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Some Obstacles on the Way of Constructing Triangular Inequality

Üçgen Eşitsizliğini Oluşturmada Karşılaşılan Bazı Engeller

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Abstract

This study was conducted to evaluate the activities designed to enable pupils to discover triangular inequality. The article reports the behaviors occurred while the pupils were doing these activities. It was a qualitative study utilizing participant observation, probe type questions, and worksheets. The participants were 8th grade pupils in a primary school. According to the data obtained in this research, some behaviors related to pupils' conceptions of triangle, caused problems on the way of constructing triangular inequality. These behaviors are explained and some implications for teaching are discussed in the paper.

Key Words: triangular inequality, learning geometry, primary mathematics education

Öz

Bu çalışma, öğrencilerin üçgen eşitsizliğini keşfetmeleri için oluşturulan etkinlikleri değerlendirmek için yapılmıştır. Geliştirilen bu etkinlikler üzerinde öğrenciler çalışırken ortaya çıkan davranışlar makalede rapor edilmiştir. Araştırma, katılımcı gözlem, kanıtlayıcı sorular ve bazı çalışma yapraklarının kullanıldığı nitel bir çalışmadır. Araştırmanın katılımcıları, bir ilköğretim okulunda öğrenim gören 8. sınıf öğrencileridir. Bu araştırmada elde edilen verilere göre, üçgenleri kavrama ile ilgili bazı öğrenci davranışlarının üçgen eşitsizliğini oluşturmada sorunlara yol açtığı görülmüştür. Makalede bu davranışlar açıklanmış ve öğretimsel açıdan bazı önerilere yer verilmiştir.

Anahtar Sözcükler: Üçgen, öğrenme, ilköğretim matematik eğitimi

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Introduction

In mathematics classrooms, activities are designed in order to allow pupils to construct the concept themselves or let them discover the relationships rather than to transmit the knowledge. Fox (2001) claimed that learning is an active process and knowledge is constructed rather than innate, or passively absorbed. Some researchers tell us that if we want quality learning, the learner must participate in his or her learning process. When the learners construct their own knowledge, the quality of learning can be improved. This does not mean that learners must be alone during the learning process as indicated by Orton and Frobisher (1996: 18 cited from Richards), 'students will not become active learners by accident but by design'. This design can be shaped by examining how pupils construct the knowledge and develop the concepts.

The activities that allow pupils to actively involve in their learning may be developed by analyzing the pupils' thinking ways and how they form the concepts in their minds. In mathematics education, several researchers have tried to describe how mathematical knowledge is understood and formed (Gray, Pinto, Pitta, & Tall, 1999; Hejny, 2003) by conceptualizing this process from different perspectives. Dubinsky and his colleagues describe this process as encapsulation and formulate APOS (action-process-object-schema) theory (Gray, Pinto, Pitta, & Tall, 1999). Sfard described this process in three steps: *interiorisation* of the process, then *condensation* as a squeezing of the sequence of operations into a whole, then *reification*- a qualitative change manifested by the ontological shift from operational thinking to structural thinking (Gray, Pitta, Tall, 2000). Hejny developed a model that consists of six stages: motivation, isolated models, generalization, universal models, abstraction and abstract knowledge (Hejny, 2003).

Our team designed a number of activities that would enable pupils to discover triangular inequality. In this research in order to evaluate these activities, the researchers examined the pupils' construction of mathematical knowledge and their thought processes (see Turnuklu and Yesildere, 2005). During the study it was observed that some behaviors related to pupils' conceptions of triangle, caused problems on the way of constructing triangular inequality. This article addresses and discusses these behaviors.

The triangle is one of the geometrical shapes that are introduced to children in early years. Children learn to recognize and distinguish triangles from other geometrical shapes. The triangle may be introduced to children by defining it in a traditional way or by doing manipulative activities and classification of shapes in a contemporary way. Teachers may define triangle as a three-sided and closed shape or one that has three straight lines and a closed shape (like in Turkish primary schools before 2005). Another way of defining and introducing the triangle is as follows: teacher starts with polygons and names it according to the number of sides like in Italian primary schools (Vighi, 2003) or Turkish primary schools (since 2005, after the education act). Aristotle defines a triangle as an imminent shape upon real objects. 'Euclid defines the rectilinear figures those bound by straight lines, trilateral shapes are bound by three straight lines and classifies them according to sides or angles' (Vighi, 2003: 2).

Definition of a concept can be formed in different ways in pupils' minds. According to Tall and Vinner (1981: 152), 'a personal concept definition can differ from a formal concept definition... It may be a personal reconstruction by the student of a definition'. A research study by Vighi (2003) reports on a personal meaning of the triangle. Although this research was carried out on pupils of various ages (between 7 and 11 year olds), he found that the pupils have a meaning of triangle that differs from its intuitional meaning. For a child, a triangle is equilateral or sometimes isosceles, and has one side horizontal. This research reports that almost all triangles that the pupils drew had acute angles. These results tell us what images pupils have in their minds concerning the triangle. It is possible to observe the same results in some other research studies. Although their aims were not directly to look for personal meaning of triangle, they come up with the same results (Hanibal, 1999; Marchini and Rinaldi, 2005; Oberdorf and Taylor-Fox, 1999; Sandt, 2001).

A number of studies have investigated the influence of drawing orientation in perception and conception of geometrical shapes (Marchini and Rinaldi, 2005; Orton and Frobisher, 1996). Marchini and Rinaldi (2005) investigated preconceptions of isosceles triangles of 8 years old pupils. They found some evidence that drawing orientation affect the perception. This kind of research tells us that some other factors influence pupils' conception of triangles and that certain position of a geometrical shape may affect pupils' conceptions because of orientation affect. As Singer (2001:95) states 'student recognizes the isosceles or right angled triangle only if it is in a certain position, any other position is perceived as a new learning element'.

The Current Study

The aim of this study is to investigate some pupil behaviors occurred when they are discovering triangle inequality. As a qualitative study, in order to investigate the original objective of the research, participant observation was chosen as the main data collection technique. In addition to this, probe type questions and some developed worksheets were used in order to elicit pupils' ways of thinking in constructing the concept. The main purpose of designing the worksheets was to allow pupils to discover the rule of forming a triangle considering the length of sides. The probe type questions "why", "how" or "what" were used interchangeably according to the context of the research. To capture all events during the research as much as possible, video camera and audio recorder and researcher field notes were used.

Participants

Twelve pupils (4 girls and 8 boys) participated in the study. They were 14 year-old students in a primary school (they were at 8th grade, the last class of the upper primary education) which is located in an under privileged, low socio-economic area in Turkey. According to the mathematics teacher, the pupils were all average in mathematics. The pupils had not been taught triangular inequality before, but had some basic concepts related to triangles.

Materials

In the study, 21 sticks at different lengths (between 5cm and 25cm) were labeled according to their lengths and 5 worksheets were used. Additionally, a guideline explaining the aim of the activity was also prepared. Although most of the data that were taken into account in this paper was obtained through the task that the pupils filled in the first worksheet, tasks in other worksheets were also explained in order to give an idea of the whole research process.

In the first worksheet, the pupils were requested to make triangles by using the sticks and to write down their trials. In the second worksheet, they were requested to choose 5 samples from the list and to show which ones form a triangle and which ones do not, and explain why. In the third worksheet, the pupils were asked to determine;

- The relation between the sum of the lengths of the first and second sides and the third side,
- The relation between the sum of the lengths of the first and third sides and the second side, and then,

- The relation between the sum of the lengths of the second and third sides and the first side.

In the fourth worksheet, as in the third worksheet, the pupils were requested to determine the relation of the absolute value of the difference of the lengths of the sides. In the last one, they were requested to arrive at a generalization depending upon the measurement results they learned.

Procedure

The pupils who participated in the study were divided into groups. Each group consisted of four pupils. In the beginning of the study, the guideline and the sticks were given to the groups. Then, the five worksheets were given subsequently. The researcher and the mathematics teacher took turns in helping the pupils to understand the activity and eliciting the pupils' thinking process. A video camera and audio recorder recorded the work of each group.

Data Analysis

Each videotape was completely transcribed into verbal data (accompanied with the actions of pupils). In transcribing the conversations from the videotapes, the non-verbal interactions between pupil-pupil, pupil-researcher and pupil-material were also described in detail. Audiotapes were also fully transcribed and supported by the researcher's field notes for describing the non-verbal interactions. The following results were obtained after the interpretation of the content and context of observations of the pupils.

Results and Discussions

According to the data, there were similarities among the behaviors of the pupils in discovering the relationship among the sides of a triangle. General points are discussed taking those similarities into consideration. At the beginning of the tasks, the pupils had triangle shape in their minds. They had previous knowledge related to triangles, but until their classroom activities, they had not thought what the conditions were to form a triangle related to its sides. They knew a triangle has three sides and it is a closed shape.

The participants spent most of their time to complete the first worksheet. Taking selections that were made by the pupils into consideration, it can be seen that they especially chose the sticks with lengths close to each other. For example one group used 5, 6, 7 and 8, 9, 10 and 11, 12, 13 in their first three attempts. Then they continued the same way. The rest of the groups worked in the same manner. It is difficult to explain the reasons of these kinds of behaviors. In some cases, they reveal their reasoning in the conversations between pupils and the researcher as in the following conversation extract (the descriptions of classroom context are given in italics).

[1]: (*The group is working together and they are choosing sticks in order to make a triangle).*

[2] Nur: Let's take 10, 9, and 13. *(She arranges the sticks in order to make a triangle. The end points of the sticks are not touching. She cannot manage to form it).*

[3] Nur: It does not form a triangle.

[4] Researcher: How did you reach that conclusion?

[5] Çağlar: Because they are not touching *(he points to the sticks' end points).*

[6] Nur: *(She is replying the researcher's question)* These must be equal. *(while pointing two sides).*

[7] Researcher: *(by pointing to two sides)* Do these have to be equal? This and that one? If you make a different one, does it not form a triangle?

[8] Nur: But this time its base does not fit. *(she means that a base must horizontal according to her position).*

[9] Çağlar: It does not form a triangle.

Researcher [10]: Why do you think it does not form a triangle?

[11] Nur: Because these right sides (*by pointing at two sides*) are not equivalent.

(Group 2, from videotape)

In [6], Nur has an image of triangle that has two equal sides. For this pupil (in [8]) a triangle is isosceles and has one side horizontal. Çağlar [5] knows a triangle is a closed shape and he agrees with his friend's view.

In another case, at one point during the task, the researcher asked "why are you choosing these sticks" by referring to the nearest sticks. Two pupils replied as;

Orçun: "All are the same (*he is referring to the triangles' sides*) and different triangles also exist. All these are different (*referring to the sticks*) now we are choosing one and we cannot form an equilateral triangle by these, can we? So, we try to put them in order (*he is referring to the sides*)".

Gökmen: "Now we did like that. Because they must look like equilateral triangles and the side lengths must be sequential (*he is referring the lengths of sticks they used*)".

The pupils have an image associated with an equilateral or isosceles triangle standing on horizontal base so that they tend to choose the nearest lengths of the sticks. Additionally, in the task, the pupils were requested to write down the lengths of sticks which do not form a triangle. At the beginning of the task, some groups formed triangles in their every attempt. Because, as mentioned above, they made triangles with the small difference in the sides' length and so their trials ended successfully. After a while, the pupils realized that they also have to write down unsuccessful trials. Then they start to choose the sticks which they thought did not form a triangle. They produced a hypothesis, based on their image of a triangle, about which lengths do not form a triangle. According to their reasoning, if the sides' lengths have large differences, it is not possible to form a triangle. The pupils' view can be seen in the following conversation extract more clearly.

[12] Researcher: If you do not want to form a triangle which sticks do you choose?

[13] Erdinç: For example 1-25-15.

[14] Researcher: Let's try them. (*They are trying to make a triangle, at the end they did not manage to form it. Then, they noted the lengths as an example of not forming a triangle.)*

[15] Researcher: Tell me why they did not form a triangle?

[16] Erdinç and Aytaç (*they reply together*): Because, the lengths are different.

[17] Researcher (*by pointing to the lengths of the sticks which the pupils used to make a triangle*): These are also different.

[18] Erdinç: They are sequential (*referring to the lengths of the sticks which the researcher pointed*) but, those are not. One from the beginning and one from the end (*referring to the sticks which formed a triangle*).

[19] Aytaç: But these are the nearest lengths (*replying the researcher's question*).

(Group 3, from audiotape)

Some pupils tend to form triangles in a certain position like the one in Figure 1. The image of the triangle that the pupils have sits on a horizontal base, which explains the difficulties pupils' have in forming different types of triangles. They did not accept different positions even though they formed a triangle. They said, and noted; "this does not form a triangle".

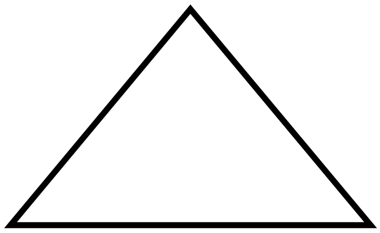

Figure 1. The image of a triangle

Some pupils tend to form a triangle without placing the sticks end to end. This appeared in two forms like those in Figure 2 and Figure 3.

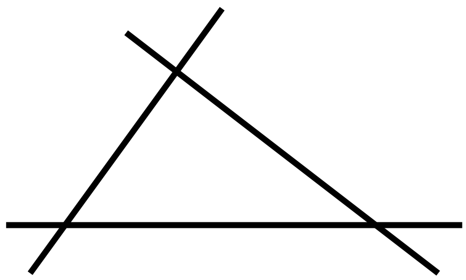

Figure 2. A sample that pupils called triangle (I)

In Figure 2, the pupils did not take into account the lengths of the sticks that they used in forming this kind of triangle. At the end of the process, however, they noted the lengths that were labeled on the sticks themselves. For example, in one case, the group used 12, 13, 15 and formed a shape like that shown in Figure 2, and they recorded that the sticks formed a triangle by these lengths.

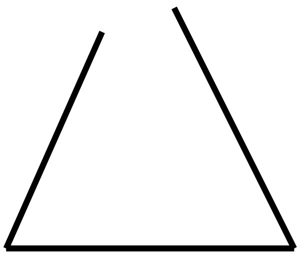

Figure 3. A sample that pupils called triangle (II)

The pupils did not care about constructing a closed shape like that in Figure 3. They thought it was a closed shape, but it is not, even though it looks like a closed shape. Although the pupils formed a shape that looked like a triangle, whereas it was not, they recorded it as a triangle. Almost in all groups these kinds of similar behaviors occurred.

Another type of behavior we come up during the tasks was that the pupils selected the sticks that could possibly form a triangle in terms of their lengths, but the pupils could not manage to form a triangle. So, they came to the conclusion that it was not possible to make a triangle by these lengths. They did not change the angles in order to fit the sides, or change the original position of the shape. When faced with this kind of situation, no one attempted to change the position of the sticks, even though the pupils were working in groups.

As it is seen above in the presented data, some obstacles were faced on the way of completing the tasks. These were related to conceptions and/or image of triangle in the pupils' minds.

Four major patterns observed when forming triangles:

1- Tendency to form only equilateral or isosceles triangles.

2- Tendency to form triangles in a certain position.

3- Tendency to form triangles without fitting the edge end to end.

4- Failing to manipulate the material in order to form triangle.

All these behaviors described above affected the process of forming different triangles, which in turn, affected the possibility of reaching the rule of triangular inequality. For example in the last worksheet when asked to write their generalization related to forming a triangle, some of the pupils wrote the followings:

"I think when the numbers are consecutive they form a triangle"

"The absolute value of the difference of the lengths of the sides must be 1"

Additionally, some of the pupils, who worked on the lengths of the sides which cannot form a triangle, did not manage to reach a conclusion, because they could not reach a pattern that may allow them to see the generalization.

Conclusions and Implications

In the primary schools, the triangle is defined as a closed shape with three straight lines. In later years of primary school, pupils are taught about triangle as having much more properties than its definition. They are taught lots of other properties of triangle and relations with other geometrical shapes. They learn any three straight lines do not form a triangle and there are some properties between angles and lengths of the sides. On the way of learning these concepts and their properties, some factors such as pupil's pre-conceptions, perceptions and the way they were taught may affect this process.

In this paper, we discussed some mathematical behaviors of 14 year-old pupils which were displayed when working on some activities related to triangular inequality. In this research, it was believed to be beneficial for pupils to be involved in their learning actively by building triangle model with the sticks; however the results indicated not exactly what was expected.

These behaviors sometimes created obstacles, by leading pupils to different directions in the way of our main target. Especially, a certain triangle image in pupils' mind; equilateral or isosceles triangles affected pupils' strategies that they used when they were forming triangles and their reasoning during the activities. These results are consistent with Vighi's (2003) and Singer's (2001). Although the participants of this study were older than the ones in Vighi's study, similar results were obtained. Another interesting result is that the pupils did not name a shape as a triangle when it was shown in unfamiliar positions, as Singer (2001) found out.

When all these mathematical behaviors are taken into account, it is better to reorganize the activities that were used in this study. For example, in the stage of forming triangles it is better to give pupils drawn triangles instead of asking them to make their own triangles. These triangles must display different types and also have strange positions. Then, pupils should be allowed to discover the relations between the lengths of sides through manipulative activities.

This way, it is possible to allow pupils to examine different types of triangles and, so, they may reach correct conclusions on the way of discovering triangle inequality.

In closing, it might be a good idea to cite Clements and his colleagues' words, 'one tenet of teaching for understanding is that one should build on a child's existing ideas' (Clements, Swaminathan, Hannibal, Sarama, 1999:192). If we restate this idea from a different perspective, we may say that if we don't teach according to pupils' existing ideas or prepare tasks accordingly, they may have trouble understanding or completing the tasks successfully. On the other hand, it may not be possible to know a child's existing ideas before teaching starts. In this case, teacher should use probing questions to learn about the child's mathematical knowledge and level of understanding so that s/he can adjust his/her teaching in accordance.

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