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## Investigation of Structural Features of Examples Which Textbook Located and Lecturers' Preferred in Calculus

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## Abstract

The aim of this study is to analyse the structural features of examples which textbook located and lecturers' choosed in teaching of engineering calculus. The study had interpretivist paradigm in qualitative research approach and the data collection process was conducted through content analysis method. The course content of calculus, which are lectured by different instructors in engineering departments, are followed during a semester within the context of the study. Examples in the textbooks and the lecture notes are analyzed with document analysis method based on their structural features of representation, language and knowledge. Besides, data are presented with descriptive statistics method. Semi-structured interviews are conducted to detect any possible components affecting lecturers' exemplification behavior, and the records are interpreted by using the inferential content analysis. The findings show that the examples of both textbooks and lecture notes have a formal language and procedural knowledge. It is also found that lecturers, unlike the content of textbooks, use more algebric representations than graphical ones. The results of the study indicate that the structural features of examples which were choosed by lecturers and which were located in textbook are similar. Besides epistemological belief, the components of the teaching environment and the type of used sources have a significant role affecting the choices of the lecturers. It has been made some suggestions for authors and researchers, which may contribute to the teaching practice for further studies.

## Keywords

Exemplification process Engineering calculus Lecturer preference Textbook

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## Introduction

Many different teaching theories are focused on the interaction between teacher, student and knowledge, as the three components, for explaining the efficiency of teaching-learning process. In particular, Theory of Didactical Situation is drawn attention to the role of teachers as an organizator to which knowledge to be taught by students (Brousseau, 1986). In the process of acquiring knowledge to be taught from the scientific knowledge (Didactical Transposition Theory), the teacher and the student are not in the position of making decisions on their own (Chevallard, 1985). The official

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framework of the content to be taught in the teaching environment is restricted by such sources as the curriculum and textbooks (external didactical transposition); lecturers may decide how to use these sources for achieving instructional objectives. In didactical transposition theory, the noosphere describes a small world which contain institution and persons who will the reorganize the knowledge and two significant elements of the noosphere are textbook and teachers (Chevallard, 1992). Due to the fact that the textbooks are regular sets of knowledge describing how the formerly-arranged lectures, activities, examples and practices should be presented to the teaching environment, they are also of great significance in terms of the mathematics courses to be given in the primary and secondary education levels (Thomson & Fleming, 2004). On the other hand, textbooks can be considered as a sort of guidance for teachers and students on account of the fact that they provide the students with their own knowledge and allow the teachers to have the chance to control the educational process at the higher education level (Brandstrom, 2005). Different from the primary education level, the students in the mathematics classes of higher educational level are free to choose the materials and textbooks they are going to follow. The knowledge in textbooks reaches the students through the filtering process performed by the teacher. Indeed, while the lecturers focus on certain topics more by taking their own teaching methods as references, they may touch on some other topics superficially. Undoubtedly, the messages conveyed through the textbooks and the way the students use and interpret these messages, which also known as internal didactic transposition process (from the knowledge to be taught to the knowledge to be learnt), are the study topics that require to be greatly focused on (Raman, 2004). The interaction between knowledge, teacher and students were evaluated in various studies in the field of mathematics education that takes into consideration the reflections of these study topics on the teaching practices (Mesa, 2004; Lithner, 2004; Rowland, 2008; Zodik & Zaslavsky, 2008).

It is necessary to detailed analysis the examples which the teachers introduce to the class in order to explain the process of internal didactic transposition. The examples, taking part in textbooks and being used for teaching process, are important for concretization and internalization of an abstract concept (Rowland, 2008), because the equivalent of definition, theorem and proofs which takes part in the context of instruction can be summarized with the help of examples. So, a particular reflection of this rule is taken as a reference to teach a general rule. Indeed, the role of example would be varying as "practice-oriented" or "generalization'" according to teachers' preferences (Watson & Mason, 2002). Yet, what must be kept in mind in the educational process is that the messages conveyed through the textbooks or by students are not indicative on their own, and that there is an interaction between these components. The aim of this study is to characterize the types of examples provided by the textbooks and lecturers, also to determine the situations affecting the example choices of the lecturers in the process of teaching engineering mathematics. This study is of great importance in terms of the fact that it analyzes the structural features of examples which textbook located and lecturers' choosed in teaching of undergraduate mathematics.

#### **Theoritical Framework**

Either textbooks or teachers make use of the examples for the subject involved to not only put forward the equivalent of the mathematical knowledge through practice but also to make sense of the theoretical nature of the knowledge, such as generalization, proof and association (Lithner, 2004). For this reason, it is important to determine the characteristics of the examples used in the teaching-learning process. In the sub-titles given below, a theoretical framework regarding the importance and the role of the examples in mathematic didactics will be presented through the component, such as the source of knowledge and the teacher, respectively.

#### The Role of Examples in Mathematics Education

Examples have a central role in the process of the development and teaching of mathematics as a discipline (Bills et al., 2006). Generalization, abstraction and conceptualization required in the process of advanced mathematical thinking along with proof, argumentation and analogies that contribute to the development of mathematical theory are the outputs to be obtained with the help of

the use of examples in mathematic didactics (Dubinsky, 1991; Sandefur, Mason, Stylianides, & Watson, 2013). The examples are not only used for putting forward the equivalent of the mathematical knowledge through practice but they also contain within the characteristics of the theoretical nature of knowledge. For instance, while the counter-examples we give in our classes explain that a mathematical judgement may not be valid in every case, they contribute to the theoretical nature of the mathematical concepts with their "generalizability" role. It is possible to encounter many studies that establish the theoretical framework upon 'exemplification' in the researches conducted on the education of mathematics (Rowland, 2008; Zodik & Zaslavsky, 2008). In these studies, there have been numerous definitions coinciding with one another for the concept of "exemplification", some of which are: a particular presentation selected to discover or describe a general principle (Chick, 2009), a material used to explain the mathematical knowledge (Bills et al., 2006), a sort of practice technique that enables a conceptual communication to be established between the student and the teacher (Peled & Zaslavsky, 1997), and the content that allows what is general to be illustrated within what is particular (Mason & Pimm, 1984). However, among these, the one regarded as the most valid and practical is by Watson and Mason (2002), and this definition is as follows: anything used as raw material for generalising, including intuiting relationships and inductive reasoning; illustrating concepts and principles or motivates a particular topic in mathematics; and particular solutions where several are possible and practising technique. Considering the content of the definitions above, the basic criteria that are used in common and that can be characterized as the key words indicate that examples are summative, descriptive and/or reflective presentations, contents or systems. There are various concepts used as if they were synonymous with the concept of example, yet, they have differences in essence. Two of them are exercises and or problem situations. Although it is difficult to distinguish such concepts from one another conclusively, a mathematical task is an exercise to an individual learner if, due to the individual's experience, the learner knows what sequence of mathematical actions should be applied to achieve the task, while the problem situation can be useful in putting the information into action in incomplete tasks, formulating an appropriate sequence of actions or strategy and applying the strategy to produce a solution (Powell et al., 2009). Examples can be beneficial in arousing curiosity in an individual as well as exercises and problem situations for implementing the teaching targets through scenarios (Chick, 2009).

#### The Structural Features of Examples in Mathematics Textbooks

It has been focused on the various evaluation characteristics such as type of mathematical knowledge and reasoning, used representations in the study that analyzes exemplification process in textbooks and lecture notes (Lithner, 2004; Patterson & Norwood, 2004; Rowland, 2008). In one of the studies, Lithner (2004) evaluated the exercises within the calculus textbooks according to the types of reasoning they contained within. The study suggested that almost 70% of the contents within the textbook had required algorithmic reasoning, and that they could be solved through similar routine operations (ibid). While Mesa (2004) states the fact that the source used in the mathematics of higher education is of importance for the quality of educational outputs, also he draws the attention to the fact that the contents requiring creative thinking are limited. The necessity to improve the teaching contents in order to eliminate such limitations and difficulties confronted in the mathematics of higher education was first articulated through the reform approach known as the Calculus Reform Movement (Hughes-Hallett, 1991). The reformers stating that the textbooks needed to be prepared and arranged over again in the way that they would support competences such as conceptual understanding, relational thinking and modelling (Hughes-Hallett, 1991). For the acquisition of these competences, it has been proposed that the contents expressible through multiple representations based on the interdisciplinary relations be applied (Ostebee & Zorn, 1997). Raman (2004), investigated how the epistemological messages conveyed through the calculus textbooks varied, and he also detected several contradictory contents in the examples of the matter of consistency in the textbooks, considering the school level (high school or college). Comparing the primary education mathematic course books of some countries in Europe within the context of the examples given within them, Bierhoff (1996) determined that the judgements contained in the examples of the English course books were rather poor in terms of pedagogical aspects (cognitive demands level) when compared with German and Swiss course books (as cited in Rowland, 2008). It is important to know why the teachers prefer which examples besides the structure of examples in the textbook. Because, teachers play vital role in the process of acquiring the teaching contents from the students within the textbooks (Brousseau, 1986). Many studies have drawn the attention to the fact that teacher/lecturer practice is the deterministic component to understand the charecteristic of examples used in the classroom (Chick, 2009). Since the lecturers' behaviours in giving examples in particular will be characterized in this study,, the literature focusing on the teacher practice will be shared among the researches themed, "examples in mathematics education".

#### **Example Preferences of Mathematics Teacher**

Teachers are responsible for the examples they prefer to use in the classroom, because while, on one hand, the preferred example or the way it is used in the classroom may facilitate the learning process, on the other hand, it may makes the structured knowledge more complicated. In order to increase the quality of educational outputs, it is necessary to prefer certain goal-oriented examples with higher cognitive level (Bills et al., 2006). At this point, it is important to determine the characteristics of the examples the teachers prefer, their purpose in using examples, the way they present their examples and the references they benefit from in the process of exemplification. Although, there is no study reviewing the structure of examples preferred by the teachers in related literature, teachers' exemplification behaviors and difficulties while generating examples were evaluated in some studies. Zaslavsky and Zoddik (2007) state that teachers' process in selecting examples is not a random act and that there are several pedagogic and epistemological factors hidden within this process, which need to be revealed. Whereas epistemological factors dealing with the nature and scope of scientific knowledge; pedagogical factors is concerned with usage of this knowledge in teaching-learning processes (Schommer, Duel, & Huter, 2005). For instance, although there are different definitions of continuity concept in different textbooks (formal versus informal), teachers' beliefs about practicality of this definition is related with epistemological factor and teachers' knowledge about how to use this definition in classroom is evaluated from a pedagogical point of view (Raman, 2004). The study of Alcock and Inglis (2008) also showed that the same example can be used differently by two mathematics educator; one of them uses the example to reach a general conclusion while the other uses it to examine a particular case in detail. In some other studies, the ability of the teachers to select the appropriate example is associated with the pedagogical content knowledge (Chick, 2009). The circumstances affecting the teachers' example preferences and the researches regarding the qualities of the preferred examples are restricted. In one of these studies, Rowland (2008) tried to ascertain the hardships experienced by the pre-service mathematics teachers in their exemplification process at primary education level. The results of the study suggested that the pre-service teachers went through similar challenges with respect to the selection of appropriate examples during teaching, the use of spatial representations compared to symbolics and the preference of the right example to perform an educational goal (ibid). According to Zodik and Zaslavsky (2008), what students really want to see in an example, in addition to the pedagogical content knowledge of the teachers, in other words, the epistemological beliefs also affect the behaviour of exemplification. Hence, the elaboration in the preference of examples is considered to be an important indicator of expertise (Rowland, 2008).

It is thought that the teachers' example preferences and tendencies in the classroom also affect the students' tendencies in the subject (Kendal, 2002). In one of the studies supporting this inference, Patterson and Norwood (2004) researched into the effect of teacher beliefs on student beliefs as regards the use of representations, and the results showed that the students of the teachers who had used multiple representations while teaching a concept also tended to use similar examples and representations. At this point, the teaching practices of teachers and the experiences of their students are the other respects that affect the example preference (Zaslavsky & Zodik, 2008). Rowland, Thwaites and Huckstep (2003) list the three factors in the inefficient example preferences of primary education mathematics teachers as follows: (1) The fact that they prefer the examples that obscure the role of variables, (2) Heir carelessness in selecting the numbers to describe certain arithmetical rules, (3) The fact that they create/produce haphazard examples while they are able to make more elaborate choices. The reason why the examples are given by the teachers and their interpretations by the students sometimes differ. For example, it can be thought that while the teacher is using an example to explain a general situation or a principle, the student may use that example only under certain conditions (Mason & Pimm, 1994), which means that the message given by the teacher and the message received by the student do not coincide with each other. In this case, such an output indicates the fact that educational targets have not been reached in full. Besides, the lecturers have used the available examples in textbook instead of generating new examples suitable for the target group and this has been interpreted as a lack in the literature (Zaslavsky & Zodik, 2007; Chick, 2009).

When the above-mentioned studies and those included in the field of mathematics education are analyzed, it is seen that the researches dealing with the teachers' example preferences and their behaviours in exemplification generally remain on the primary and secondary education levels. In the related literature, it has been ascertained that various researches aiming at the knowledge and skills of the students in generating examples and using the counter-examples as a proof argument in the mathematic classrooms of the undergraduate level have been performed (Peled & Zaslavsky, 1997; Watson & Mason, 2002; Sandefur et al., 2013). Hovewer, it is important to evaluate that which type of example used and that what is the role of textbook in the exemplification process to understand the message given by mathematics lecturers.Within this context, the answers to the following three research questions are sought for: (*a*) What are the structural features of the examples given in the textbooks in engineering calculus? (*c*) What effect does a textbook have on the exemplification behaviour of the lecturers in engineering calculus?

#### Method

In this study with a paradigm of qualitative interpretation, the content analysis was benefited from as data collection technique. The process in which the examples contained in the textbook and course notes were evaluated according to their characteristics was termed as the content analysis. In order to analyze the teaching contents used in the teaching process, the mathematics courses given in different engineering curriculums during the fall semester in 2013-2014 academic year were focused on. The reason why this study is conducted in engineering mathematics is that most of the engineering students have diffuculty in mathematic classes which are known as the language of engineering (Güner & Çomak, 2011) and this diffuculty is originated from the inconsistencies between the instruction contents and student expectations (Coutis, Farrell, & Pettet, 1999; Felszeghy, 2010). The research was carried out in an engineering faculty placed on the top list according to the preferences made in the transition exam to higher education. The common courses included in several engineering programs/curriculums are Calculus, Linear Algebra and Differential Equations. The fundamental mathematics course provided in the first year of engineering curriculums and usually abbreviated with the code MAT101 is Calculus. In Turkey, calculus called by different names, such as Higher Mathematics, Mathematics-1 or Fundamental Mathematics and this course is generally given by the lecturers of mathematics department as a service course. The primary topics included within the course scope (for single variable functions) are limit, derivative and integral, and these subjects are similar in the majority of the engineering faculties. In the weekly course schedule, four hours were spared for the "Calculus" course, and the course contents of the first 9 weeks were followed up within the scope of the study.

#### **Data Collection Process**

The main data sources in this study are the textbooks, course notes and the views of the lecturers. In the data collection process, the qualitative research techniques, such as participant observation, interview and document analysis, were benefited from. In this way, the reflections of the interaction between the knowledge and the lecturer in engineering mathematics, which are the two basic constituents of teaching environments (Brousseau, 1986), were evaluated.

*Features of the Lecturers:* While the characteristics of the examples preferred by the lecturers during the lesson were being evaluated, interviews were performed with five lecturers in total who had been teaching calculusin the departments of electronics, computing/computer, industry, aviation and astronautics in engineering faculties, and the course notes of the lecturers were used in that process. The lecturers participated in the study on a volunteer basis. They had teaching experiences varying between 8-15 years, with at least a doctorate degree. One of the lecturers who took part in the study was an assistant professor, whereas three of them were associate professors and the other was a professor. The classes were performed through the method of direct-instruction and question-answer (qa) technique in the amphitheaters. Two out of five lecturers made use of reflections in the teaching-learning process, while the other three used no educational technology whatsoever. The lecturers presented the source, the characteristics of which was given under the heading "selection of the textbook", reviewed in the scope of the study among the basic sources in the course schedules. Some of the lecturers stated that they benefited from other calculus textbooks or the notes they compiled during the class hours.

The data obtained over the lecturers are the course notes and interview recordings. The course notes were obtained not from the documents of the students but from their notebooks in order to clearly see the reflection of the given information on the classroom environment. The course notes were provided by the students who had fully taken part in all the classes and had documented the information given on the board in a completely written-form. Observations were made in the process of determining the students in question. Semi-structured interviews were used to determine the types of examples preferred in the classroom by the lecturers and to ascertain the other constituents that could affect their behaviours in exemplification. It was also tried to be determined in what way the behaviour of exemplification mentioned in the interview questions raised to the lecturers was affected by the source used in the educational process, the education group for which the lesson was taught, the targets of the course and the educational approach.

Selection of the Textbook: In the literature, it is stated that one of the variables to affect the lecturers' exemplification behaviours could be the textbook itself (Mesa, 2004). Within the scope of this study, the behaviours of the lecturers in the exemplification of engineering mathematics will be evaluated through the textbooks, and thus, it is of significance how the textbook to be evaluated at this point will be selected. There are numerous study books to be used for the fundamental mathematics course also known as "Calculus" in the mathematics of higher education. These books usually go under the name of their authors; six of them can be listed as follows: Adams, Apostol, Edwards & Penney, Larson, Stewart and Thomas Calculus. The book with its original name, "Calculus, Early Transcendentals", which was prepared by Henry Edwards and David Penney among the calculus sources cited above, has the Turkish version that is often preferred. The 5th edition of this book was translated into Turkish by a committee specialized in their own field, with the approval of the authors in 2008. Of all the books to be used in engineering mathematics, the reasons why "Edwards and Penney-Calculus" is that this book is the main textbook preferred in the engineering departments involved for the purpose of complying with the data to be obtained from the instructive dimension of the study. Additionally, comments in he relevant literature also influenced the choice of Edwards and Penney-Calculus: (a) It is particularly prepared for Science-Engineering faculties and the students of higher education, (b) It is one of the main sources to be used in the mathematics of higher education, the Turkish translation of which is already in use and (c) It is also one of the final editions despite the fact that it was used for a particular time period in the educational practice (Jungic, Kent & Menz, 2013; Edwards & Penney, 2008). In the succeeding sections of this study, the short expression, "The Calculus Textbook", was attributed to "Edwards and Penney-Calculus" textbook used in engineering mathematics. Within the textbook are the contents in compliance with the traditionalteaching cycle (Definition  $\rightarrow$  Theorem  $\rightarrow$  Proof  $\rightarrow$  Practices) along with the technology-aided tasks and project practices. In the Calculus textbook, the subjects like limit, derivative and integral have been described through the single variable functions, and the book consists of 7 units and 474 pages.

#### Data Analysis

In this study, there are two types of data, which are written documents and interviews. The written documents are composed of the textbook and the course notes, while the interviews consist of the sound recordings performed together with the lecturers. In the data analysis process, the consistency of the examples contained within the lecture notes with those in the textbook is primarily evaluated. Afterwards, the examples contained in the written documents are classified according to their characteristics (structural features). The evaluation characteristics used in the research are the representation, language and knowledge. In deciding on these structures, it has been benefited from studies in related literature. In this context, assessment framework of Kendal and Stacey (2003) has been used in data related to representation and language; in the process of the examination of the examples according to the knowledge structure, operation steps which Hiebert and Lefevre (1986) take care for mathematical understanding has been followed.

*Representation:* The reviewed teaching contents were compared according to the types of representations used in the presentation of the examples. While the representations used in the example presentation were being analyzed, each example within the teaching contents was scrutinized at full length, and the representative approach of the example that was required to be explained on the concerned topic was also taken into consideration. At this point, the content assessment framework by Kendal and Stacey (2003) was taken into account, and which of those representations (those that affected the meaning (internal) and that were used in a presentation (external)) the example was characterized for was investigated. For instance, an example highlighting the tangent-slope relationship regarding the derivatives was evaluated under the category of graphical representation, whereas another example emphasizing the instantaneous rate of change was evaluated under the category of numerical representation. Yet, not every example could be characterized by structures of internal representations. In this case, the external representations were taken into consideration, and the equations, graphics, tables or verbal expressions used in the presentation of the example were evaluated under the category of algebraic, graphical, numerical and verbal representations.

Language: The topics in the Calculus course, by axiomatic nature of the mathematical knowledge, is the basic course of higher education that can be associated with the daily life problems at most besides the fact that it has an algebraic language (Hughes-Hallet, 1991; Felszeghy, 2010). At this point, there are also differences among the languages used in teaching contents. Kendal and Stacey (2003), while classifying used language in mathematics topics, utilized contexts in the contents of the examples and collected problems under these two categories: "contextual-structured" or "formal-structured". The contents in which an example on a certain topic in the Calculus course is presented in association with real-world problem like sciences, economy or engineering disciplines are evaluated under the "contextual-structured" category, whereas the contents in which examples are presented through a purely mathematical language are evaluated under the "formal-structured" category. The use of an algebraic language in the statement of example does not make it formal example (Kendal, 2002), because the aim of this classification is determining used context, not used syntax.

*Knowledge:* What type of knowledge the examples were used to support and their functions in the educational process were evaluated, as well. Hiebert and Lefevre (1986) explain that the mathematical knowledge may be evaluated on conceptual or procedural aspects. The occasions which include deep and rich mental networks regarding the concept are called conceptual, and the ones where the result is reached through following a routine of rules is analyzed under procedural knowledge. In this process, the root of the question within the problem phrase was also taken into account in the study. Accordingly; the examples which considered conceptual relations and which comprised inquiry-based expressions in the problem phrase (research, examine, discuss, interprate etc.) were evaluated in the category of "conceptual knowledge", whereas the examples which considered routine solutions and which used for repeating and practising the previous informationin

the problem phrase (find out, calculate, solve etc.), were evaluated in the category of "procedural knowledge". While the functioning of the examples in the educational process was being evaluated, the theory practice balance was also taken into account. In this sense, if the knowledge within the examples had a purpose of identifying and generalizing a mathematical object or proving a rule or a theorem, it was evaluated in the category of "theory-based"; yet, if it exemplified the reflection of the knowledge in practice, it was evaluated in the category of "practice-based". Within the scope of this study, how an example is evaluated according to its characteristics is summarized in Figure 1. In this example, a problem situation that features the derivative which is related to the daily life (contextual language) has been approached with the help of verbal expression (verbal representation), and in the content of solution it is intended to research the possible dimensions by relating the geometrical interpretation of derivative with the optimization techniques (conceptual knowledge/practice-based).

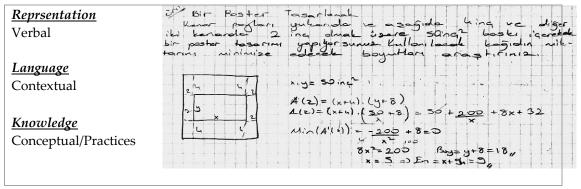


Figure 1. An Example Evaluated in Accordance with Its Structural Feature

Analysis of the Interview Data: The interview data documented in written form were analyzed through a descriptive approach. In the interpretation of the interview data, the method of inferential content analysis was used (Hsieh & Shannon, 2005). In order to promote the data obtained from the document analysis and to be able to characterize the lecturers' tendencies in exemplification, citations were taken by sticking to the original form of the data, and they were shared under the related title.

Validity and Reliability: The process of obtaining the study data took place within a year's period. The researcher investigated the engineering schedules in different universities to perform the pre-field research within the context of targets and acquisitions at undergraduate mathematics classes. The university where this study was conducted was determined within this scope. The researcher took part as a participant observer in the classes of some of the lecturers who participated in this study. The content validity was referred to for evaluating whether or not the lecture notes obtained from the students's notebooks literally reflected the student preferences. In this context, the notebooks of more than one student were compared for each lecturer, and the most comprehensive student notebook was decided to be the lecturer's course notes. Separately, it was aimed that the interpretation validity be ensured by taking into consideration the same topics involved. In the process of the classification of the examples in the textbook and in the lecture notes according to their structural features, two experts along with a researcher were asked to assist in this matter. Inter-rater reliability analysis was performed to examine the consistency in the classification process. In this context, the coeffcient of the consistences of the three encoders using the same evaluation rubric for the randomlyselected 40 examples were: .82, .87 and .92; and these result ssuggest that the classification performed is quite reliable.

#### Results

Before the structural features of examples are subjected to the content analysis, it is important to know the distribution of the examples according to the topics involved. In this context, the distribution of the examples contained in the calculus textbook (Edward & Penney, 2008) and the lecture notes evaluated within the scope of the study according to the topics involved are presented in Table 1 in terms of percentage and frequency.

	Resources				Consistency	
	Textbook		Lecture Notes			
Topics	(f)	(%)	( <i>f</i> )	(%)	( <i>f</i> )	(%)
Limit (and Continuity)	40	16	27	13	12	44
Derivative (and its Applications)	117	47	84	39	41	49
Integral (and its Applications)	94	37	104	48	66	63
Total	251	100	215	100	119	55

**Table 1.** The Frequence and Percentage Distribution of the Examples According to the Topics and Resources

The distribution of the examples contained in the textbook and preferred by the lecturers varies according to the topic. Whereas the total number of examples in the textbook was more than those in the lecture notes (the notes presented in the classroom by the lecturer), there were more examples spared for the derivatives in the textbook and for the integral in the lecture notes. In both the textbook and the lecture notes, there are examples included with the least percentage regarding limit and continuity (consistency). The lecturers presented the subject of integral through more examples in comparison to other topics and almost one of the two examples given in the calculus course was selected from the integral topic. Moreover, no apparent difference was determined between the textbooks and lecture notes for the example distribution according to the topics. The consistency of the examples contained within the lecture notes with those in the textbook was also evaluated, and the exact citations were taken into consideration in this process. In comparison to the subject of limit and derivative, the students were determined to have made use of the examples in the textbook more regarding the subject of integral (Table 1). Accordingly, 55% of the examples used in the classroom by the lecturers were directly and literally obtained from the source used as the main textbook.On the other hand, lecturers stated that 45% of the examples they used in the classroom were formed with the help of other supplementary resources, former lecture notes and their own mathematical content knowledge. Two of the interview findings providing this inference have been shared below:

**Lecturer-1**: My resource for obtaining examples is myself. The textbook provides a planning for me; however, I usually create the examples during the lesson. I have several examples concerned with the topic, which I quite like and the didactical quality of which I believe in, and in addition to the textbook, I often refer to the my own course notes.

<u>Lecturer-4</u>: There are already standard examples in the calculus courses, which can be used when required. However, we are preparing a infrastructure for engineering, so I present as many examples as the department specified during the service courses and stick to the resources suggested by them.

#### Representations

When the examples in the textbook and in the lecture notes were compared in terms of the frequency of the used representations, it was observed that along with several distinctions, there were generally similar representation characteristics that were used. In this context, it was determined that the algebraic and graphical types of representations were used more frequently in both sources (textbook and lecture note) and in the all topics (limit, derivative or integral). The least commonly-used type of representation independent of the topics and the sources is the numerical representation. The use of verbal representation is seen to have varied according to the topics (Table 2). While the verbal representations on limit (19%) are often preferred in the lecture notes, the verbal

representations on derivatives (20%) are preferred more frequently in the textbooks. Independent of the topics, the contents presented through algebraic representations in the examples contained in the lecture notes are at higher rates than the contents presented through the graphical representations that prove to be at lower rates in comparison to those in the textbook. While passing from the subject of limit on to the derivatives and integral, respectively, in the textbooks, the frequent use of the graphical representation increases. Also in the lecturers' preferences on representations, similar tendencies could be observed, and while the topicsin which the algebraic representation is used in the least is limit, it is the integral in which it is used at most. The total findings obtained from the topicslike limit, derivative and integral were used to evaluate the general representational characteristics of the examples contained within the textbook and the lecture notes (Figure 2).

	Limit		Der	ivative	Integral		
<b>Representation</b>	<u>Textbook</u>	Lecture <u>notes</u>	<u>Textbook</u>	Lecture <u>notes</u>	<u>Textbook</u>	Lecture <u>notes</u>	
Algebraic	35	48	38	60	46	68	
Graphical	49	26	39	18	33	21	
Numerical	8	7	3	5	4	6	
Verbal	8	19	20	17	17	5	

Table 2. The Distribution of the Used Representations in Examples According to Topics

In this context, the types of representations contained in the textbook examples according to their frequent use were determined to be graphical (41%), algebraic (39%) and verbal (15%), respectively; whereas, the most frequently used representations in the lecture notes were found to be algebraic (62%), graphical (20%) and verbal (12%), respectively. It was observed that 5% of theexamples in the textbook and 6% of theexamples in the lecture notescontained the numerical representations. There are notable differences between the textbook and the lecture notes in terms of the representations used in presenting the examples. The examples used by the lecturers in the calculus content are structured mainly through the algebraic representations. Without making any changes, the lecturers brought 55% of the examples in the textbook directly into the classroom environment (Figure 2), and while they preferred examples according to the type of representation, they used the graphical representation less (20%).

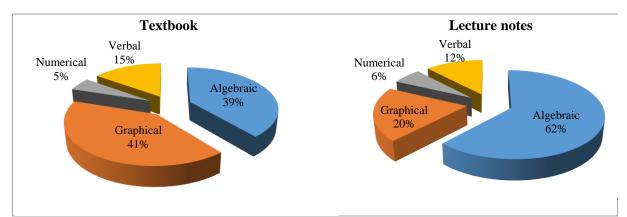


Figure 2. The Frequency of Used Representations According to the Type of the Resource

In order to determine the reason why the graphical representation has less space in the lecture notes in comparison to the textbooks, the lecturers were asked to respond to the question, "which type of representation would you include more in the teaching content, and why?". Some of the participants' views reflecting the general characteristics is shared below:

<u>Lecturer-1</u>: Although this course is taught to the students of engineering departments, it covers the subjects of the Analysis course by its actual title. While calculating integrals and finding derivatives, the engineers must primarily have a command of the symbolic language of mathematics in the first place.

<u>Lecturer-3</u>: I'm doing my best in trying to draw the graphic of a function to be calculated, but trying to draw the graphic of each equation on the board is both time-consuming and pointless, because we should not neglect the abstract nature of mathematics while trying to use a visual language.

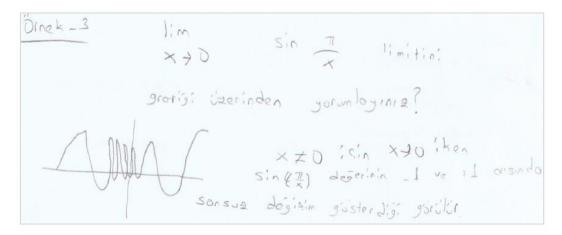


Figure 3. An Example That Encourages the Presentation with Graphical Representations in Lecturer-3's Course Note

In the views of the lecturers that are not included above but comprise similar judgements, it was commonly pointed out that there were algebraic representations underlying the calculus course. The lecturers also emphasized that it was not easy to integrate the graphics within the textbook into a traditional classroom environment and if they did, it consumed a great deal of time. Whereas, in the preface of the textbook, it was stated that much importance was attached to the use of visual representations like graphics and diagrams for this edition of the book and that the practices into which technology could be integrated in this matter were also of great significance; in addition, it was also stated that various examples and tasks had been presented in the contents of the book to encourage the students in that direction.

**From the Preface of the Edwards and Penney-Calculus:** For this edition of the book, a review at full length, from the very beginning till the last page was performed...In effect, the current new spirit and varieties regarding the graphics in the calculators and computer systems will be noticed in this edition. Hundreds of different updated computer graphics have been added into the book in accordance with the graphical emphasis of today's restructured calculus (Edwards & Penney, 2008, pp. 9).

#### Language

The examples in the textbook and in the lecture notes were evaluated under the categories of "contextual-structured" or "formal-structured" according to the language used in the content presentation. Among the examples presented in the teaching content, Figure 4 shows the proportion of the contextual and formal structured examples to all the examples in terms of percentage. The findings have suggested that the topics are indicative in terms of the languages used in the example presentation. The examples in regard to derivatives were associated more with daily life in both the textbook and the lecture notes. The examples given on integrals by the lecturers are mainly of formal-structured (91%), and in one of each of the five integral examples in the textbook was a contextual language used. All the examples on limit in the textbook were expressed by using a formal language.

When the percentages of the contextual-structured examples were compared over the topics involved, the examples in the textbooks can be observed to have a more contextual structure in comparison to those in the lecture notes (Figure 4).

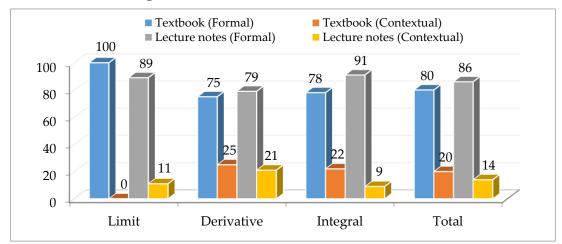


Figure 4. The Percentage Distribution of the Contextual and Formal-Structured Examples

It was observed that the examples in the lecture notes were associated much more with the physical concepts, such as momentum, speed and distance (Figure 6). However in the textbook, there are examples reflecting the practices of derivatives and integrals in different fields, such as economy, physics and statistics. Whereas the American measurement units were used for the same physical quantity in the examples of the textbook, it was observed in the lecture notes that the International System of Units (SI) were preferred in general. In order to examine the tendencies of the lecturers to use examples associated with daily life in their classes, the question, "what sort of examples (with contextual/formal language) do you prefer more often in your classes, and why?" was raised, and some quotations were taken to express the most common views. Accordingly, similar to the preferences made in the lecture notes, the lecturers usually give priority to the formal-structured examples (See Figure 5) and base their reasons as to why they use the contextual-structured examples less on the grounds, such as the importance of calculation, the restriction of the examples associated with daily life, the measurement-unit incompatibility and the students' expectations.

Örnek-11: Monoton Fonksiyonlar iain Birine's Terev Testini  
Kullannak  

$$f(x) = x^3 - 12x - 5'$$
 in Kritik noktolonni bulun ve frin arthigi ve asoldigi  
araliklari belirleym.  
Goselm: f fonksiyons her yerde sürekti ve tërevlenebilirdir.  
Birine' tërevden  
 $f'(x) = 3x^2 - 12 = 3(x^2 - 4) = 3(x+2)(x-2) = 0$   
 $= 1 \times 1 = -2 \wedge 1 \times 2 = 2$  bulunur.  
 $\frac{x}{f'(x) = 3x^2 - 12} = \frac{-2}{12} + \frac{2}{12} - \frac{2}{12} + \frac{$ 

Figure 5. An Example That Usages Formal-Structured Content in Lecturer-1's Course Note

<u>Lecturer-1</u>: Ultimately, these students will not be mathematicians; what really matters is their ability to cope with the problems they encounter in their daily lives and to create mathematical models over the data. However, in consequence of my experiences of 12 years, I can easily say that the students get bored more and more with the examples that are presented in the form of problem scenarios. That's why, I prefer examples that can help students comprehend the subject and improve their computing skills. The examples in the textbook, however, are usually about either finance or physics...

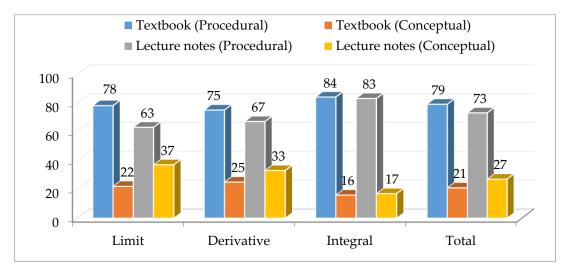
Brock-U almosi 8 En strijorsa 8 soniglik stredekt artalana har 352 24 ft/sn'dir. 8 Hizlanna sirecindeki bir anda 200. SERU ortolomo deger teoremine give 320. him gusterges; tom 30 mill soot 240. ( 44 r+1 sn) gostere cektir, 160 B. noktodo or 80 nin hizi 30 mil formants

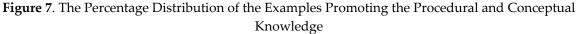
Figure 6. An Example That Usages Contextual-Structured Content in Lecturer-3's Course Note

<u>Lecturer-3</u>: The ability of the students of engineering to perform advanced mathematical calculations comprises the main targets and acquisitions of the course. And "Calculus" already means calculation! I'm trying to associate the subject with the daily life examples when needed; yet, it is not easy to find the appropriate example for each chapter. On the other hand, some of the daily life examples in the book do not contribute to the involved mathematical subject whatsoever. Several examples involved are included in the translation books, and the American System of Measurement units, such as mile, feet and inch are used...

#### Knowledge

Within the scope of the study, the applicability of the examples contained in the teaching content were evaluated by considering what type of knowledge (conceptual or procedural) would be promoted by those examples. The data presented in Figure 7 are the percentages of the use of the examples promoting the conceptual knowledge and procedural knowledge according to the type of resource. The findings, in each of the three topics, suggested that the examples in the lecture notes supported the conceptual knowledge more than those in the textbooks (Figure 7).





The examples in the subject of limit have a more conceptual content in the lecture notes, whereas those in the subject of derivatives have a more conceptual content in the textbooks. For both of the resources, the subject of integral has been the topicin which the operational proficiencies remain in the forefront. When the total results are analyzed, it can be said that almost one out of each of the four examples in the lecture notes can be used to promote the conceptual knowledge, whereas in the textbooks, only one out of each of the five examples can be used in the same way. The findings showed that the examples in both the textbook and the lecture notes mainly had the characteristic of procedural knowledge. While the roles of the examples within the teaching content were being evaluated according to the type of knowledge, the balance between theory and practice was also taken into consideration. The findings have shown that the examples in the form of practice are included more in both the textbook and the lecture notes when compared with those in theory. Accordingly, 84% of the examples in the textbook and 80% of them in the lecture notes are given for putting knowledge into practice. The examples with theoretical content, such as proof, generalization and the verification of a mathematical rule were determined to have been used more frequently in the subject of limit in the textbook (28%) and in the subject of integrals in the lecture notes (23%). The solution examples which were gathered from lecture notes for theory-practice content are shared in Figure 8. In the preface of the textbook, it was stated that some of the theoretical information and proofs could be skipped by the initiative of the lecturer and that the content of the book could be updated according to the requirements of the science-engineering majors (Edwards & Penney, 2008, pp. 8).

An Example in Lecturer-4's Course Note
Ornet!
Sxdx integralinin & recrit old.
Ciozim Sxdx = lim & f(s/n) 1 Ciozim Sxdx = noos int noi
$Gozim   S \times dx = 100 Gill 1$
$= \frac{1}{1-100} \frac{1}{1-1} \frac{1}{1-1} \frac{1}{1-1} \frac{1}{1-1} = \frac{2}{1-1}$
$= \lim_{n \to \infty} n^2 (1 - 2)$

Figure 8. Theory-Practice Examples in Lecture Notes

Two lecturers responded to the question as to why the examples promoting the procedural knowledge were preferred at most with the answer, "to put theory into practice", and three lecturers, replied as "to increase the operational proficiencies". The interviews suggested that the textbook was used as a framework by the lecturers while deciding on the theory-practice balance. The dominant opinion among the lecturers was in the direction of focusing more on the practice-based examples in the departments that apply mathematics as a complementary discipline. However, the interview findings indicate that the epistgemological beliefs adopted by the lecturers could also affect their example preferences in the undergraduate mathematics. Two of the interview data supporting this inference given below:

<u>Lecturer-2</u>: I teach these subjects also to the students of mathematics department under the Analysis course. The classes are 60%-70% theoretical. However, in the other departments except for the mathematics department, practice is focused on more, which is already supposed to be like that anyway. For instance, in the department of electronics engineering, the definition of epsilon-delta, in the subject of limit, can be taught superficially, and rather than giving examples to be solved by using the definition of limit, the examples in which the rule to take limit can be applied should be preferred.

<u>Lecturer-4</u>: Here, what needs to be focused on is not the department where the course is taught but the ability to apply the knowledge in a functional way. It is not enough to only know the derivative-integral calculation; we already have this calculation performed via the programs like Mathlab, Maple in the engineering departments. We have to explain the theory well in order to talk about the concepts, so I am giving priority to the examples that pay attention to the occurrence of the involved rule.

#### **Discussion, Conclusion and Suggestions**

Engineers, who known as the person who create solution to the problems encountered in reallife situations; have to use mathematics, which is called the language of engineering, in this process (Güner & Çomak, 2011). The students of several departments in the field of engineering faculty take courses on basic mathematics in their first or second educational years, and there are numerous textbooks to be used for this course, as well. Even though the the credits, targets and acquisitions of the course share a resemblance for the same schedules of different universities, the main or the supplementary resources to be used during the classes vary according to the lecturer's preferences. There is a textbook recommended as the main resource for engineering mathematics in the higher education institution where this study was conducted. Yet, the lecturers are also free to use whatever resources they want provided that they stick to the target acquisitions of the course. At this point, while the fact that the mathematics teachers of primary and secondary education stick to the textbooks in the process of example selection (Raman, 2004; Brandstrom, 2005), can be explained by the fact that the textbook followed is common and compulsory. The findings in this study have shown that more than half of the examples used by the lecturers in their classes are obtained directly from the textbooks even though the lecturers are free to choose textbook at the higher education level (Table 1). These findings showed that the textbooks can be evaluated as the basic and fundamental resource for students in the exemplification process of higher education mathematics, as well. Previous studies reported that instead of using examples of verbatim as quoted in the textbook, it is suggested that lecturers should generate new examples suitable for the target audience (Zaslavsky & Zodik, 2007; Chick, 2009).

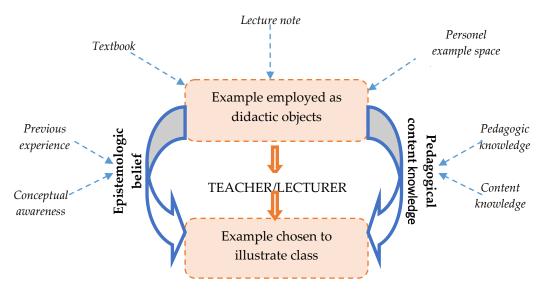


Figure 9. Internal Didactic Transposition Model for Exemplification Process

The textbooks are primary resource; however, lecturers' pedagogical intent, implementation knowledge and their epistemological beliefs affect the benefit rate of the textbook examples and the intended purpose of examples (Watson & Chick, 2011). On the other hands, the findings that the examples in the lecture notes are low in number when compared with those in the textbooks can be explained as the time restriction, the content preferences of the lecturers and ignoring the sets of examples that serve for the same purpose. Along with examples in the textbook, personal example spaces formed as a result of previous teaching experience and the examples prepared from other resources aids constitute the teaching objects which can be used in the teaching environment. The model presented in Figure 8 developed with the compilation of the internal didactical transposition model (Chevallard, 1992) and the trajectory of example use model (Watson & Click, 2011), and this model has been used for the interpretation of the findings. According to this model, the factors such as personal example space (in the transition process from knowledge to be taught to knowledge to be learnt), pedagogical content knowledge and epistemological belief seem to be determinative in the example preference. The impact of these factors in the example preference of lecturers is discussed over representation, language and knowledge structures, respectively.

When the characteristics of the representations given by the textbooks and used by the lecturers in the exemplification process are analyzed, it was determined that the examples containing the graphical representations were used more often in the textbooks, whereas those containing algebraic representations were used more often in the lecture notes. In the interviews performed with the lecturers, much attention was drawn to the fact that drawing graphics was rather a challenging and time-consuming practice. This finding was interpreted in the way that the technological proficiencies in the educational environment could affect the exemplification process of the lecturers. In other words, lecturersmay prefer the dynamic and visual examples more for the courses performed in technology-assisted classrooms or in smaller classes. The technological restrictions in the educational environment may affect the lecturer's preferences on representations (Patterson & Norwood, 2004). On the other hand, while some of the lecturers were explaining the reason why they most frequently preferred examples based on algebraic representations, they drew the attention to the abstract and theoretical nature of undergraduate mathematics. At this point, the lecturers can be said to shape the example preferences of their previous experience regarding a course or a concept. Even though the lecturers give the textbooks as the main references, they still reflect the examples they choose from the textbooks by evaluating them according to their own pedagogical content knowledge or epistemological beliefs (Zaslavsky & Zodik, 2007). For instance, while Duval (2006) claims that one mathematical concept should be stated at least two representations, he also remarks the importance of representational awareness (convert between representation) on pedagogical content knowledge. Kendal (2002) also states that the lecturers prefer the numerical representations less in comparison to the algebraic and graphical ones in teaching the subject of derivatives, she explains the reason for this, with the fact that the belief in teaching numerical representation is at rather low levels, which suggests that in the lecturers' beliefs, the teaching aspect of the types of representations according to the topic show differences. Similar inferences can also be true for this study, and only 6% of the examples used in the subjects of limit, derivatives and integrals by the lecturers cover numerical representations. The lecturers who found the numerical interpretations and meanings (the rate of change in the derivative, the accumulated change in the integral, etc.) of the subjects or concepts epistemologically (according to the order of importance) less valuable may have reflected these beliefs they have on their example preferences. Whereas, as also shown in Mesa's (2004) study results, the use of multiple representations must be considered important in the teaching contents in order to develop the problem-solving, reasoning and metacognitive skills. In the classes where this study was carried out, teachers followed the traditional approaches and students write down the examples on the blackboard. But, if engineering students use the mathematic programs such as MATHLAB and MATHEMATICA that they will use also in their professional life in the classes, they will be able to use different representations in a more dynamic and easier way in the classroom environment (Coutis et al., 1999; Kendal, 2002).

Another finding in the study indicated that the examples associated with daily life (contextual-structured) were limited in both the textbooks and the lecture notes. Whereas, the contextual examples comprise solving the real problem cases by using the mathematical language (mathematicalization), which is the process of modelling, the modelling skills are required, particularly for solving optimization problems regarding the derivatives. The necessities of developing the modelling skills of the students are urgently stated in the syllabols of calculus course at industrial and computer engineering departments (Felszeghy, 2010). In fact that the participants less prefer contextual example can be also related to the characteristics of personal example space. Because, the lecturers benefit from only a part of the contextual examples contained in the textbooks. While Sandefur et al. (2013) mention that teachers and students have personal example spaces about a mathematics concept, they assert that the various example types (i.e., generalization and counterexample) looking familiar to individual and to be considered as prototype in this space. The participants of this study are composed of tutorials who have lectured calculus course to the mathematic majors in previous years and with this aspect, example spaces of participants contain a reflection of the previous teaching-learning experience. As these examples presented in the class have been chosen from the personal example space, participants may have made their choices according to their experiences instead of the examples which reflect the features of the target audience. At this point, it is likely that the lecturers have inadequate pedagogic knowledge as to the interests and needs of their students. The fact that the contextual examples necessitate further preparations and the thought that a general framework rather than a particular event be presented may have led the lecturers to the preference of formal-structured examples. Whereas, the preference of the examples associated with the professional field in higher education mathematics is of great significance in terms of the fact that it will positively affect the cognitive and affective competences, such as relational thinking, modelling and motivation (Cavallaro & Anaya, 2011). In the interviews, the lecturers stated that the units like mile, inch or feet did not apply to the context of Turkey; therefore, they did not use some of the contextual examples in the textbooks. The original language of the textbook reviewed within the scope of this study is in English, and its Turkish version is also in line with the American System of Measurement units given within the original book. In effect, the students of engineering should have the proficiency to make conversions among different measurement units. However, in order to eliminate the prejudices of lecturers on the matter, the book translators can prefer the measurement units that conform to Turkey's context.

Another point that attracts attention in the findings of the study is the fact that the lecturers lay more emphasis on the examples promoting the procedural knowledge compared to the conceptual one in the teaching-learningprocess. In the process of creating or developing the contents aiming at certain targets that will provide a relation with the former knowledge of the student, the instructors (teachers or lecturers) must be aware of the fact that they are responsible for this process, for such sort of contents cannot be expected to be created randomly during the teaching process. At this point, the contents of the textbook can be said to shape the lecturers' preferences. Indeed, it was also determined in the reviewed examples of the textbook that the contents promoting the conceptual knowledge are rather limited. It is known that the types of examples referred to as " worked example" in the literature and through which references are made to the types of contents in which a number of examples can be solved with a routine algorithm are often included in the textbooks (Lithner, 2004). It is known that the examples inadequate in terms of the conceptual knowledge, which does not support the advanced mathematical thinking skills, such as relational thinking, reasoning and creating alternative solution ways (Hiebert & Lefevre, 1986). The examples to promote the cognitive proficiencies on the lower level like memorizing, repeating and applying the newly-acquired knowledge occupy a larger part within the examples of the textbooks and this situation has led to the quest for reform. The pioneers of the Calculus Reform Movement pointed out that the teaching contents had to be revised to increase the efficiency in the teaching-learningprocess (Hughes-Hallet, 1991). Indeed, while the researches carried out in USA suggest that more than half of the students who took engineering calculus in the traditional classes proved to be unsuccessful, the reason for this

situation was based on the instrumental understanding, the use of symbolic language and the theorybased instruction (Ostebee & Zorn, 1997). The American Society for Engineering Education (2014) draw the attention to the fact that the lecturers of mathematics and engineering must work in coordination with one another in the educational process in order for the engineering mathematics to be more efficient (as cited in Lam, Danforth & Hughes, 2014). The lecturers who participated in this study are from the mathematics departments and teach engineering mathematics as a service course. The fact that the lecturers most include practice-based examples instead of theoretical examples can be interpreted as example preference suitable for target audience (pedagogical knowledge). The results of the interview analysis revealed that most of the participants give importance to the use of computational skills while exemplifying a calculus concept. More frequent use of the examples that emphasize procedural skills by participants can be explained with beliefs regarding usability of knowledge to be taught (Raman, 2004; Schommer et al., 2005).

The results of this study indicated that the examples given in the calculus course by the lecturers who took part in the study were generally cited directly from the single source of knowledge (textbook). The study also showed that two of factors which shape example preference of lecturers are pedagogical content knowledge and epistemological belief. The graphical and algebraic representations are included more often in the examples of the textbook, and the lecturers mainly used the algebraic representations in the examples they preferred. The reason behind the lecturers' single representation preferences is more related with the capabilies of teaching environment and lecturers' epistemological beliefs about didactic quality of algebraical representation. Also in accordance with the contents of the textbook, the lecturers, in the teaching-learning process, also preferred the formal-structured examples more than the contextual-structures ones as well as preferring the examples that promoted the procedural knowledge more than the examples that promoted the conceptual knowledge. Lecturers' previous learning experience and pedagogical knowledge have restricted the preference of appropriate examples (in the context of language and knowledge structure). Although the lecturers have different approaches regarding the theory-practice balance in the engineering calculus, they tend to select the contents that exemplify the reflections of knowledge in practice rather than the examples containing proof or generalization. Even though the results of the study may be giving some idea as to the general exemplification characteristics of the lecturers in an engineering faculty, they are still inadequate in depicting the reflections of the instructional differences in example preferences, since the teachers' or lecturers' proficiencies in selecting and presenting good examples may vary according to their knowledge and experiences (Zodik & Zaslavsky, 2008). At this point, a research to be carried out to examine the epistemological beliefs of the lecturers relative to the didactical quality of knowledge, their experiences in teaching and how their example preferences vary according to their educational approaches will fill another gap in the literature. Also, the knowledge and teacher component of the didactical environments were taken into considerationin this study, however, the students' expectations and perceptions over the subject at issue were not included in it. The study to be conducted in the near futureabout investigation of students' exemplification expectation would be contributed to the literature.

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