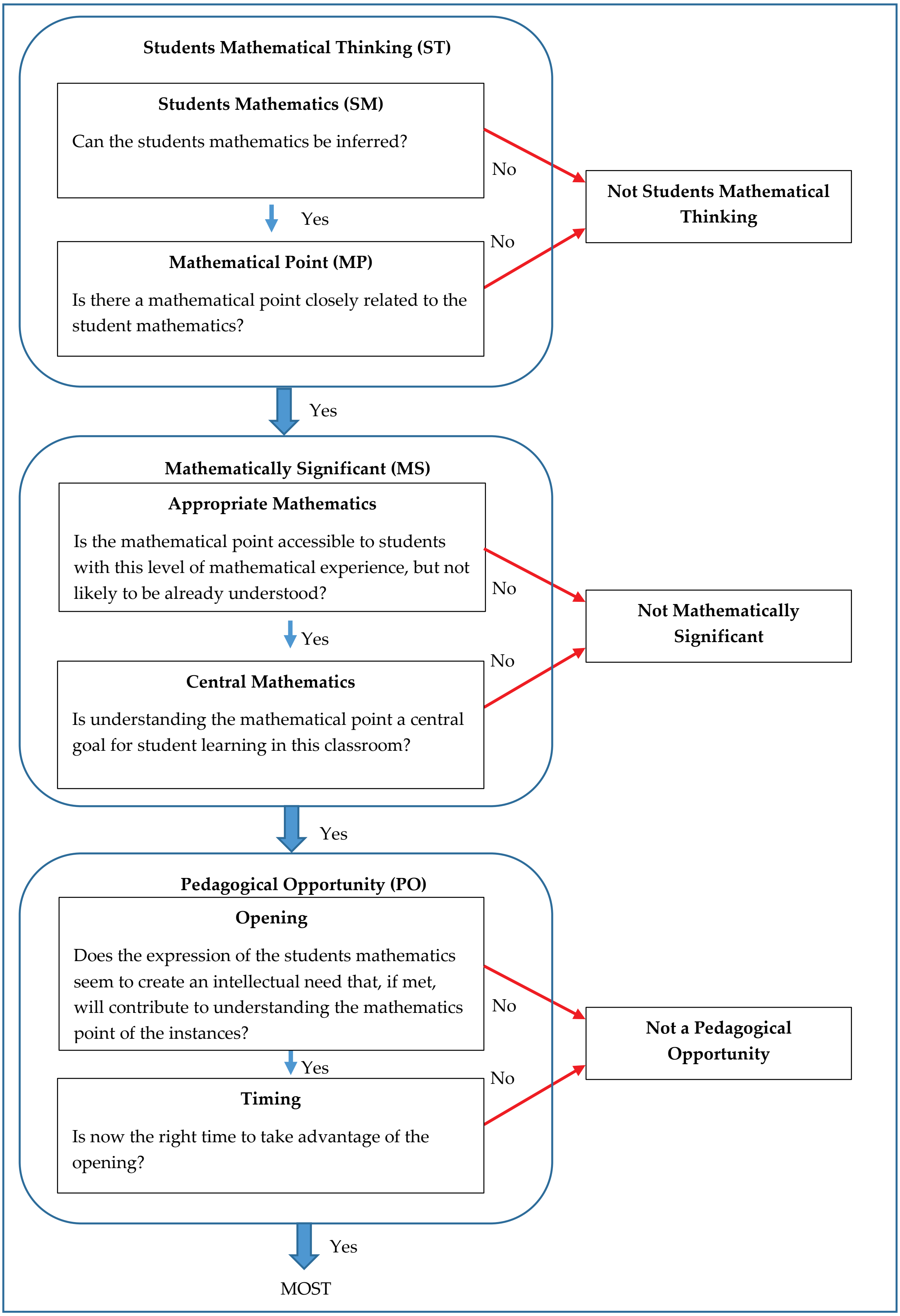


**Figure 1.** The MOST Analytic Framework (Leatham et al., 2015, p.103)

In addition, Barnhart and Van Es (2015) evaluated teacher candidates' attention, interpretation and deciding how to respond skills together and classified this joint skill as low, medium and high level of sophistication. The low-level responses include the teachers' answers that explain what will be done in the next lesson without taking into consideration students' thoughts. In medium-level responses, teachers take students' thoughts into consideration while planning next lesson. In the high-level responses, teachers describe what will be done in the next lesson with logical connections to the teaching-learning relationship. Teuscher et al. (2017), on the other hand, examined pre-service teachers' in-the-moment noticing skills based on their written reports. They interpreted pre-service teachers' references to students' mathematical thinking in their responses to MOST cases and classify these references in three categories; no clear connection, elaborated and facilitated.

Although research on noticing skills have a history, it is quite new to study what are the important classrooms moments that should be noticed and how to respond these moments so as to contribute students' mathematical thinking and understanding (Nickerson, Lamb, & LaRochelle, 2017). In this context, this study brings together the frameworks of noticing and MOST to analyze and classify noticing and responding skills of pre-service teachers who had chance to work with the middle school students for one year.

## Method

This paper is based on data emerged from a study aiming to investigate pre-service teachers' *mathematical knowledge for teaching* and noticing skills. Grounded theory approach was used to analyze the data about pre-service teachers' noticing skills. Grounded theory is used to explore or understand a concept or a construct when there is limited information about it in the literature (Cohen, Manion, & Morrison, 2011; Grbich, 2007). Thus, it enables to enrich an existing theory as well as development of a new one (Strauss & Corbin, 1998). Although there are different perspectives used in grounded theory, the scholars agree that it is emerged from the data, it involves in systematic data collection and analysis, patterns are implicit in data and it is both inductive and deductive (Cohen et al., 2011). In grounded theory, data is collected through observations, interviews or documentations and it is analyzed by using a coding scheme (Cohen et al., 2011; Grbich, 2007). The coding or classification scheme is achieved as a result of an iterative process (theoretical sampling, open-axial-selective coding, constant comparison, the core variable and saturation) such that it might be the one used in earlier studies or a revised version of an earlier coding in the line of new findings or a newly developed one (Cohen et al., 2011). The coding and classification scheme discussed in this paper passed through such a process as well as it was constructed upon relevant theoretical framework discussed in the literature.

The primitive version of the scheme discussed in this paper was based on the data collected in 2013-2014 academic year in the form of pre-service teachers' lesson videos and reflection reports which were analyzed in the line of Jacobs et al. (2010) definition of noticing (Kilic & Tunc-Pekkan, 2017; Tunç-Pekkan & Kılıç, 2014). The classification of pre-service teachers' noticing was then enriched by taking into consideration of Leatham et al. (2015) description of MOST instances occurred in a classroom. We reformed the classification based on the findings of a pilot study held in 2015-2016 academic year and then we revised and finalized it in the line of the data collected in 2016-2017 academic year.

### *Research Setting and Participants*

This study was conducted under a faculty-school collaboration between a large university in Istanbul and a local middle school in the neighborhood. The aim of the collaboration is to support students' mathematical thinking and understanding as well as pre-service teachers' professional knowledge and skills. The collaboration program was launched during 2011-2012 academic year such that pre-service teachers and students participated in the program voluntarily. Initially, it was set up as an after-school program but then we took the responsibility of carrying out elective mathematics courses for 6th graders and 7th graders in the middle school since 2014-2015 academic year. We also offered an elective course for pre-service teachers in the university such that we discussed student-centered teaching approaches, task design and implementation and scaffolding practices in this course. We also discussed the tasks that would be implemented by the pre-service teachers in the school.

A total of 17 pre-service teachers participated in the pilot study during 2015-2016 academic year. They were all senior undergraduate students who participated in the study voluntarily such that 10 of them participated in study during Fall 2015 and 7 of them participated in Spring 2016. However, 10 pre-service teachers participated in the study held in 2016-2017 academic year such that two of them were senior, four of them were junior and the rest were sophomore students. Seven of those pre-service teachers attended to the study for two semesters while three of them just attended in one semester. One of the seventh-grade classes was determined as a project class by the school administration. It was noted that the class was representative of other seventh grade classes in the school in terms of achievement level and ratio of male-female students. As the research team, we made a group of four students and assigned a pre-service teacher for each group for implementation of tasks. The pre-service teachers worked with the same group of students throughout the year.

We designed the elective course for pre-service teachers in a way that they had opportunity to learn about basics of task design and implementation process and different ways of scaffolding students' understanding. At the beginning of the semester, for a couple of weeks, we discussed how they would implement the tasks in the school and how they should interact with students to both elicit their thinking and also scaffold their understanding. After such preparation process, we began to go to the school for implementation. For each week, just after the implementation we met with the pre-service teachers to discuss how the implementation went, whether unpredictable instances occurred or not and how they addressed to students' difficulties (oral reflection). In the same week, two days later than the implementation, we met again with pre-service teachers to discuss the following week's tasks as well as whether we should revise the tasks or not based on students' performances on earlier tasks (post-reflection). Furthermore, we (research team) watched pre-service teachers' videos and looked at students' worksheets to comment on pre-service teachers' and students' performances during the implementation to take pre-service teachers' attention to important issues that we recognized. We followed the same cycle (implementation, oral reflection, post-reflection) throughout the study. Moreover, we asked pre-service teachers write a reflection report about each implementation by watching their own videos and analyzing their students' work.

We followed a 3-step process for task implementation in the school: 1) students worked on the given tasks individually, 2) they discussed their solutions within their groups, and 3) pre-service teacher joined in their discussion and asked for their solutions and reasoning. As students were working individually or making group discussions, pre-service teachers observed them and took some notes about their performances. Sometimes they managed the group discussion to justify each student's contribution to the discussion. We expected that pre-service teachers would respond to the MOST instances that they observed during individual work and group discussion. However, some of the tasks did not have individual work session but designed as a group work. When it was the case, implementation was carried out as group discussion followed by pre-service teacher's intervention.

Indeed, the goal of faculty-school collaboration was to provide opportunities for pre-service teachers to improve their pedagogical content knowledge and noticing skills through the process of getting prepared for implementations, implementing tasks at school and reflecting on implementations. Thus, pre-service teachers learned about middle school students' mathematical abilities and they experienced how they could support their mathematical understanding. Moreover, because they worked with the same group of students throughout year, they got better in estimating their students' performances on the tasks and they got prepared for what to attend and how to respond to students during implementations. In other words, the faculty-school collaboration enabled pre-service teachers to adopt the phases of noticing (attending, interpreting, responding) naturally.

### *Data Collection*

The qualitative data was collected through videos, reflection reports, assignments and students' worksheets. We videotaped all pre-service teacher-students' interactions, and oral reflection and post-reflection sessions. Pre-service teachers wrote a reflection report for each task implementation. Furthermore, they prepared and implemented 5 tasks for their own group. We collected students' worksheets and extra sheets at the end of the implementations. We analyzed the data to determine MOST instances occurred during implementations, pre-service teachers' noticing skills, and the effects of pre-service teacher-students' interactions on students' performance.

### *Data Analysis*

**MOST instances occurred during implementations**: Each member of the research team watched pre-service teachers' videos to determine MOST instances occurred during the implementations by using Leatham et al.'s (2015) framework. The research team discussed MOST instances in weekly meetings. Because pre-service teachers implemented the same tasks in their groups, except the ones prepared by themselves, almost all MOST instances were common in the groups. Therefore, the research team initially achieved .98 agreement on whether an instance was a MOST or not. Then they discussed the discrepancies and achieved a fully agreement about MOST instances.

In this study, we prepared mathematical tasks in the line of mathematics curriculum such that we both paid attention to students' prior knowledge and also the seventh-grade mathematics objectives for particular content area. In this paper, we used two sample MOST instances to discuss pre-service teachers' responding actions. These MOST instances occurred while implementing tasks on *data analysis* and *percentages* and are explained briefly below.

In the seventh grade, students are expected to represent given data as a pie chart (MoNE, 2013). In relation to this objective, we prepared a task where we gave a list of 20 foods and beverages and asked students to choose 4 of the foods and one of the beverages that they liked to have. Then we asked for their choices and made a frequency table for the items in the list. Then we asked them to determine which one had the highest frequency or the lowest frequency. Then we asked them to classify the given foods and beverages and then represent their classification as a pie chart. We recognized that most of the students did not pay attention to accurate size, in terms of degrees, of each slice but determined them roughly. Furthermore, some of them put the frequencies of the items as percentages on the graph instead of calculating the degrees of slices for each group of items. In Figure 2, sample student work for such instances are given.

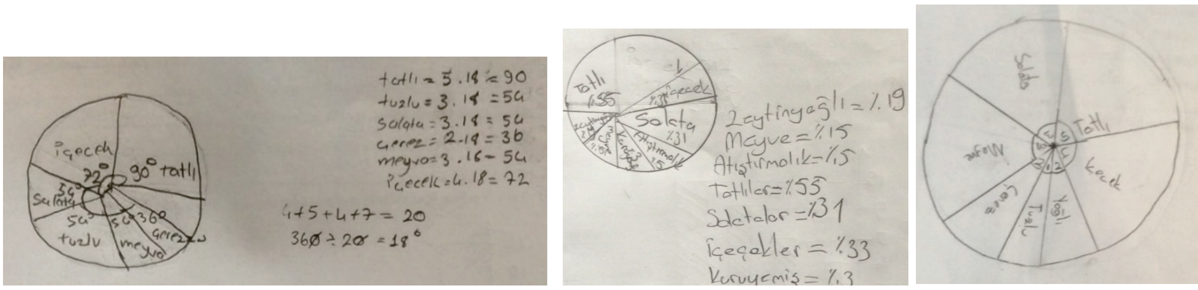


**Figure 2.** Sample Student Work for Pie Chart Task

We identified those instances as MOST in the line of Leatham et al. (2015) framework. In this case, construction of pie chart is a MOST instance. The first student sample in Figure 2 revealed that the student determined the degree of each slice by using the fact that measure of a circle is 360° but he did not construct the chart accurately by using a protractor but roughly. The second student thought that frequency of choice is the same with percentage of choice and he also replaced the items on the chart roughly. The third student did not determine any measurement of slices but divide the circle roughly in terms of the items in each group. The mathematical point for this instance can be stated as "*In a pie chart, each part of the data is represented as a proportion of 360° and the size of slices is determined by protractor*". Since we have both student mathematics (SM) as shown in Figure 2 and mathematical point (MP) for this instance, we can conclude that we achieved the first criteria for being MOST that is, being student mathematical thinking (ST). To construct pie chart is one of the objectives in the seventh-grade curriculum and it was already discussed in the regular mathematics lesson of the class. That is, they had prior knowledge for constructing pie charts and we expected them to use that knowledge to achieve the goal of our task. Thus, we justified the second criteria for MOST which is to be mathematically significant (MS). Finally, a MOST instance should be a pedagogical opportunity (PO) for teachers to discuss the instance to address students' intellectual need when it occurs. As seen in Figure 2, students had lack of knowledge about pie charts and therefore, pre-service teachers should take an action to eliminate students' confusions and support their understanding. That is, this instance also satisfies the third criteria of a MOST. As a result, students' answers for pie chart task revealed that it is a MOST instance. Hence, we used this instance to analyze pre-service teachers' noticing skills.

The second MOST instance used in this paper is about determining percentages. In the seventh grade, students are expected to find a percent of given amount and solve problems related to percentages (MoNE, 2013). In our task we asked students to compare the cost of a shopping when it was done in a regular day versus prices cut-off day. Then, we asked them to calculate the percentage of saving by shopping during the prices cut-off day. Most of the students found out how much they would save but they failed to determine the percentage of saving. Some of the students thought that the amount of saving is the same as the percentage of saving as shown in Figure 3. In this question, the cost of items on prices cut off day is 33 liras while it is 44 liras on a regular day. The student found how much he saved but he failed to calculate percentage of saving.

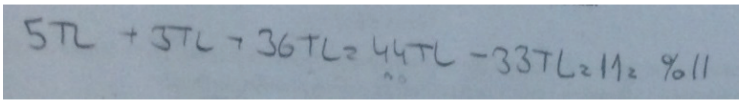


**Figure 3.** A Sample Student Work for Saving Percentage Task

The student work given in Figure 3 satisfy all criteria of MOST such that it is emerged from student mathematical thinking, it is mathematically significant and it is a pedagogical opportunity. The student thought that saving amount is the same as saving percentage (SM). However, to find the saving percentage, the difference between the prices is divided by the highest price (MP). To calculate percentages and solving related problems are involved in the curriculum (MS) therefore, teachers should address to students' thinking (PO) when they did such a mistake.

**Noticing:** Jacobs et al. (2010) described three interrelated components of noticing as attending, interpreting and responding. They noted that teachers' decisions about how to respond students' thinking emerge from their attention to and interpretation of student's mathematics because they have to make sense of that instance in their minds before acting. However, when an attention or interpretation is not finalized in terms of a decision for responding or an action then the teacher is not accepted as having noticed that instance. In this study, we investigated teachers' in-the-moment noticing and noticing after task implementations (oral and written reflections) in terms of MOST instances occurred during implementations. We analyzed in-the-moment noticing in terms of attending and responding components while we used oral and written reflections for evidences for attending and interpreting components. We did not analyze responding component in oral and written reflections since we evaluated it in action. Thus, we were able to analyze all components of pre-service teachers' noticing (Kilic, Dogan, Arabaci, & Tun, 2019). However, we will discuss pre-service teachers' in-the moment noticing in this paper.

Data about pre-service teachers' in-the moment noticing was collected though task implementation videos. When pre-service teachers took an action for the MOST instances in terms of telling that the answer was wrong or opening a discussion about the MOST, we accepted such actions as an evidence for attending to the MOST. Pre-service teachers' responding actions after attending to MOST instances was analyzed in the line of grounded theory premises.

The research team has been investigating pre-service teachers' noticing of MOST instances and their responding actions since 2013-2014 academic year (Doğan, Yılmaz, Kılıç, Ergül, & Arabacı, 2017; Kilic & Tunc-Pekkan, 2017). A coding and classification scheme were attempted to be developed through the process transcribing videos, open-axial-selective coding, comparison, and classification (Cohen et al., 2011). The scheme was revised as a result of new data such that final version of scheme was achieved by analyzing data collected in 2016-2017 academic year. The consistency and validity of scheme was checked by discussing it by mathematics educators as well as reviewing relevant literature constantly (Doğan et al., 2017). The research team discussed their coding for pre-service teachers' responding actions in their weekly meetings. Although the interrater reliability was .89 for initial coding, the team discussed the differences and a consensus was achieved for final coding.

## Results

We identified two foci in pre-service teachers' in-the-moment responding actions: answer-focused and mathematical understanding-focused. We categorized pre-service teachers' actions under *answer-focused* responding when their major aim was to make students to find or understand the correct answer or solution. When pre-service teachers attempted to elicit and support students' conceptual understanding rather than only making them to find out correct answer, we classified their actions under *mathematical understanding-focused* responding.

The sample MOST instances given in data analysis section occurred during implementations in the spring term of 2016-2017 academic year. In each group of students that pre-service teachers worked with, there were students who struggled with understanding the given tasks such that they could not construct pie chart or failed to calculate the saving percentage. Except in one group, pie chart task was discussed during pre-service teacher-student interactions. In the other group, one of the students explained how to construct pie chart to her peers therefore, the pre-service teacher was not able to address that MOST directly. A similar case was observed during the discussion of saving percentage task. In three of the groups, one of the members of the group explained the solution to the others during the group discussion. In such a case, the pre-service teachers asked the others to repeat and explain the solution. Briefly, all pre-service teachers attended to these MOST instances however they differed in terms of responding actions.

***Answer-Focused Responding***

In some cases, pre-service teachers attempted to encourage students to find the correct answer rather than eliciting and fostering their conceptual understanding as a respond for the attended MOST instance. We observed cases where pre-service teachers only declared that students' answers were wrong even though the MOST instance emerged from a misconception. We named such responding actions as *No Attempt* and classified them under *Level 0*. For the MOST instances that we used as a sample in this paper, we did not observe such a respond most probably because students discussed the correct answers during their group discussions as mentioned above.

We observed that pre-service teachers attempted to explain the procedure or solution themselves or ask one of the students to explain it as a responding action for the MOST instance. Such an action might help students to remember or understand the solution of the problem however it might not be permanent and students might experience the same difficulties in similar problems in the future. We gave a vignette below as a sample for such responding action of pre-service teachers. In this case, the students determined the degree of each slice in pie chart but they placed them roughly on the circle.

**PST 1:** In a pie chart, if you have 2 nuts out of 20 [items], how many degrees would it be out of 360? You all determined the degrees [of each slice].

**Zeynep:** We know the degrees but our pie charts are different from each other.

**PST 1:** But, don't you find how many degrees are there [for each slice]? For example, your 54 degrees is different than Murat's and Duygu's. However, they have to be congruent because they are all 54 degrees.

**PST 1:** What do we use for this? We use a tool for geometric constructions. (She took the protractor) What is the name of this?

**PST 1:** (simultaneously with Duygu) Protractor.

**PST 1:** To determine the angles, we use protractor.

**Zeynep:** But, how?

**Duygu:** We count from here. (She showed benchmarks on the protractor)

**PST 1:** Do you know how to use it?

**Duygu:** I know.

**PST 1:** We need to use protractor to determine the angle measures in a pie chart. Because yours are not clear enough (She pointed students' pie charts).

**PST 1:** Now, construct a circle.

**PST 1:** (On a piece of paper, she showed how to use protractor). First, we determine the center. (She drew a radius and then constructed a 90° slice)

PST 1 noticed that students calculated the degrees of each slices correctly but they placed them on the circle roughly (Lines 4, 5, 6). She asked students whether they knew how to use a protractor. Although one of the students told that she knew, she did not let her (Line 10) or others to explain what they know (Lines 9, 14). She, herself, explained how to use protractor and construct a pie chart (Lines 17, 18, 19). We classified PST 1's responding as *Level 1* and named it as *Explanation*.

In some cases, pre-service teachers attempted to make students recognize their mistakes either by asking them short answer, yes/no type, prompting, no-follow up or non-specific type of questions or make them to re-read, re-do the problem. The major aim of such questioning was not to address students' confusions in their minds but to encourage them to find out the correct answer. During the discussion of pie chart task, the pre-service teacher-oriented students to correct answer as follows:

**PST 2:** How do we construct a pie chart?
**PST 2:** What is the sum of angles here? (She pointed the angles at the center of the circle)
**Kerem:** 360.
**PST 2:** OK. How do we determine the slice of each? For instance [slice for] desserts?
**PST 2:** How many foods and beverages do we have?
**Kerem:** 20.
**PST 2:** How many of them are desserts?
**Musa:** 3. (There were 5 desserts but 3 salads in the list)
**PST 2:** Or, how many of them are salads?
**Nisa:** Salads, 3.
**PST 2:** OK. If we have 3 salads out of 20 foods, how do we construct pie chart when you take this [circle] as a whole?
**Kerem:** According to degrees.
**PST 2:** According to degrees. We determine it according to 360, to find one of them, slice for salad. Would you please write here, how many salads, how many desserts we have?

PST 2 aimed to make students to set up a proportion between foods and angle measures of slices to construct a pie chart. She asked measure of a circle in terms of degrees (Line 2). Then she took students attention to elements of proportion. She reminded that out of 20 foods 3 of them were salads and 360 degrees corresponds to 20 foods (Lines 14, 15). We named such responding actions as *Orientation* and classified as *Level 2*.

***Mathematical Understanding-Focused Responding***

In some cases, pre-service teachers attempted to elicit and foster students' mathematical understanding instead of only scaffolding for correct answers and procedures. They asked students to explain their thinking and reasoning through probing questions. However, such pre-service teacher-student interactions failed to be completed when time was up or other students intervened the discussion. Furthermore, in some cases, pre-service teachers used inappropriate examples or representations to address gaps in students' thinking or even they led more confusion in students' mind. Below, we gave two vignettes to exemplify such responding actions. The first sample is taken from pie chart task and the second one is from percentage task.

**Sevgi:** We divide 360 by 20, it is 18.
**PST 3:** Why do you divide 360 by 20?
**Sevgi:** To construct the pie chart. All is 360 degrees. To find the slice for salads we have to divide 360 by the total.
**PST 3:** Total of what?
**Sevgi:** Types of foods.

PST 3 attempted to understand whether the student knew the reasoning behind the procedure that she used to solve the problem. Therefore, PST 3 asked her to explain why she divided 360 by 20 and what number 20 was referred to. However, PST 3 did not ask further questions to elicit student's deeper understanding of pie charts. Although students' responses (Lines 3, 4) revealed that she understood how to determine the degrees of slices, it is not inferable whether the student also knew about how to use that information to construct pie chart. The following sample vignette is taken from percentage task.

| | | |
|---|---|---|
| 1 | **PST 4:** | He saved 11 liras. OK, here my question: What is the percentage of his saving? |
| 2 | **Selim:** | 11 percent. |
| 3 | **PST 4:** | 11 percent? He saved 11 liras. How much he saved as a percent? |
| 4 | **Selim:** | 89. |
| 5 | **PST 4:** | 89. Is that much? OK, I have a question. His saving is 11 percent. We know how to |
| 6 | | find a percent [of a number]. Is 33, 11 percent of 44? Sorry, is 33, 89 percent of 44? |
| 7 | **Selim:** | I said 11 percent, not 89 percent. |
| 8 | **PST 4:** | 11. OK he saved 11 percent. We know that his cost is 44 [on a regular day]. If I find 11 |
| 9 | | percent of 44, then is it 11? |
| 10 | **Selim:** | Yes. |
| 11 | **PST 4:** | Is it? Try it. |
| 12 | | *Students did calculations.* |
| 13 | **PST 4:** | Bora found 11 percent [of 44]. Reduced cost is 39.16. Is it 33? What is it asked? If I |
| 14 | | decrease 11 percent of 44 then I should find 33, but it isn't. |
| 15 | **PST 4:** | We have understood that we saved 11 liras. OK, 11 lira is saved from "what"? |
| 16 | **Bora:** | He saved 11 liras, then what is the percentage of saving? |
| 17 | **PST 4:** | We had such a discount that the cost is now 33 [liras]. 11 lira is saved from "what"? |
| 18 | **Bora:** | "Prices cut off" day |
| 19 | **PST 4:** | No, as a number value, it is discounted from? |
| 20 | **Bora:** | Isn't it 33 discounted from 44? |
| 21 | **PST 4:** | He saved 11 liras from 44 liras. I am asking for the percentage of saving. |
| 22 | **Bora:** | I have understood your question but I don't know how to do. |
| 23 | **PST 4:** | You saved 11 liras from 44 liras. How much do you save from 100 liras? |

PST 4 asked students to calculate 11 percent of 44 to make them realize that amount of saving is not the same as percentage of saving (Lines 8, 9). One of the students, other than the one who claimed that they are the same, calculated 11 percent of 44. Then PST 4 explained what he did (Lines 13, 14) to show that the claim was wrong. Then PST 4 tried to take students' attention to what the numbers 11, 44 and 33 represented for (Lines 15, 19, 21). The student noted that he did understand the problem but he did not know how to calculate it (Line 22). Then PST 4 attempted to simplify the problem by rephrasing it (Line 23). PST 4 initially encouraged the students to recognize their mistakes and then she tried to make students to understand the problem and figure out a way to solve it. Although students realized their mistakes eventually, they could not figure out the solution of the problem. The pre-service teacher was not able to fully eliminate their struggle with finding percentage of saving.

PST 3 and PST 4 aimed to elicit students' understanding of mathematical concepts in the context rather than supporting them to find the correct answer. However, their interactions with students were ended up without tying to a conclusion. We classified their responding as *Level 3* and named it as *Exploration*.

We observed cases where pre-service teachers aimed to elicit students' understanding by using appropriate examples and representations and also ending up their interactions with a conclusion. Below, we presented PST 4's such interaction with students during the implementation of pie chart task.

| | | |
|---|---|---|
| 1 | **PST 4:** | OK, you all constructed the pie chart. But I have a question for you. Selim stated that |
| 2 | | there are 2 in each of pastry and nuts. How did you draw them as congruent? |
| 3 | **Selim:** | Roughly. |
| 4 | **PST 4:** | You drew them roughly. But how I know that they are congruent? Show me that they |
| 5 | | are congruent. |
| 6 | **Selim:** | We construct angle bisector. |
| 7 | **Bora:** | We cannot show it without a protractor. |
| 8 | **PST 4:** | Say it again! |
| 9 | **Bora:** | We cannot show it without a protractor. |
| 10 | **PST 4:** | We cannot show it without a protractor. You have learned about angles, haven't you? |
| 11 | | Then you can draw these by using protractor. OK, what will be the degree of this |
| 12 | | angle? (She pointed to one of the slices) |
| 13 | **Bora:** | It would be better if we rewrite them as a percent. This is 360. Wait a minute. |
| 14 | **PST 4:** | Bora stated that we can write it as a percent. (She pointed to the center of circle) Is the |
| 15 | | degree of this angle 100? |
| 16 | **Bora:** | 360. Can we use 360? Yes, we can. |
| | | *Bora began to solve it.* |
| 17 | **PST 4:** | Do we know the angles in the circle? |
| | | *Bora did the following calculation: 360:20=18.* |
| 18 | **PST 4:** | OK. I want you to draw it [pie chart] again. The important point here is the angles in |
| 19 | | the circle. |
| 20 | **Bora:** | Their sum has to be 360 [degrees]. |
| | | *Bora, enlarged the fraction 3/20 by 18.* |
| 21 | **PST 4:** | You have to find the values [on the circle] in terms of 360 not 20. |
| 22 | **Bora:** | Salad is 54 [degrees] |
| 23 | **PST 4:** | How many salads we have? 3. That's right. |
| 24 | **Selim:** | Salad is 15%. |
| 25 | **Bora:** | No. Look, this is 360. Therefore, you will do it by 360. |

PST 4 recognized that students paid attention to how the pie chart looked like rather than the reasoning behind constructing a pie chart. Therefore, she asked them to justify their drawings (Lines 4, 5). PST 4's such attempt showed that she wanted to address students' conceptual understanding of a pie chart such that she asked them how to find the angle measures of each slice (Lines 11, 12). She recognized each student's ideas and encouraged others to contribute to the discussion (Lines 14, 15). Thus, she created an environment for students to learn from each other (Lines 22-25). The interaction between pre-service teacher and students revealed that the MOST instance was addressed completely such that after finding the angle measures of each slices by setting up a proportion between the given items and 360 degrees, slices are determined by using a protractor. We named such responding actions as *Elaboration* and classified them as *Level 4*.

As a result, we analyzed the videos of pre-service teachers' interactions with students to describe their in-the-moment noticing in the line of Jacobs et al (2010) definition of professional noticing of students' thinking. We initially analyzed the data collected during 2013-2014 academic year to determine the significant issues about noticing. Then we developed the first version of our classification for in-the-moment noticing. In the light of the data collected through 2016-2017 academic year we revised the classification. The final version of classification framework is given in Table 1. Because all components of noticing are interrelated with each other and attending and interpreting are embedded in teachers' responding we did not take interpreting component of noticing separately in this classification (Barnhart & van Es, 2015; Jacobs et al., 2010; Simpson & Haltiwanger, 2017). However, data about interpretation component was collected via oral and written reflections and analyzed accordingly. The findings about analysis of noticing after implementations is not given in Table 1 since it was not in the scope of this paper.

**Table 1.** Classification of In-The-Moment Noticing

| | | |
|---|---|---|
| Attending | 0: | Missed the MOST |
| | 1: | Attend to the MOST |
| Responding | **Answer-focused** | |
| | 0: | ***(No Attempt)*** Only tells to the students that their answers/ solutions are wrong; no guidance for students |
| | 1: | ***(Explanation)*** S/he or other students tells/explains the procedure or solution |
| | 2: | ***(Orientation)*** Attempts to make students find out the correct answer through a) short-answer, yes/no type, prompting (directs students to correct answer like "…,isn't it?), no-follow up, non-specific type of questions or b) asking them to re-read, re-do, re-think |
| | **Mathematical understanding-focused** | |
| | 3: | ***(Exploration)*** Attempts to elicit students' thinking by asking probing questions (why, how, what if, …) but either conversation is not concluded or in case of existence of misconceptions/ misunderstandings she fails to address the gap in student's mind because her guidance involves partially incorrect issues such as lack of terminology, inappropriate examples or representations |
| | 4: | ***(Elaboration)*** Attempts to elicit students' thinking by asking probing questions and guiding students through appropriate examples, representations, connections between concepts and representations |

## Discussion and Conclusion

This paper provides examples of MOST instances through real classroom dialogues and a classification of how pre-service teachers responded to these instances. The classification of pre-service teachers' responding actions was constructed upon Jacobs et al. (2010) definition of three components of noticing. We specifically focused on pre-service teachers' in-the-moment noticing in terms of attending to and responding for MOST instances occurred in the classroom. Thus, mathematics educators would have opportunity to learn about students' thinking-based learning opportunities that could be encountered in mathematics classes as well as how to respond these opportunities more resourcefully.

Teachers' or pre-service teachers' noticing and responding skills in-the-moment of teaching has yet to be investigated (Nickerson et al., 2017). For instance, Teuscher et al. (2017) attempted to analyze pre-service teachers' responding actions in terms of their written reflections about own teaching. On the other hand, Amador, Carter, Hudson, & Galindo (2017) observed a teacher while she was a pre-service teacher and then first-year teacher and described patterns and changes in her responding actions narratively. Therefore, this 5-level classification discussed in this paper has potential to construct comparison criteria in this research area since it was obtained as a result of years of study on noticing. We think that this classification, including the categories, *no attempt, explanation, orientation, exploration and elaboration* is capable of covering almost all possible "responding" moments. However, in the light of the future results obtained from similar studies of teachers' in-the-moment responding behaviors, there can be some reorganizations and improvements in this classification.

When we compare this classification with the results of research that examines pre-service teachers' feedback through video analysis (Barnhart & van Es, 2015; van Es & Sherin, 2008), some similarities and differences arise. As in the case of the related literature, a hierarchical classification for responding actions has been followed in this study. For example, parallel to the classification of van Es and Sherin (2008), the level of responses given by the pre-service teachers is considered to be at lower levels if they only say "Right" or "Wrong" as a response to what the student says or writes. If their responses have potential to reveal or support the conceptual understanding of students, then such responses are considered to be at higher levels. In addition, similar to Barnhart and van Es study (2015), teachers' responses are examined comprehensively and the interpretation component of the noticing skills (Jacobs, et al., 2010) was evaluated together with the responding component.

On the other hand, the difference of this study from the studies in the field is that this classification revealed two main categories for teachers' response behaviors; answer-focused and mathematical understanding-focused. In a framework or classification used to describe teachers' noticing, it is an important distinction whether teachers' responses are focusing on students' thinking or only correct answers. For this reason, pre-service teachers' attempts for eliciting students' mathematical thinking and supports for their conceptual understanding are accepted to be higher level responding actions in contrast to their attempts to make students just figure out the correct answer. In that sense, the responding actions in the form of *exploration and elaboration* aim to foster students' mathematical thinking beyond getting correct answers and solutions.

Another difference between the findings of this study and related literature was about MOST instances. When the MOST instances in this study was examined, it can be concluded that student thinking-based learning opportunities are mostly grounded on students' misconception. However, Leatham et al. (2015) listed the classroom moments that are likely to create a pedagogical opening as "(a) a correct answer with novel reasoning, (b) an incorrect answer that involves a common or mathematically rich misconception, (c) a mathematical contradiction, (d) incomplete or incorrect reasoning, and (e) why or generalizing questions" (p.100). Among these instances, 'an incorrect answer that involves a common or mathematically rich misconception' cases were mostly observed in our activities. While students' correct answers with novel reasoning were very rare, there were almost no cases that includes 'why or generalizing questions' or 'mathematical contradiction'. This finding was different from what van Zoest et al. (2017) found in their recent study. They investigated the nature of MOST by analyzing 278 instances and found that approximately in 40% of the MOST instances emerged from students' correct explanations of some mathematical facts. Besides being the subject of another study, the reason of this can be the nature of the tasks, the nature of the implementation or students' lack of prior knowledge (Kilic, Dogan, Tun, & Arabaci, 2018). Because it is necessary for the students to have a prior knowledge about the content and clear understanding of the relation between the concepts in order to reach a correct answer with novel reasoning. However, since the overall academic achievement of the participant 7$^{th}$ grades was very low, the likelihood of MOST cases emerged from students' alternative solution strategies or generalization questions decreased (Kilic et al., 2018). In addition, the number of the MOST cases that emerged in the conceptual knowledge-based activities can vary from the procedural knowledge-based activities. Moreover, the number of the MOST cases in individual works of the students and in group discussions also likely to differ.

As student-centered teaching approaches has become popular in recent years, teachers' noticing skills as well as their pedagogical content knowledge has been discussed more in the literature (Jacobs et al., 2010; Mason 2011; Stockero et al., 2017). Because teachers' ability to understand students' mathematical thinking and addressing the gaps in their thinking effectively is one of the factors influencing the quality of teaching (van Zoest et al., 2017). Therefore to investigate mathematically significant pedagogical opportunities and how to respond these opportunities has a potential to affect both the quality of the learning environment in mathematics classes and the productivity of teacher training program positively. The studies revealed that teachers' ability to attend mathematically significant opportunities occurred in the classroom and use them to support students' learning and understanding is not developed naturally but entails special efforts to be improved (Barnhart & van Es, 2015; Baş, 2013; McDuffie et al., 2014; Stockero et al., 2017). Therefore, pre-service teachers should be given opportunities to improve their noticing skills though specific teacher training programs like specific courses and faculty-school collaborations as carried out in this study. Thus, pre-service teachers learn about as well as experience what instances are significant to elicit or support students' mathematical understanding and how they can respond for students' mathematics other than orienting them for correct answers.

The combined use of noticing and MOST theoretical frameworks in this study can provide both teachers and prospective teachers with guidance on what moments they should be aware of in the classroom and how to respond to these moments. The classification that emerged in this study supports teachers' and pre-service teachers' closer consideration of the students' original responses or critical consideration of students' incorrect answers in future mathematics classes. Through this classification, it is recommended that mathematics teachers should look for ways to effectively communicate with students in the moments where there is an educational opportunity rather than giving superficial feedbacks, such as "be more careful here" or "it is not true". In order to adopt this viewpoint by pre-service teacher, MOST cases that may arise in real classroom environment and how to respond these opportunities should be integrated into teaching method courses in education faculties. Specifically, micro teachings in method courses and presentations in teaching practice courses could provide opportunity to examine MOST cases and develop pre-service teachers' noticing skills. In this context, exploring how different mathematical tasks or different teaching strategies can uncover different pedagogical opportunities could provide a wealth of research opportunities. Similarly, studying how specific teacher training programs that aim to develop noticing skills can improve prospective teachers' noticing skills could be a productive research area.

As a conclusion, to have a high-quality mathematics instruction, the importance of personal and authentic ties that students have with mathematics must be highlighted as well as the importance of technology, manipulatives, school environment and curriculum. As mathematics educators and teacher educators, in order to be able to recognize and respond the pedagogical opportunities more efficiently we need to pay more attention to the student ideas, what they think, why they think and how they express their thinking. Re-conceptualizing 'student-centered education' as the education that centralizes students' thinking can strengthen the concept itself.

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