Additional Operation Learning Process to the Mild Intellectual Disabilities Students by means of Virtual Manipulatives: A Multiple Case Study

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Abstract
It is frequently emphasized that technology integration to the education of the students have intellectual disabilities has great importance. The researches performed show that using virtual manipulatives consist positive effects on learning mathematics on the students have intellectual disabilities. This study was done to investigate the addition operation learning process of the students have intellectual disabilities. Two students have mild intellectual disabilities and are 14 and 17 years old participated to this research conducted by means of multiple case study is one of the qualitative research designs. In this study was realized during five schedules data were collected through interviews, observations and documents. Content analysis were used for data analysis. At the end of the analysis, three categories of findings on “Learning of the addition operation”, “Opinions on the use of computer and virtual manipulatives” and “Individual differences”, were structured.

Introduction
To raise child due to the social requirements and to equip them by knowledge and skills proper with modern life necessities with adapting them to their social networking are the main goals of the education (Ministry of National Education [MNE], 1973). Thus, knowledge and skills which can be used for adapting to the society and making life easier are taught to individuals. This state is not different for the students have intellectual disabilities. As much as possible providing the education in the same training environment to both students have intellectual disabilities and healthy students and developing the life of students have intellectual disabilities to make them independent is so significant (Kaplan, Öztürk, Doruk, & Duran, 2016). For this reason, from preschool to higher education phases, integration of technology, which offers diversity and differentness to learning and training process,

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materializes learning and increases the motivation level (Kose, 2009) to the education has an important role (Davies, 2011; Kaleli Yılmaz, 2015; Olçay Gül, 2016; Perry & Dockett, 2008, s. 93). Hence, recently, one of the important reforms on education is technology integration (Hwang, Su, Huang, & Dong, 2009; Lim & Chai, 2004; Lim & Khine, 2006).

As a result of the continuous improvement on the computer technology, both quantity and quality of learning software, virtual manipulatives, learning objects are increased and alternatives are always getting accumulated. Especially, virtual manipulatives (Moyer, Bolyard, & Spikell, 2002, p. 373), occurred through becoming widespread of manipulatives using in mathematics can be defined as physical manipulatives transferring to computer environment (Bouck & Flanagan, 2010; Mildenhall, Swan, Northcote, & Marshall, 2008). Contrary to physical manipulatives, virtual manipulatives have mobilizing specification (Langrall, Mooney, Nisbet, & Jones, 2008). Clements (1999, s. 100) states that transferring manipulatives from physical environment to virtual environment dynamises learning; because, virtual manipulatives are flexible and can be reused and changed. It can ease generalization and passing to symbolic comprehension (Clements & McMillen, 1996).

In the literature, there are many researches produced that using virtual manipulatives influences mathematics education positively (Perry & Dockett, 2008, s. 95; Peltenburg, Heuvel-Panhuizen, & Doig, 2009; Akkan, 2012). Moyer-Packenham and Westenskow (2013) examined 32 studies in a meta-analysis study of the effectiveness of virtual manipulatives in mathematics teaching. According to this, they compared virtual manipulatives with other teaching methods, stated that the results of 12 studies were meaningful for virtual manipulatives, 8 were not meaningful differences, 7 were mixed results, and 5 were not. In fact, technology integration to education can be present some new opportunities to increase the comprehension levels of the students have mental deficiency in the education and training process by means of materializing the regarded concepts. Davies and West (2014, p. 841) emphasized the importance of K-12 education and special education technology integration to education to increase the academic achievement of the students and to increase the technology literacy in the mathematics, science and reading courses in the special education. Using virtual or visual manipulatives instead of the intangible representations for especially the education of students have intellectual disability (Junge, 2013, p. 210). Thus, students have intellectual disability meet with difficulties on reasoning, generalizing (Camadan, Özer, & Şen, 2011; Howard, Williams, & Lepper, 2011, p. 478; Hunt & Vasquez, 2014; Pierangelo & Giuliani, 2004, p. 177), and have special training can overcome these difficulties. On the other hand, virtual manipulatives make the comprehension of mathematics and mathematical symbols easy for the students have intellectual disability by means of materialization (Langrall et al., 2008; Peltenburg et al., 2009; Pierangelo & Giuliani, 2004, p. 150). One of the reasons of the comprehension difficulties (Cebeci, 2013, p. 173; Wearmouth, 2009, p. 113) is the short attention span (Özak & Avcıoğlu, 2007) and having difficulty on generalization (Pierangelo & Giuliani, 2004, p. 156). Moreover, students have mental deficiency meet difficulty on fine motor skills such as computer mouse and keyboard control (Choi & Chan, 2015). Technology integration to this kind of individuals’ education can provide them skills (Wearmouth, 2009, p. 188; Doğan & Akdemir, 2015); to gain eye hand coordination (Pierangelo & Giuliani, 2004, p. 154), playing, socializing, being active through making them eager to learn (Howard et al., 2011, p. 479). On the other hand, virtual manipulatives can contribute to mathematics learning process of the student have intellectual disability through right feedbacks (Wearmouth, 2009, p. 144) and guidance and by providing to materialize intangible objects (Özak & Avcıoğlu, 2007) during the education process.
Particularly, functional academic skills are required in order that students have intellectual disability can be successful in their lives and live independently (Çifçi Tekinarslan, 2014, p. 154). Erbaş (2008), indicates these functional academic skills are aimed to gain to these students such as telling the time, reading, writing, fundamental arithmetic processes. Getting functional academic skills as summing and eliminating through the students have intellectual disability is seen the basic requirement (Çifçi Tekinarslan, 2014, p. 155; Tournaki, 2003). Because addition and subtraction operations provide the students have intellectual disability to overcome functional academic skills to daily issues and obtain similar academic skills, be able to communicate with each other and socialize (Çifçi Tekinarslan, 2014, p. 155). However, addition process is seen as an easy activity for rhythmic counting, it is not easy as considered because that realizing conceptual comprehension requires some intangible processes (Erdoğan & Erdoğan, 2009, p. 31). Carpenter and Moser (1984) specified two basic strategies as adding and combining are used in addition process. In spite that adding strategy is defined as adding one number to another one by an object group or rhythmic counting method, combining strategy is identified as composing a new set by combining two different sets (Carpenter & Moser, 1984; Haylock & Cockburn, 2014). Olkun and Toluk Uçar (2012, p. 105) have also labeled these strategies as unification and part-whole. On the other hand, showed mental abilities are separated as three phases: adding by completing to 10 (Olkun & Toluk Uçar, 2012, p. 102; Pesen, 2008, p. 135), adding small number to the large number (Albayrak & Şimşek, 2010, p. 242; Cengiz, 2013, p. 60), separating the object as the equal groups (Van De Walle, Karp, & Bay-Williams, 2013, p. 154). Savaş (2005, p. 122) is stated that to be understood of equality signal for number pairs of which total values are the same is very important to comprehension addition process. To be learned of addition process conceptually, composing and assorting decades (Pesen, 2008, p. 135), comprehending number preservation (Olkun & Toluk Uçar, 2012, p. 68) and being able to make one to one correspondence between numbers and objects (Soylu, 2006, p. 127) should be gained.

The representation of a part of the conceptual learning of the addition operation of students with intellectual disabilities in the field of writing related to the teaching of the addition operation is presented in Fig 1.

![Fig 1. A Model for Conceptual Learning of Addition Operation](image-url)
In recent years, many researches about technology integration to the education were performed (Akyüz, 2014; Blackwell, Lauricella, & Wartella, 2016). These researches are generally oriented K-12 training. Therefore, researches for individuals need special education are international and limited availability (Fuchs et al., 2006; Tournaki, 2003; Eliçin & Tunali, 2016). Since using of education technologies during learning teaching process of students need special education is effective to provide equality of opportunity (Pierangelo & Giuliani, 2004, p. 155), social (Howard et al., 2011, p. 479), emotional (Chen, Lee, & Lin, 2016) and educational (Peltenburg et al., 2009) development, performing researches investigate process of technology integration to the learning and teaching activities for students have mental deficiency ha a great importance. Fien et al. (2016) study showed that students who were learning disability at the elementary level of elementary school level were learning to teach integers so that computer use makes it easier for students to understand the concept of integers. Butler, Miller, Crehan, Babbitt, and Pierce (2003) concluded that middle school students with math learning difficulties were more successful than the control group of groups that were practiced through virtual manipulatives in their experimental work on teaching the concept of fraction. Bakker, van den Heuvel-Panhuizen, and Robitzsch (2016) conducted a study on computer-assisted multiplication and division of their work on special education students. They aimed to teach multiplication and division processes by playing 16 mini games in 10 weeks. As a result of experimentally conducted studies, it was concluded that the teaching of computer assisted multiplication and division training to special education students was not useful and there was no significant difference between the group with computer assisted instruction and the group without computer assisted instruction.

Especially, because describing addition process as one of the functional academic skills (Çifci Tekinarslan, 2014, p. 155; Tournaki, 2003) and using virtual manipulatives are very effective tool for addition process (Bouck & Flanagan, 2010; Junge, 2013, p. 215) learning for students have mental deficiency, teaching addition process by means of virtual manipulatives were investigated through different perspectives in this research. Also, qualitative researches will be done oriented with searching the process of technology integration to teaching-learning activities of the students have mental deficiency have great importance in respect to clarify the interaction of process’s positive and negative characteristics with virtual manipulatives of the students. By this research, it was aimed that addition learning process of two students have mental deficiency was investigated by means of virtual manipulatives thoroughly.

Method

Research Model

In this research, one of the qualitative research designs, multiple case study method was used. Multiple cases study is a method based on investigating more than one conditions by both in itself and also together with other conditions (Stake, 2005, p. 39). In this study, every student was taken as one condition; thus, students’ processes were investigated by both in itself and comparing with each other.

Participants

The quota sampling method was used in determining the students who participated in the study. The quota sampling method is based on the selection of individuals who represent a group according to certain characteristics. In this method, the individuals to be included in the study are determined according to certain criteria (McMillan & Schumacher, 2006, p. 126). In this study, two students who are able to do numerical recognition, rhythmic counting, collecting with numbers not exceeding 5, but have not achieved complete learning for the achievements required by their peers in
the collection process, have the psycho-motor ability to use computer skills. This research was held with a girl student is 14 years old and boy student is 17 years old, who go to special school for the students who have mild mental deficiency in Bayburt City places in the Northeast of Turkey. These two students were subjected to some tests such as Wechsler Intelligence Scale for Children and two intelligence tests, Stanford-Binet and Leiter. According to the results of these tests, and IQ results, they were defined as having mild mental deficiency. In the informal interviews before the work, it was determined that Arda can perform counting by finger, and Nisa can do addition operation together shape. Both students can read; but they were unable to read fluently and therefore had difficulty in understanding what they read. All required formal permissions were taken from governmental authorities, behalf of these two students and from their families to participate them to the research. And also no payment was done or received for this research. Participation to this research was completely voluntary basis. You can see the characteristics of the students at the Table 1. Informal interviews with the teachers and observation in the first lesson were effective to determine these characteristics.

### Table 1. Characteristics of the Students Participated to the Study

<table>
<thead>
<tr>
<th>Features</th>
<th>Female Student (FS)</th>
<th>Male Students (MS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grade</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>Special education program</td>
<td>Education support</td>
<td>Education support</td>
</tr>
<tr>
<td></td>
<td>Occupational support</td>
<td>Occupational support</td>
</tr>
<tr>
<td>Graduation on K-12 education</td>
<td>Middle school</td>
<td>Middle school</td>
</tr>
<tr>
<td>Computer opportunity</td>
<td>She has got not a computer</td>
<td>She has got a computer</td>
</tr>
<tr>
<td>Basic Computer Skills</td>
<td>She has psychomotor capability to use computer mouse</td>
<td>He has psychomotor capability to use computer mouse</td>
</tr>
</tbody>
</table>

School where the research was held includes K-12 education system which combines basic education and occupational training for the students who have mild intellectual disability (Range of intelligent point is 50/55-70) and completed middle school (K-8). There is one computer, projection device, tables and white board in every class. In addition to them, there are four computers in the school hall. All computers in the school have internet connection.

**Data Collection Tools**

In order to obtain in-depth data, it is important to use multiple data collection tools together (Merriam & Tisdell, 2016). In this research which managed by case study method, three data collection tools, interview, observation and document, were used. Interview were semi-structured during the study were realized in two different time interval, there are during the application process and after the application. In the process, some questions to find out the mathematical process skills of the students were asked to students. Such as; “You have five candies. If I give you three candies more, how many candies have you got?” , “How did you find the result?” , “How did you find the result?” And then some questions to find out their behaviours to use computer were asked to the students. Such as; “Do you enjoy to use computer?”, “How did you spend time by computer sooner?” On the other hand, after the application, some questions to evaluate the application were asked to the students. Such as; “Did you enjoy the activities done by computer?”, “What did we do in these activities?”, “Did you like to learn Mathematics by computer?” “What does it come to your mind when you think addition?” “Does making addition activity by computer contribute to you?”, “Do you want to take the further lessons supported by computer”. In the preparation of the interview forms, the first researcher prepared the questions to reveal the cognitive and affective domain skills. The prepared form was sent to three researchers in the research group and the cognitive skill questions for mathematical skills were accepted in line with the opinions of the researchers. Since the questions were directed to the process, language compliance with the form could not be determined by interviewing the students. In order to overcome this problem of linguistic validity, the students were
asked to evaluate the suitability of the form so that students could understand it by presenting three forms of teaching that entered the course for two years. Teachers have indicated that the form is appropriate as a result of their examination. In addition, the intelligent disability students entering the lessons of the students suggested that the students Arda liked “şahin trademark” car from the students and that Nisa liked the chickens and therefore asked questions involving these objects in talks during the study period. In this direction, the forms are finalized. All interviews done were recorded through a voice recorder by taking permissions from the parents of the students. On the other hand, in the study unstructured observation type was used and some situations being interested by students, immediate responses and the behaviours on learning were recorded. However, all observations were recorded by video camera; those scenes could not have been taken in the content of the research because their family did not give permission. For this reason, all scenes saved during the study only used in order not to become data loss while data analysis. In the document analysis, work papers used in the learning process were analyzed and some passages from document data were given place in the presentation of the study.

Process
Virtual Manipulatives

In the study, some of the virtual manipulatives take place in the web site of NVLM (National Library of Virtual Manipulatives-http://nvlm.usu.edu) were used. The reasons to select some of virtual manipulatives from NVLM web site are that these manipulatives have continuous regenerating structures; they can be put on the web site after expert confirmation (Utah University NLVM Project team teaching members); systems can be used free by mathematics teachers in order to enrich the courses and systems were established on the idea that students can play while learning. In this study, while selecting the virtual manipulatives were used during the lesson time interval as 3 hours, opinions of two mathematics education experts, one special education experts and one special education teacher were taken. Mathematics education experts were asked to evaluate the appropriateness of the selected manipulatives for teaching the addition operation, to assess the suitability of the special education specialist for students with intellectual disabilities, and to assess the suitability of these manipulatives for the selected students from the special education teacher. In view of the opinions received from the evaluators, it was concluded that the selected manipulatives are appropriate for the purposes of the study. Virtual manipulative used in the study and their intended purposes can be seen at the Table 2. Study was realised within 5 lessons completely. Researchers joined to the first lesson as being only observer. By this method, it was provided that students trusted and got used to the researchers. In the next 3 lessons, practices for addition process by means of virtual manipulative and some interviews with the students were performed. And in the last lesson, interviews about the practice with the students were realised. In the study, two of the researchers in the observer role in the first lesson, and participator observer role in the other lessons.
Table 2. Manipulatives Used in the Study and their Intended Purposes

<table>
<thead>
<tr>
<th>Lesson 1</th>
<th>Lesson 2</th>
<th>Lesson 3</th>
<th>Lesson 4</th>
<th>Less. 5</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Virtual Manipulatives</strong></td>
<td><strong>Intended Purposes</strong></td>
<td><strong>Researchers only observed in the first lesson.</strong></td>
<td><strong>Interviews about the practice with students were realized.</strong></td>
<td></td>
</tr>
<tr>
<td>✓ Knows base blocks</td>
<td>✓ Generates and Separates decades</td>
<td>✓ Add the large number to small number. (Counts up)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>✓ Matches one-to-one.</td>
<td>✓ Notices a decade base block equals ten ones’ base blocks.</td>
<td>✓ Writes given process as addition process by means of manipulatives</td>
<td></td>
<td></td>
</tr>
<tr>
<td>✓ Gains number preservation.</td>
<td>✓ Constitutes a new group block by combining different blocks (Combines two sets.)</td>
<td>✓ Estimates the result of the addition process and checks the results through virtual manipulative.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>✓ Write the number formed by base blocks.</td>
<td>✓ Shows the written numbers by base blocks.</td>
<td>✓ Comprehends that addition process means combining and reproduction.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Worksheet

It frequently emphasized by many researches that well-designed worksheet provides easier comprehension positively effects the students’ behaviours for the courses (Uslu & Akgün, 2012; Kaleli Yılmaz & Güven, 2014). For this reason, worksheets were prepared in compliance with gains about the addition process and virtual manipulatives in order that computer aided virtual manipulatives can be used effectively. In the prepared worksheet, information was not presented to the students directly, it was aimed the students discover the information by comprehensible and clear instructions and reach the various results. Besides, questions on the work are enable to make practice in the computer environment and take feedback and develop operational skill. Examples of worksheets image is presented in Fig 2.

![Fig 2. Sample Images from Worksheets](image-url)
In the worksheet development phase, opinions of two mathematics education experts and two special education experts were taken. And then those worksheets were applied to another student have education in the same school. That student was observed and feedbacks were taken. After those all phases, work papers were put into final form by considering the informal interviews with teachers of intellectual disability.

**Teaching Process**

Study was carried out by using two computers allocated for the students in the school hall. During the study two computers assigned for two students named Nisa and Arda (these names are nicknames.) These two students worked on the same computer during the study. In first, virtual manipulatives were introduced to students, what they can do by those virtual manipulatives were stated and some instructions about base blocks takes place in the virtual manipulatives were made. Then work sheets were given to the students and students were direct to study with virtual manipulatives by the instructions in the work papers. And it was requested from the students to use the computers himself. Noticing the made mistakes made by the student firstly was expected and it was provided the students did not notice fault to find the result by warning through some hints (When the right answer was reached, answer was written on the right side of the paper). Student reached to the right answer was awarded by reinforce (Chocolates were given as primary reinforce; As secondary reinforce, as if I knew you could do wonders). Thinking aloud and voice response were requested from the students during the practice. Data in-depth collection was aimed by asking some questions (Why?, How?, vb.) about the operations they did both on the work papers and on virtual manipulatives.

**Data Analysis**

In the study, content analysis were used for analyzing the collected data. In the content analysis, voice records were analyzed (transcript) Observation data were added to the analyzed data. The data is then coded by the first researchers. The resulting codes are categorized according to their common characteristics. Finally, the categories are named so as to overlap with the field text, taking into consideration the features of the categories. Data from interview and observation were compared to determine whether if there is difference or not. Data were coded by first researcher and control list was constituted. Semi of made codes were checked by the second researcher and signed by “I agree / I do not agree”. According to the quantity of “I agree” or “I do not agree” results, The formula. [(Result of “I agree” / Total answer) x100] was used. Then the conformity between two researchers is determined as 84%. Güler and Taşdelen Teker (2015) suggested that the percentage of numbness should be at least 75%. In addition, some of the participants’ statements were presented through direct transfer.

**Results**

**Learning of Addition Operation**

According to informal interviews with Nisa and observation for her, the following results were taken: She could not have separated the number of objects equals 20 or less by decade or ones and fixed the corresponding values. But Nisa could have determined the corresponding values by separating the numbers by decade and ones groups through the reusable characteristic of virtual manipulatives in the second lesson. Nisa could have combined the ones groups to obtain decade groups and on the contrary, separated decade groups to ones groups. It can be said that Nisa comprehended combining ones groups to decade - separating decade to ones groups according to the dialog in the Fig 3 realized between researcher and her up to the end of the second lesson.
Before the practise, it was determined that Arda could not have comprehended concepts of ones and decades according to the observations taken in the first lesson and results from the interviews with Arda. Also, in the second lesson according to dialog (Fig 4) established between Arda and researcher about a question takes place in the work paper showed on the right side of the paper, it was seen that Arda encountered difficulties to comprehend decades and ones concepts. Then, Arda showed that questions on the work paper by helping of virtual manipulatives. And he gave right answer after this practise. Arda could have defined the numbers corresponding to a set separated to decades and ones with some similar operation through virtual manipulatives.

In the question asked on the work paper presented in the third lesson Nisa made addition by adding small number to the large number instead of adding the large number to the small one. Because Nisa said that “[21.09] by adding three to the six from … [22.00] the nine... [22.45] After the first six numbers, 1-2-3-4-5-6, she paused then added to the three numbers on the first six ones, 7-8-9” According to Nisa’ this behaviour, it was determined that Stopping after she said “six” means she added small number to the large number. Also, it was seen from this dialog that Nisa made addition process by counting method with beginning from 1.
In the fifth lesson, it can be seen that Nisa did one of the addition processes by completing to ten according to the dialog on the current dialog (Fig 5). However, this addition strategy is effective for one digit numbers addition Nisa encountered some difficulties to make mental addition process during the study.

It was determined that Nisa, who was specified she could not gain number preservation according to the informal interviews between teacher and Nisa, and performed observation, can make addition process by the numbers up to five. Hence, it was found out that she could not have gained number preservation due to the answer she gave for the question written on the work paper and in the first stage of the dialog on the right side. But it can be said that she obtained number preservation by means of virtual manipulatives according to the remained part of the same dialog (Fig 6). Moreover,
however she started to count from 1; according to the dialog, it is seen that she started to counting on
the number given first after she studied by the different activities include virtual manipulatives. So, it
means that Nisa gain the skill of making addition by adding on a number during the process.

Fig 7. Dialogue Showing that Arda Won the Number Protection with the Help of Virtual
Manipulators

At the beginning of the study, it was determined that Arda could not have gained number
preservation. And up to fourth lesson, he had been kept this situation. Hence, in the first minutes of the
fourth lesson, as seen from the dialog (Fig 7) on the right side established by Arda and researcher, It
can be said that Arda could not have gained number preservation because he answered as “six” to the
question asked on the work paper. Then, Arda gained number preservation as Nisa by operations tapes
place in the work paper were performed through virtual manipulatives; because Arda could have given
answer “10” to the question on the work paper by helping of virtual manipulatives. Especially, the
sentence of Arda as “[35.03] There are five birds and in there also there are five. [35.38] Five plus five
equals ten” shows that he can make mental addition.

At the beginning of fifth lesson, According to the dialog with Arda on the right side, Arda can
make addition process by method of “adding on” While Arda was making addition operation, he
contributed from the image he drawn on the Figure 8. As seen from the dialog, especially, Arda gave
answer easily to the questions includes his interests like car model Şahin. In the interview realised before
the study, teacher stated that Arda was interested in Şahin automobiles. After that dialog a different
question includes “Şahin car” asked to Arda as “Arda, you have eight Şahin automobiles. If I give three
Şahin automobiles more to you, how many Şahin automobile have you?” Arda answered to this
question as “11” by mental calculation in a short time. But Arda encountered difficulties to answer the
questions include different objects (For example, “You have seven flowers, if I give four flowers more
to you, how many flowers have you?”).
In the third lesson, due to the dialog with Arda, it can be understand that Arda comprehended the number “zero” Arda said “[12.10] We should write zero because there is no decade… [12.22] If there is not any decade, we should write only one digit.” Due to these speeches, we can say that Arda comprehends that zero means absence and nothing and also if there is no block on the decade block, number should be only one digit. In the same lesson, it was seen that Arda could have added both small number on the large one and large number on the small one. Adding the large number on the small number was defined by Arda with this speech: “[24.11] If we add five and six, [24.18] 1-2-3-4-5… [24.25] 6-7-8-9-10-11”. So, this statement proved that Arda can add large number on the small number.

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| [11.19] A: So, what is the result of eight plus four? | ![Fig 9. Model in which Arda Equally Groups Objects](image) |
| [11.22] A: Peki… What is the result of seven block plus five block? | |
| [11.34] A: Why twelve? | |
| [11.35] Arda: There are one decade and two ones at the every of two groups. So 8+4=12 and 7+5=12 | |
| [11.36] A: Can it be different addition operation whose sum is the same result | |
| [11.38] Arda: Of course. It can be. [by self-assuredly] | |

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**Opinions on the Use of Computer and Virtual Manipulatives**

At the beginning of the second lesson, Virtual manipulatives was introduced Nisa and instructed how they would be used. Nisa tried to make addition with virtual manipulatives by emulating the instructed movements, such as moving unit cubes, taking into the frame. But she encountered difficulty to make the similar operations by virtual manipulatives. Then, Nisa, who learned to make operation by virtual manipulatives, said “[26.11] Of course I can do again on the computer. It is pleasure.” in the dialog done up to the end of the second lesson. Her response by smiling shows that she enjoyed making operations by computer and she is willing to study by computer. In the first part of the third lesson, it was seen that Nisa took feedback when she made a mistake during the mathematics learning process by the help of virtual manipulatives and she continue the operation according to the feedbacks. According to the dialog placed on the right side of the page, it was fixed that Nisa checked her answer by seeing the right answer on the display, if she could not see the answer on the display, she automatically checked her answer by taking feedback and she noticed her mistake. On the other hand, Nisa did not regard computer as a learning tool, she used the computer as a reinforce. In a dialog established at the end of third lesson, Nisa stated that “[25.44] If we keep quiet and do what our teachers want, teacher brings us to computer and we play computer.” So, it shows that she regarded the computer as a reinforce.
A: Arda, is computer useful to learn mathematics?
Arda: Yes, it can be.
A: Why? Do you think that today’s lesson given by computer has an effect for you?
Arda: Yes, I think.
A: What was kind of effect performed?
Arda: I learned Mathematics.
A: All right, how did you learn Mathematics before?
Arda: Normally, I was learning by written.
A: What kind of things were contributed to you after learning by computer?
Arda: Mathematics on the computer is so enjoyable.
A: In the future, do you want to learn by computer?
Arda: I wish; I will be happy.

Fig 10. Dialogue Showing that Arda is a Tool in Computer Mathematics Teaching

At the beginning virtual manipulatives presentation was made to Arda and information about how the practice would be done gave to him. Also Arda tried to make operation by imitating researchers while using virtual manipulatives like Nisa. In the dialog with Arda established in the second lesson, it was determined that Arda thought the computer as a tool to learn mathematics. Besides, it can be understood from the dialog (Fig 10) that Arda enjoys and is so happy to learn Mathematics by computer.

A: Could you please show twelve by base blocks, Nisa?
Nisa: Twelve?.
A: But we request 12, do not we? Your answer is true?
Nisa: No (She is upset).
A: How do you know your result is not twelve.
Nisa: Twelve is not written. When I find the true answer, true answer is written here.

Fig 11. Dialogue Showing that Nisa has Decided According to the Turnaround She Received from the Screen to Reach the Correct Ambition

In the first part of the third lesson, it was seen that Nisa took feedback when she made a mistake during the mathematics learning process by the help of virtual manipulatives and she continue the operation according to the feedbacks. According to the dialog (Fig 11) placed on the right side of the page, it was fixed that Nisa checked her answer by seeing the right answer on the display, if she could not see the answer on the display, she automatically checked her answer by taking feedback and she noticed her mistake. On the other hand, Nisa did not regard computer as a learning tool, she used the computer as a reinforcer. In a dialog established at the end of third lesson, Nisa stated that “[25.44] If we keep quiet and do what our teachers want, teacher brings us to computer and we play computer.” So, it shows that she regarded the computer as a reinforcer.

In the fourth lesson, Nisa stated that “[11.23] Teacher explains to us everything. We show what he asked to us. But I learned the mathematics first time... [11.40] Learning Mathematics by computer is very enjoyable. [11.47]” In other saying, it was comprehended that she is happy and have first experience to learn mathematic by computer. In the fifth lesson, according to the dialog established between Nisa and researcher, she said that “[12.27] I enjoyed playing with cubes. I like to play game so much. Besides, I enjoyed to play game while studying lesson.” It was found that Nisa is glad to work with virtual manipulatives and she sees virtual manipulatives as a game or playing tool.
Due to the informal interview with Arda before the practice, it was noticed that Arda liked to spend time by computer. Arda said that he enjoyed playing games on the computer, but he had not enough experience to learn mathematics by computer. But Arda is aware that he can learn mathematics by computer; because, in a part of the dialog with Arda realised in the second lesson, Arda said that “[34.30] We learned addition process on the computer. … [34.36] We learned decades and ones”. So, this speech shows that he is aware that he learned decades and ones blocks and addition process by means of virtual manipulatives. Also, Arda stated that “[37.41] Adding by computer is very nice; because, it is so easy. Using pencil is more difficult.” Thus we can understand that Arda has an idea that making adding process by virtual manipulatives is easier than using paper and pencil to make addition.

**Individual Differences**

At the beginning of the second lesson, it was determined Nisa encountered difficulties to take the base blocks in the virtual manipulatives into frames. On the other hand, in the next lesson, it was observed that Nisa had some difficulties to take unit cubes into the frame. Also, it was seen that Nisa could not have established eye-hand coordination because her fine motor skills were not developed sufficiently. Moreover, Nisa has problem to complete the instruction in time. For example, Nisa needs rest time as 12-20 minutes averagely. If the study time is over than her rest time need, it was observed that she meets difficulties to answer the question. So, her attention span is not long time. Nisa could have performed the operation she should do by watching how other friends’ do their operations. For example, in the fifth lesson, after following Arda, Nisa said that “[21.01] I did it, my teacher! He should add four ones more”. According to her this speech, we can understand that she contributed the instruction given to Arda and she performed the learning by this way. So it means that Nisa can learn by interaction.

At the beginning of the dialog with Arda, Arda encountered problem to reach to the generalization. Because Arda stuck to the details, he cannot make generalization. Two questions were asked to Arda sequentially. Arda gave the true answer for the first question. But for second question, He gave the same answer. When why he responded to the same answer was asked to Arda, Arda had been thinking for a short time. Then Arda changed his response. Moreover, it was requested Arda to check his answer by virtual manipulatives. Then Arda could have reached the right answer quickly. Consequently, it was determined that Arda firstly stuck to the details, then, he could have reached the result by helping of virtual manipulative practices. As it was stated above, it was seen that he could have given answer quickly and cheerily for the question includes the objects interested by Arda (For example: Şahin automobile).

According to the obtained three categories from the cases of both Arda and Nisa, Findings about the reflections from the teaching addition, opinions on the use of computer and virtual manipulatives, personal characteristics were searched and the following codes were formed.
Table 3. Codes Obtained from Two Different Cases

<table>
<thead>
<tr>
<th>Category</th>
<th>Cod</th>
<th>Nisa</th>
<th>Arda</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reflections from the Teaching Addition</td>
<td>He/She comprehends number preservation.</td>
<td>●</td>
<td>●</td>
</tr>
<tr>
<td></td>
<td>He/She makes addition by “adding on” method.</td>
<td>●</td>
<td>●</td>
</tr>
<tr>
<td></td>
<td>He/She creates decade by bringing together of ones and ones by separating decades.</td>
<td></td>
<td>●</td>
</tr>
<tr>
<td></td>
<td>He/She makes mental addition process.</td>
<td>●</td>
<td></td>
</tr>
<tr>
<td></td>
<td>He/She makes addition by adding the small number on the large number.</td>
<td>●</td>
<td></td>
</tr>
<tr>
<td></td>
<td>He/She makes addition by both adding the small number on the large number and adding the large number on the small number.</td>
<td>●</td>
<td></td>
</tr>
<tr>
<td></td>
<td>He/She makes addition by completing to 10.</td>
<td>●</td>
<td></td>
</tr>
<tr>
<td></td>
<td>He/She classifies the objects by the equal quantities for making mental addition operation.</td>
<td>●</td>
<td></td>
</tr>
<tr>
<td></td>
<td>He/She gives meaning nonstandard bilateral operations (3 + 4 = 5 + 2).</td>
<td>●</td>
<td></td>
</tr>
<tr>
<td></td>
<td>He/She gives meaning “zero” number.</td>
<td>●</td>
<td></td>
</tr>
<tr>
<td>Opinions on the Use of Computer and Virtual Manipulatives</td>
<td>He/She performs virtual manipulative practises by imitation method.</td>
<td>●</td>
<td>●</td>
</tr>
<tr>
<td></td>
<td>He/She does the computer aided operation by a pleasure and happiness with a great interest.</td>
<td>●</td>
<td>●</td>
</tr>
<tr>
<td></td>
<td>He/She accepts computer as a reinforcer tool.</td>
<td>●</td>
<td></td>
</tr>
<tr>
<td></td>
<td>He/She accepts computer as a tool to learn mathematics learning.</td>
<td>●</td>
<td></td>
</tr>
<tr>
<td></td>
<td>He/She gives meaning base blocks easier by helping of virtual manipulatives.</td>
<td>●</td>
<td></td>
</tr>
<tr>
<td></td>
<td>He/She queries whether if he/she did right or not by receiving immediate feedback from the computer.</td>
<td>●</td>
<td>●</td>
</tr>
<tr>
<td></td>
<td>Integration of mathematics learning with computer is the first experience for him/her.</td>
<td>●</td>
<td></td>
</tr>
<tr>
<td></td>
<td>He/She is glad to perform by virtual manipulatives and learn mathematics by computer.</td>
<td>●</td>
<td>●</td>
</tr>
<tr>
<td></td>
<td>He/She sees the computer aided courses like a game.</td>
<td>●</td>
<td></td>
</tr>
<tr>
<td></td>
<td>He/She performs different activities about addition process by means of virtual manipulatives.</td>
<td>●</td>
<td></td>
</tr>
<tr>
<td></td>
<td>He/She thinks that making addition process is easier by using computer.</td>
<td>●</td>
<td></td>
</tr>
<tr>
<td>Personal Characteristics</td>
<td>He/She gives answer to the question about the objects he/she is interested in truer and more quickly.</td>
<td>●</td>
<td>●</td>
</tr>
<tr>
<td></td>
<td>He/She cannot provide eye-hand coordination while using computer mouse as required (fine motor skills).</td>
<td>●</td>
<td>●</td>
</tr>
<tr>
<td></td>
<td>He/She encounters difficulties to answer the questions after completing 15-20 minutes’ study, because his/her attention span is less.</td>
<td>●</td>
<td></td>
</tr>
<tr>
<td></td>
<td>He/She learns easily by interaction method.</td>
<td>●</td>
<td></td>
</tr>
<tr>
<td></td>
<td>He/She encounters difficulties to make generalization because he/she stick to the more details.</td>
<td>●</td>
<td></td>
</tr>
</tbody>
</table>

As it was determined from Table 3, both students learned to combine ones and separate decades, calculate by adding on and gained number preservation by means of virtual manipulatives. While checking the differences on the table, it can be seen that Nisa learned addition by completing to 10 and she can make addition by only adding small number on the large number. But Arda can manage both adding small number on the large number and large number on the small number. Also Arda made addition process mentally and for doing it, he classified the objects as equal quantities. Arda can give meaning to “zero” number and nonstandard bilateral operations (3 + 4 = 5 + 2). It was founded that both students performed practise on the computer by the imitation way and they enjoyed making computer aided activities. However Nisa accepts computer as a reinforcer after the lesson, Arda accepts
the computer as a tool to learn Mathematics. Regarding mathematics learning experience on the computer is new for Arda, it is the reason to accept computer as a tool to learn Mathematics. Moreover, it was fixed that both students gave meaning to and used base blocks easier by helping of virtual manipulatives. Also they enjoyed learning Mathematics by computer and study with virtual manipulatives. When differentness is evaluated, it was seen that when Nisa learned addition process by virtual manipulatives, she can notice whether is she made the mistake or not by receiving immediate feedbacks. So she sees mathematics as a game. Also, Nisa and Arda stated that using computer was the first experience to learn mathematics for them. It was determined that Arda can make different addition operations by virtual manipulatives and he emphasised that making addition by computer is easier than by using pencil and paper. If the findings related with the students were searched, it can be seen that they can more quickly answer to the questions include the subjects which they are interested in.

Nisa could not have established eye-hand coordination to use computer mouse because her fine motor skills were not developed sufficiently. She encounters difficulties to answer the questions after completing 15-20 minutes’ study, because his/her attention span is less. Also Nisa learns easily by interaction method. It was found that Arda encounters difficulties to make generalization because he/she stick to the more details.

Discussion and Conclusions

In this research performed to investigate the mathematics learning process of the students have mild intellectual disability by means of virtual manipulatives, findings obtained from two students were checked by three categories: Reflections from the teaching addition, opinions on the use of computer and virtual manipulatives, personal characteristics.

When the findings of the teaching of addition process were investigated, it was seen however students had not gained number preservation completely, they gained it during the practice. Olkun and Toluk Uçar (2012, p. 68) and Soylu (2006, p. 127) states that one-to-one correspondence between objects and numbers should be done in order to gain the number preservation. Hence, since it was recorded that students could have made one-to-one correspondence between objects and numbers during the practise, it can be said that they gained number preservation. Sufficient tangible objects should be used in order to constitute concept of number (Olkun & Toluk Uçar, 2012, p. 72). For this reason, virtual manipulatives are accepted as intangible objects (Durmuş & Karakırık, 2006) are useful for repeatability and presenting different examples and important to develop the number preservation in the addition process. Students participated to the research were determined to make calculation by counting method. Haylock and Cockburn (2014, p. 65) defined that primary school students can make addition operation by counting on the number. Tarım and Dinç Artut (2012, p. 43) remarked students can make calculation by two steps; firstly, counting then applying adding on the number. Baykul (2009, p. 150), Cengiz (2013, p. 60) and Pesen (2008, p. 132) is defined that counting on the number while addition is used as a strategy. In this context, calculating addition operation by counting on the number shows parallelism with the current researches. It was determined participated students comprehended combining and separating decades during the study. Pesen (2008, p. 105) emphasized that learning concept of decade and combining decades for being able to make addition should be adapted to the education system from the first class level. As Pesen’s study statement, students who can create decade concept can make addition operation by completing to 10. While making addition operation by Completing to 10 in this study proves that thesis. Olkun and Toluk Uçar (2012, p. 102) and Pesen (2008, p. 135) stated that students make addition by completing to 10. Obtained data about making addition operation by completing to 10 overlapped with body of literature. Making addition operation by completing to 10 can be caused that students get used to and try to take base the number near to 10. It can be related with it that students who make addition operation by completing to 10 add small number to the large number. Albayrak and Şimşek (2010, p. 242) determined that adding small number to large number is more fundamental skill and easier. Cengiz (2013, p. 60) defined that addition operation coming from large number to the small number is proper strategy for the addition operation. Finding about making addition operation by adding small number to large number was supported by the current studies.
Another finding obtained in this study is that one of the students noticed the results of the two different addition processes were equal and was sure it could be different variations. In this case, student’s well comprehension about nonstandard bilateral operation \((3 + 4 = 5 + 2)\) can be effective on structuring relational meaning of the equation symbol. Cengiz (2013, p. 60), Savaş (2005, p. 122) and Van De Walle et al. (2013, p. 149) states that to be able to make the bilateral operations which are nonstandard and their sums are equal are important to be learned addition process as students should comprehend relational meaning of equality symbol. For this reason, it can be said that students took a pointed step forward to comprehend relational meaning of the equation symbol by helping of virtual manipulatives. One of the students participated to the research performed addition process by separating the objects by classifying with equal groups. It can be said that the student, who can use this strategy can be accepted as the preparation of the quick addition (Ölkün & Toluk Uçar, 2012, p. 72) and multiplication strategies (Cengiz, 2013, p. 68; Van De Walle et al., 2013, p. 154) could have comprehended addition process by virtual manipulatives. Making addition operation by classifying groups can be evaluated as the strategy of the addition process. One of the students is aware that zero means absence and if there is not number on the tens digit, it means the number has only one digit. However, Albayrak and Şimşek (2010, p. 228) stated that awareness of “zero” meaning which is accepted to be gained in the primary school (Pesen, 2008, p. 118) is semiabstract; Soylu (2006, p. 135) declared that learning of “zero” number is more difficult than the other numbers learning and more intangible. It can be said that considering that students have mental deficiency encountered difficulties (Hunt & Vasquez, 2014) giving meaning to “zero” has great importance. As well as these skills, it was seen that when questions based the objects interest in by students were asked to the students have mental deficiency in the learning process, they response more quickly. Öztürk, Akkan, Kaplan, and Oktay (2016) and Howard et al. (2011, p. 67) stated that using the objects were enjoyed by the students and similar in the daily life makes them be happy and willing to learn and encourages them to learn. Obtained finding related with using the objects are enjoyed by students in the learning process overlaps the results provided from the current studies.

When the finding includes students’ opinions about using virtual manipulatives are investigated, it was defined that both students made practices by imitation and imitation is a tool for learning process. In the body of lecture, it was determined that individual has mental deficiency learned by imitation (Ökcün Akıçmuș & Turan, 2015). In this case, it can be stated that learning by imitation overlap with the body of lecture. It was seen that students participated to the study were interested in the performed activities by virtual manipulatives and enjoyed to make activities. Wearmouth (2009, p. 188) defined that technology usage calls attention of individuals have mental deficiency and make them be willing to learn. The statement is, which using virtual manipulatives attracted the students’ attentions and made the learning process be enjoyable, similar with the current study. One of the students saw the computer as reinforcer. To accept the computer as a reinforcer can be consider a natural situation because it is known that computer is a tool attracts the students’ interests and attentions. Doğan and Akdemir (2015) defined that when learning process is supported computer, students can digress from the education’s aims as students see the computer is a reinforcer. The male student participated to the study stated that computer was a tool to learn Mathematics but also computer is not only a game player. This finding shows that that student noticed that computer is a tool to learn Mathematics with apart from the game feature of the computer. Moreover, it was determined that students could have defined and used base blocks by virtual manipulatives. Bouck and Flanagan (2010) specified that base blocks are great tools on the addition and subtraction learning and students can use them well. Obtained finding about the using of virtual manipulatives is in accord with the results of the current study. Students used base blocks can see their true answers on the display includes virtual manipulatives. It was fixed that this situation has great important for students to take feedback. In the research performed by Doğan and Akdemir (2015) related to training supported by computer to students take special education, it was seen that taking feedback from the computer constituted positive effects on the students. In the literature, it is written that taking feedback is important for learner (Altun, 2008, p. 57; Çakırörgü, 2015). Kabaca (2016, p. 821) emphasized that if computer is used as teacher, feedback facility of the computer is very important for learning. Obtained finding related to giving immediate feedback
of the computer shows parallelism with the body of the lecture. It was determined that students participated to the study enjoyed to learn Mathematics by computer and see the Mathematics course as the game. Doğan and Akdemir (2015) stated that when the students have mental deficiency learned Mathematics by helping of computer; they considered the lessons like the game according to their research. Arslan (2006, p. 352) defined that learning Mathematics by computer of the students made them be happy. The factors like student’s happiness by learning computer aided Mathematics and considering the lesson as a game overlap with the body of literature. Experience of Mathematics learning by computer was the first for the both students. On the other hand, the boy student stated that learning Mathematics and making addition process by computer is easier than using pencil and paper to make calculation. Junge (2013, p. 210) defined that learning Mathematics by virtual manipulatives of students have mental deficiency is easier.

In the findings obtained from the student’s characteristics, it was seen that one of the students could not provide eye-hand coordination and has not fine motor skills as required. Choi and Chan (2015) define that student have mental deficiency encounter difficulties to control computer keyboard and mouse in their research. Doğan and Akdemir (2015) determine that computer aided education contributes to the fine motor skills developing of the students have mental deficiency in their research. Finding related to not be able to provide eye-hand coordination while students have mental deficiency perform operation by computer overlaps with the body of literature. It was fixed that studying time of a student participated to the research is 15-20 minutes and over than this time interval she encountered difficulties to answer the questions asked. Wearmouth (2009, p. 182) states that ideal learning time interval should be 20 minutes. It was seen that one of the students participated to the study could have provided learning by interaction method during the learning process. Avcoğlu (2012) and Howard et al. (2011, p. 475) specify that students have mental deficiency provide the learning by interaction way. In this regard, research conducted proved that students have mental deficiency encountered generalization problem (Camadan et al., 2011; Howard et al., 2011, p. 478; Hunt & Vasquez, 2014; Pierangelo & Giuliani, 2004, p. 177). Hence, in this research, the boy student was obsessed by details in the question content; therefore, he could not have reach the generalizations. In this case, the finding related to make generalization overlapped with the literature.

Findings belong to three categories obtained by cases of the both students - Reflections from the teaching addition, opinions on the use of computer and virtual manipulatives, personal characteristics- were investigated and codes for these categories were created. (See at Table 3) However, 10 codes were generated under the category of reflections from the teaching addition; 11 codes were created under the category of opinions on the use of computer and virtual manipulatives. There are 5 codes under the category of personal characteristics.

**Limitations and Suggestions**

Results of this research contains three fundamental limitations. First one is about research method. It is case study and performed with two students. Second limitation is about participator characteristics. Research was performed with the students have mild learning disability earlier and no intelligence test or brain imaging technique was applied by researchers. In the future studies, you will be able to find out in which part of brain of students learn the procedures by applying different intelligence tests and using brain imaging techniques. For eliminating these limitations in this research, using mixed method can be suggested to the researchers. On the other hand, to reach the generalization, the study should be performed with more students.
Reference


