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Relationship Between Middle School Students' Mathematical Understanding and Mathematical Attitude*

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Abstract

The purpose of this research is to develop an instrument for testing students' mathematical understanding (Study I), to search whether there is a correlation between middle school students' mathematical understanding and their attitudes towards mathematics and to present the correlation by analyzing it according to various variables (Study II). The reliability and validity studies which were conducted within the scope of Study I showed that the scale have practicable features. The Study II was carried out with 341 students who are studying in various grades of a middle school. 'Determining the Mathematical Understanding Levels Scale (DMULS)' and 'Mathematics Attitude Scale (MAS)' was used as data collection instruments. According to the results of the study; it was appeared that there was a positive and significant correlation at a high level between mathematical understandings of students and their attitudes towards mathematics and there was a positive and significant correlation at middle level between mathematical understandings of students and sub-dimension scores of mathematics attitudes scale. Besides, it was determined that the mathematical understanding of the students differed by gender but the attitudes of students towards mathematics did not differ by their genders and both mathematical understandings and attitudes of students differed significantly according to their grade levels.

Key Words

Mathematical Understanding
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Introduction

Understanding which is more important than knowledge for in-depth thinking and which is always more valuable on its own (Boylu, 2010) contrary to knowledge is the ability to transfer or connect a representation or a situation to another representation or situation (Smith, 1996). Von Glasersfeld (1987) stated understanding as; *"a well-structured mind is now the producer of cognitive concepts and is at the level of solving issues such as perception and comprehension. The structures which are situated among them and which we point out as understanding are constantly in progress"*. Usiskin (2012) evaluated understanding as something that goes on in the brain without external actions. When related literature was analyzed, it was seen that there were different types of understanding (Barmby, Harries, Higgins, & Suggate, 2007; Ghazali, & Zakaria, 2011; Hasenbank, 2006; Joffrion, 2005; Milligan, & Wood, 2010; Pirie, & Kieren, 1994; Skemp, 1976, 1987). These types are; *i) Conceptual understanding, ii) Operational understanding, iii) Relational understanding* and *iv) Mathematical understanding.*

Conceptual understanding is related with what the learners know and their learning about a concept. When concepts are processed deeply for generalizations, they become conceptual understanding and there are various conceptual understandings related with a concept (Ministry of Education, 2007, as cited in Milligan, & Wood, 2010). *Operational understanding* is; focusing on step by step operations and skills for mathematical ideas without having particular references (Ashlock, 2001). Applying rules without knowing what they do. While it allows us to remember easily, it also promotes more concrete and indirect awards and provides accessing answers rapidly (Skemp, 1987). *Relational understanding* is; understanding mathematical concepts and their components, expressing them with symbols and using their facilities; understanding the techniques in mathematical operations and expressing them with symbols; establishing relations and connections between symbols and concepts (Baykul, 2009; Skemp, 1987).

Learning by understanding is important in mathematics education (Jung, 2002) because mathematics is composed of abstract concepts (Altun, 2008) and students need to pass from abstract structures to concrete structures of mathematics (Goldin, 2002). This can be possible with students' understanding the mathematics (Boylu, 2010). In this study students' mathematical understanding was discussed within the scope of the theory which was proposed by Pirie and Kieren.

Pirie-Kieren's Model of the Growth of Mathematical Understanding

The first structure of the theory was formed in 1989 and different formats of the theory were developed in the next following three years. Pirie and Kieren (1989) shared their opinions about understanding in various conferences with other researchers and they re-structured the theory according to feedback and questions occurred in the conferences (Pirie, & Kieren, 1994). Pirie and Kieren conducted one to one interviews with students in learning environments enriched with activities, they recorded classroom activities simultaneously with the interviews and they combined these data with the written notes of the students (Pirie, & Kieren, 1992). The collected data and observed students displayed some levels regarding mathematical understanding and this resulted with the question as "*What is mathematical understanding*?" (Pirie, & Kieren, 1994). In this theory, it is suggested that students pass on different levels of understanding layers by structuring new information with their pre-cognitive knowledge (Cavey, 2002). In theory, eight potential action layers for mathematical understanding and learners' mathematical actions are defined as the growth of understanding (Martin, & Towers, 2009; Pirie, & Kieren 1989). Layers of the model are; *primitive knowing, image making, image having, property noticing, formalizing, observing, structuring* and *inventizing*. Each layer covers the previous one and is covered by all the subsequent layers for describing the integrated nature of the mathematical understanding (Martin, 2008). Information about the first four layers of the model was presented below since only the first four layers of the model were discussed in this study. According to this;

Primitive knowing layer involves physical actions, symbols and graphics etc. This layer is the starting point for the growth of understanding rather than involving low level mathematical knowledge. For example, it is the ability to divide a rectangular into equal pieces (Meagher, 2005). Primitive knowing expression does not mean low level mathematical knowledge. It is the starting point for understanding any topic in Mathematics. The first thing that an individual who performs understanding does for understanding is primitive knowing (Pirie, & Kieren, 1994). In order for an individual to understand something, everything that he/she brings into his/her mind about the subject is stated as his/her primitive knowing (Thom, & Pirie, 2006). At *Image making* layer the learner has an opinion about what the learnt concept is about, develops particular representations and engages in useful activities (Martin, 2008). Learners start distinguishing between the actions in previous layer. For example, in the case of sharing three slices of pizza among four people, saying each people will take 3 quarters from pizza may show that the student is at this layer (Meagher, 2005). At *Image having* layer the learner is exempted from any physical action. Learners at this layer have image as mental objects. Learners can say that they showed the remaining value of the fractions coming from the division in arithmetic (Meagher, 2005). The learner is not dependent to an activity for long time (Martin, 2008). For instance, students at this layer now know that $y = 3x + 4$ equation is a straight line equation without drawing the graphic of this equation (Martin, & Pirie, 2003). At *property noticing* layer learners start noticing connections and features in particular images. For example, learners can say that equivalent fractions are made of by multiplying the numerator and denominator with the same number (Meagher, 2005). It is the learner's reflection action where he/she can ask questions about his/her learning and where he/she search for what can be said about the subject generally (Martin, 2008). For example, students at this layer know that all the graphics of $y = ax$ equation pass through (0, 0) point (Martin, & Pirie, 2003).

Only seven studies about mathematical understanding were recognized (Argat, 2012; Arslan, 2013; Bal, 2006; Bike-Kalkan, 2014; Gülkılık, 2013; Kardeş-Birinci, Delice, & Aydın, 2013) when the studies which were carried out in our country were analyzed. It was specified that any scale instrument for testing mathematical understanding of the students has not been developed yet in the studies carried out abroad. Therefore, it is thought that specifying mathematical understandings of the students is one of the important research topics which stand in front of today's educators. It is predicted that the quantitative studies which will be carried out for specifying students' mathematical understandings will significantly offer an insight into the related literature. Besides, it is thought that it is an important step since understanding was discussed from a different perspective. At this point, this research is important because of that it will make possible to do qualitative, quantative and mixed researches in the future. In this regard, the aim of this study is to try to develop an instrument for testing middle school students' mathematical understanding (Study I). Besides, to search whether or not there is a correlation between middle school students' mathematical understanding and their attitudes towards mathematics and to present this mentioned correlation by analyzing this correlation according to different variables (Study II). In this sense the answers of the following questions were searched within the scope of Study II.

1. Is there a significant correlation between middle school students' mathematical understanding and their attitudes towards mathematics?
2. Is there a significant correlation between middle students' mathematical understanding and the sub-dimensions of the mathematics attitude scale?
3. Is there a significant difference between middle school students' mathematical understanding and their attitudes towards mathematics by their gender?
4. Is there a significant difference between middle school students' mathematical understanding and their attitudes towards mathematics by their grade levels?

Method

The study which was conducted according to general screening model was carried out in two stages. In the first stage, a scale was prepared and evidences were collected regarding the reliability and validity of this scale. The first stage was called as 'Study I'. In the second stage, the evidences about the practicability of this scale were collected by using this developed scale with another group. The second stage process was called as 'Study II'.

Study I

Research Desing and Study Group

The study group of this study which was carried out according to general scanning model is composed of 969 students who are studying in 6th, 7th and 8th grades of nine middle schools (total number of central middle schools) from the Burdur city. When the distribution of the study group is analyzed, it is seen that 48,40% of the study group students are female (n=469) and 51,60% of them are boys (n=500). The study group is composed of students who are studying; 37,36% of them in 6th grades (n=362), 32,51% of them in 7th grades (n=315) and 30,13% of them in 8th grades (n=292). The data obtained from the first group of participants (500 students) were used in construct validity and reliability studies and the data obtained from the second group (469 students) were used in confirmatory factor analysis studies. When the distribution of the first group is analyzed, it is seen that 49% of them are girls (n=245) and 51% of them are boys (n=255). The first study group is composed of students who are studying; 35,4% of them in 6th grades (n=177), 32,4% of them in 7th grades (n=162) and 32,2% of them in 8th grades (n=161). When the distribution of the second group is analyzed, it is seen that 47,76% of them are girls (n=224) and 52,24% of them are boys (n=245). The second study group is composed of students who are studying; 39,45% of them in 6th grades (n=185), 32,62% of them in 7th grades (n=153) and 27,93% of them in 8th grades (n=131).

Scale Development Process

Determining the Mathematical Understanding Levels Scale (DMULS): Validity and Reliability Study

During the process of development of data collection instrument, first of all both domestic and foreign studies carried out about mathematical understanding were analyzed. In this regard, Pirie-Kieren theory focused on the growth of mathematical understanding was used as a base for writing the items of the scale. By considering the structure of the theory and middle school students, it was decided to write scale items about the first four layers of the theory. The type of items that can be written about the first four layers of the theory was analyzed and an item pool which was composed of 91 expressions was created. In the pool; there were 22 items about the first layer, 21 items about the second layer and 24 items about the third and fourth layers. The first form which had 91 items presented to 5 experts who had information on the content of the study and who were informed about the research subject from teaching mathematics major in order to take expert opinions. Binary evaluation was used for taking expert opinions. Experts were expected to choose one of the options stated as 'appropriate'and 'inappropriate' in the expert opinion form. It was determined that how many experts approved probable options of each items by combining all the expert forms in one form.

The content validity of the items was determined by using content validity ratio which was developed by Veneziano and Hooper (1997) in accordance with the obtained expert opinions. The ratio mentioned above for each item was calculated as; *"(The Number of Experts with Positive Answers/Total Number of Experts)-1"*. Items which had content validity ratio below 0,80 were omitted from the study. In this sense, 3 items were omitted from the scale in accordance with the obtained content validity ratios. It was determined that 4 items had similar expressions and these items were omitted from the scale. 2 items were edited for increasing clarity. The response format of the items at the pilot/draft scale which had 84 items eventually was organized in 5 point likert type as *"Strongly Disagree (1), Disagree (2), Neutral (3), Agree (4)* and *Strongly Agree (5)"*. The highest score that can be obtained from the score is 420 and the lowest score is 84. The height of the score means that student's level of mathematical understanding takes place in the upper layers and the low score means student's level of mathematical understanding takes place in the lower layers.

The pilot/draft scale which took its final format in this way was applied to 50 8th grade students who are studying in a public middle school in Kocaeli in order to determine whether there are items/item that cannot be understood by students, to determine the approximate response time and to control whether there are misspellings or not. According to the obtained data, it was appeared that there were not any misunderstandings and misspellings in pilot/draft form. 8th grade students completed the form approximately in 17 minutes. The scale will be used together with 6th, 7th and 8th grades. In this regard, by considering the reading speeds of students in lower levels, approximate response time was specified as 25 minutes. Pilot/draft form was applied in classroom environment to the participants by explaining the purpose of the research between the dates of 05/09 November 2012. The application lasted approximately 20 minutes.

Data Analysis

Exploratory and confirmatory factor analyses were conducted for the construct validity of the scale and Cronbach Alfa, Spearman-Brown and Guttman Split-Half reliability analyses were conducted for the reliability of the scale. Obtaining consistent results with the testing instrument from application to application is another indicator of the reliability. For this reason, test-retest method was also used for the scale and necessary calculations were made.

Exploratory Factor Analysis of the DMULS (DMULS-EFA)

Comrey and Lee (1992, as cited in Tabachnick, & Fidell, 2007) stated that 300 is good, 500 is very good and 1000 is perfect for the sufficient sample size to conduct factor analysis. Bryman and Cramer (2001, as cited in Çokluk, Şekercioğlu, & Büyüköztürk, 2010) suggested that it is necessary to obtain a number which will be find with the multiplication of the number of items with five or ten for sufficient sample size. The study group with 500 students where pilot/draft scale was applied is almost 6 times of the number of items in the scale. For this reason the sample size of this study was accepted as very good for the factor analysis. Later on, Kaiser Meyer Olkin (KMO) and Bartlett tests were conducted in order to test the suitability of the sample size to factoring. As a result of the analysis, KMO value was determined as " ,959". In accordance with this finding, it was concluded that the sample size for conducting factor analysis was 'perfectly sufficient' (Şencan, 2005; Tavşancıl, 2010). According to Bartlett test results, chi-square value is significant at 0,01 levels, in short having p value which is smaller than 0,01 shows us that the data came from multi-variable normal distribution (Çokluk, Şekercioğlu, & Büyüköztürk, 2010). When Bartlett test results were analyzed, it was seen that chi-square value was significant at ($X^2 = 20984{,}991; p < ,01$). In this sense, both KMO and Bartlett test results and sample size showed that data set was suitable for factor analysis.

While determining the number of factors, eigenvalue statistics, line graphic which is drawn according to eigenvalue of the factors and variance rates explained by factors should be considered (Büyüköztürk, 2012). Factors of which eigenvalue is smaller than 1 are not considered but factors of which eigenvalue is 1 and bigger than 1 are accepted as important (Çanakçı, 2008). In this study, there was not any restriction for the number of factors at the beginning and it was seen that there were 16 factors that eigenvalue was bigger than 1. Later, line graphic which drawn according to eigenvalue of the factors was analyzed and factors with a rapid decline in graphic was accepted as important. When graphic was analyzed, it was observed that there was a rapid decline after the first factor and after that point graphic reached a plateau. Another approach for determining the number of factors is the rate of total variance explained. 30% or more of the total variance explained is seen sufficient in single factorial designs (Büyüköztürk, 2012). According to the results of the analysis from this perspective, the communality of the first factor was found as 30,972%. In this regard, the number of factor in this study was determined as 'one'.

While deciding on the size of the factor loading, it is stated that it is necessary to pay attention to the size of the sample (Şencan, 2005). Kim-Yin (2004, as cited in Şencan, 2005), suggested a particular sample size in order to be able to decide on whether an item would be omitted or not. According to this, the sample size of an item that has 0,30 factor loads should be at least 350. Also in this study, having sample size as 500 students for the explanatory factor analysis provides an opportunity to be able omit the items that factor loads below 0,30. Tabachnick and Fidell (2007) indicated that the factor load of the variance should be 0,32 and more. Besides, Comrey and Lee (1992, as cited in Tabachnick, & Fidell, 2007) suggested that the variance should be evaluated as “weak” since 10% of the variance is explained when the load value is 0,32. They stated that if the load value is 0,45, it should be evaluated as ‘average” and if it is 0,55, then it should be evaluated as ‘good’. In addition to that, they also explained that if the factor load value is 0,55, and 30% of the variance will be explained (as cited in Tabachnick, & Fidell, 2007). In this study both by considering the size of sample and references stated above, items that had factor load values below 0,55 omitted from the analysis with the assumption that 30% of the variance was explained. By considering all of these criterions, 28 items were omitted from the scale and there were 56 items remaining in the scale. When the items were analyzed according to the Pirie-Kieren theory based on this study, it was seen that there 7 items regarding the first layer, 13 items regarding the second layer, 17 items regarding the third layer and 19 items regarding the fourth layer. The last form of the Determining the Mathematical Understanding Levels Scale (DMULS) was given in paper's Turkish version (pp. 122-123). The factor loadings and total variance which were obtained as a result of the analysis were demonstrated in Table 1.

Table 1. The Factor Analysis Results of DMULS According to EFA

Item No	Factor Loading	Item No	Factor Loading	Item No	Factor Loading	Item No	Factor Loading
1	,589	15	,647	29	,603	43	,678
2	,647	16	,628	30	,574	44	,625
3	,649	17	,663	31	,570	45	,689*
4	,613	18	,552**	32	,571	46	,617
5	,556	19	,602	33	,615	47	,631
6	,611	20	,596	34	,677	48	,605
7	,567	21	,639	35	,637	49	,616
8	,572	22	,616	36	,656	50	,618
9	,610	23	,568	37	,575	51	,554
10	,611	24	,579	38	,587	52	,563
11	,584	25	,610	39	,591	53	,625
12	,661	26	,581	40	,556	54	,602
13	,621	27	,600	41	,581	55	,606
14	,577	28	,663	42	,586	56	,553
Total Variance Explained= 36,922%							

*Maximum value

**Minimum value

The total variance explained in Table 1 is seen as 36,922%. 30% and more of the variance explained in a single factor scale, is accepted. In this regard, the variance of this developed scale was accepted as valid. It is seen that factor loading values differ between “ ,552” and “ ,689”.

Item-Remainder, Item-Total Correlations and Discriminations of Items

Item–total correlation and item-remainder correlation values which gives the validity coefficient of each item was calculated. Besides, t-test was used for determining the discrimination power of the items in the scale (Balcı, 2009). With this purpose, groups below 27% and above 27% were determined by sorting the scores obtained from 500 scales from smallest to largest. Independent samples t-test were showed in Table 2 by calculating from both groups' scores.

Table 2. Item Analysis Results of DMULS

Item No	Item Remainder Correlations	Item Total Correlations	t*	Item No	Item Remainder Correlations	Item Total Correlations	t*
1	,569	,586	-12,261	29	,584	,659	-12,079
2	,628	,645	-13,177	30	,554	,602	-13,676
3	,627	,645	-13,814	31	,549	,573	-13,130
4	,594	,614	-12,591	32	,553	,571	-12,461
5	,538	,560	-12,867	33	,596	,573	-15,251
6	,591	,608	-13,151	34	,658	,614	-13,713
7	,549	,572	-13,034	35	,619	,675	-13,624
8	,551	,571	-12,267	36	,640	,636	-15,185
9	,590	,609	-14,006	37	,556	,657	-13,201
10	,588	,605	-12,739	38	,570	,575	-12,107
11	,565	,585	-13,362	39	,575	,590	-13,690
12	,641	,658	-16,095	40	,539	,595	-12,793
13	,602	,620	-14,501	41	,564	,560	-13,508
14	,557	,576	-13,086	42	,570	,583	-13,785
15	,625	,641	-14,088	43	,663	,591	-15,386
16	,607	,625	-15,109	44	,607	,680	-15,101
17	,643	,660	-15,103	45	,669**	,625	-16,191
18	,534***	,556***	-13,351	46	,600	,684**	-14,365
19	,580	,600	-13,383	47	,611	,618	-13,215
20	,575	,594	-12,875	48	,589	,630	-14,194
21	,617	,635	-14,370	49	,598	,608	-13,993
22	,598	,617	-14,831	50	,597	,618	-13,919
23	,549	,569	-12,017	51	,536	,558	-11,342
24	,558	,577	-12,639	52	,544	,567	-12,651
25	,589	,606	-13,006	53	,606	,624	-14,349
26	,561	,581	-13,855	54	,585	,604	-13,700
27	,577	,596	-12,994	55	,588	,607	-12,554
28	,642	,596	-15,661	56	,537	,558	-11,843

*p < ,01

**Maximum value

***Minimum value

When Table 2 is analyzed, it is seen that the correlation between factors and item factor total scores differ between ",556" and ",684" and item remainder correlations differ between " ,534" and " ,669". It is seen that there is a significant differences between lower and upper groups (p < ,01). This significant difference shows that items in the scale have satisfactory distinctive features. All these findings were accepted as evidence for the validity of scale items and for the fact that they measure the same structure.

Confirmatory Factor Analysis of DMULS (DMULS-CFA)

Confirmatory factor analysis was conducted in order to evaluate the validity of the structure that 56 items were determined by explanatory factor analysis. The results of the confirmatory factor analysis which was conducted for determining the mathematical understanding of the students were displayed in Table 3.

Table 3. The Results of Comfirmatory Factor Analysis of DMULS

Indexes	Value	Model-Data Fit
X^2	3054,99	
df	1482	
X^2/df	2,06	Perfect Fit
NFI *(Normed Fit Index)*	0,97	Perfect Fit
NNFI *(Non-Normed Fit index)*	0,98	Perfect Fit
CFI *(Comparative Fit Index)*	0,98	Perfect Fit
GFI *(Goodness of Fit Index)*	0,81	Sufficient Fit
AGFI *(Adjusted Goodness of Fit Index)*	0,80	Sufficient Fit
RMR *(Root Mean Square Residual)*	0,045	Perfect Fit
SRMR *(Standardized Root Mean Square Residual)*	0,042	Perfect Fit
RMSEA *(Root Mean Square Error of Approximation)*	0,048	Perfect Fit
PGFI *(Parsimony Goodness of Fit Index)*	0,75	Plain and Simple

As it can be seen from Table 3, according to confirmatory factor analysis, it was found as; X^2= 3054,99 and df =1482. The X^2 results which tested model and data fit show that the data are not fit to model. Because, X^2 value was significant (p < ,01). In addition to that since X^2 is affected by the sample size, X^2/df ratio is used while deciding on the model-data fit. In big samples having this ratio as 3 as or smaller than 3 represents perfect fit (Sümer, 2000). As X^2/df ratio of the model given in Table 3 is 2,06, model-data fit was evaluated as perfect.

It is stated that having NFI and NNFI values as 0,95 or bigger reflects that model-data fit is perfect (Hu, & Bentler, 1999; Sümer, 2000). In Table 3, NFI value is seen as 0,97 and NNFI value is seen as 0,98. In this case, it is understood that model-data fit is perfect. It was pointed out that model-data fit was perfect when CFI value was equal to 0,95 or bigger (Hu, & Bentler, 1999; Sümer, 2000). The CFI value displayed in Table 3 is 0,98 and it shows that model-data fit is perfect.

Having GFI index as 0,90 or more indicates that model-data fit is good and 0,85 and more is sufficient for model-data fit. For AGFI index 0,80 or more is accepted (Aydın, 2009). In Table 3 GFI value is 0,81 and AGFI value is 0,80. In this case, both model–data fit indexes were accepted. RMR and SRMR values differ between 0 and 1 and having the value equal to 0 shows perfect fit (Kline, 2005). Brown (2006) stated that having RMR and SRMR values smaller than 0,05 indicates perfect fit. In Table 3, RMR value was displayed as 0,045 and SRMR value was 0,042. In this case, model-data fit is perfect.

RMSEA is used to predict population covariance in non-central X^2 distribution and takes a value between 0 and 1. Having RMSEA index as 0 or smaller than 0,05 indicates that model-data fit is perfect and there is no difference between universe and sample covariances (Brown, 2006; Sümer, 2000). In Table 3 RMSEA value is 0,048. In this sense, it is accepted that model-data fit is perfect. PGFI gives us information about how plain the model is and re-interpret by considering the rate of the independent models suggested to GFI. Having PGFI value closer to 1 shows that the model is plain and simple (Sümer, 2000). In Table 3 PGFI value is seen as 0,75. In this sense, it can be said that the model is simple and plain enough. In conclusion, the items of the scale which was developed for determining students' mathematical understandings are verified as a result of the confirmatory factor analysis.

Internal Consistency of DMULS

In order to specify the reliability level of the scores obtained from "Determining the Mathematical Understanding Levels Scale (DMULS)", Cronbach Alfa which is calculated depending on the variance of the each items, Spearman-Brown calculated with the division of the test into two equal parts, Guttman Split-Half and test-retest reliability analysis were conducted. Cronbach Alfa internal consistency for the prototype of the scale with 84 items was found as " ,968". The internal consistency obtained as result of the analysis conducted after omitting 28 items as a result of the factor analysis is displayed in Table 4 below.

Table 4. Internal Consistency Coefficients of DMULS

Reliability	r	p
Cronbach Alpha	,969	p < 0,05
Spearman-Brown	,946	p < 0,05
Guttman Split-Half	,946	p < 0,05

According to Table 4, Cronbach Alpha value was found as " ,969", Spearman-Brown value as " ,946" and Guttman Split-Half value as " ,946". It can be said that the reliability of the scale is high since all the internal consistency coefficients are over 0,80. In other words, the items of the scale are testing the same feature. These values show that the scale is highly reliable (Kayış, 2009). Test-retest method was used for DMULS.

Four weeks later after the first application, the scale was re-applied with 273 persons from the first group which was created from the study group. The relation between the first measurements and the measurements obtained from the scale which was re-applied to the same group after a certain period of time was found. Besides, whether the mean scores of scales found after two applications were significantly different from each other or not were analyzed by conducting paired samples t-test. As a result of the paired samples t-test which was carried out for testing DMULS's invariance over time, it can be said that students' DMULS average scores do not differ significantly at 0,05 significance level as a result of the two applications (t= -1,619, p > ,05). In addition to that, Pearson Correlation Coefficient (r=, 878; p=, 0001) showed that the relation between scale scores obtained as a result of the two application was extremely high.

Study II

Research Desing and Study Group

The relational screening model was used in this study. The aim of the relational screening models is to determine the existence of joint variation between two or more variables (Karasar, 2003).

Purposeful sampling was preferred in determining the study group. The purposeful sampling is a sampling method which is probable and non-random (Büyüköztürk, Kılıç-Çakmak, Akgün, Karadeniz, & Demirel, 2011). There are 14 different strategies about purposeful sampling (Patton, 1990). In this study among these strategies, 'convenience sampling' was preferred. The researcher in this method studies on the most accessible sample which will provide maximum saving (Ravid, 1994). In this regard, the study was carried out with 341 middle school students who are studying in a public school in İzmit district of the Kocaeli city. The distribution of participating students in terms of their gender and grade levels can be seen in Table 5.

Table 5. Study II's Study Group

	5th grade	6th grade	7th grade	8th grade	Total
Female (F)	35	44	53	42	174 (51,03%)
Male (M)	44	42	40	41	167 (48,97%)
Total	79 (23,17%)	86 (25,22%)	93 (27,27%)	83 (24,34%)	341

When the distribution of the study group is analyzed, it is seen that 51,03% of them are female students (n=174) and 48,97% of them are male students (n=167). The study group is composed of students; 23,17% (n=79) of them 5th graders, 25,22% (n=86) of them are 6th grades, 27,27% (n=93) of them are 7th graders and 24,34% of them (n=83) are 8th graders.

Data Collecting Tools and Collecting Data

Determining the Mathematical Understanding Levels Scale (DMULS): Within the scope of Study II, first of for the reliability of the scale Cronbach Alpha, Spearman-Brown ve Guttman-Split half reliability analyses were conducted. In this sense, the data obtained from these analyses were presented in Table 6.

Table 6. Internal Consistency Coefficients of DMULS (Study II)

Reliability	r	p
Cronbach Alpha	,972	p < ,05
Spearman-Brown	,939	p < ,05
Guttman Split-Half	,938	p < ,05

When Table 6 is analyzed, Cronbach Alpha internal consistency coefficient is seen as " ,972"; Spearman-Brown internal consistency coefficient as " ,939" and Guttman Split-Half internal consistency coefficient as " ,938". In this sense, it can be said that the reliability coefficients of the scale are at desired level. Later on, in order to test the validity of factorial structure of the scale, confirmatory factor analyses were conducted. The results of the confirmatory factor analyses of the scale which was used for determining the mathematical understandings of the students were given in Table 7.

Table 7. The Results of Comfirmatory Factor Analysis of DMULS (Study II)

Indexes	Value	Model Data Fit
N	341	
X^2	2769,00	
df	1484	
X^2/df	1,87	*Perfect fit* (Sümer, 2000)
NFI	0,97	*Perfect fit* (Hu, & Bentler, 1999; Sümer, 2000)
NNFI	0,98	*Perfect fit* (Hu, & Bentler, 1999; Sümer, 2000)
CFI	0,98	*Perfect fit* (Hu, & Bentler, 1999; Sümer, 2000)
GFI	0,77	*Sufficient fit* (Aydın, 2009)
AGFI	0,76	*Sufficient fit* (Aydın, 2009)
RMR	0,052	*Good fit* (Brown, 2006; Kline, 2005)
SRMR	0,044	*Perfect fit* (Brown, 2006; Kline, 2005)
RMSEA	0,05	*Perfect fit* (Brown, 2006; Sümer, 2000)
PGFI	0,72	*Plain and simple* (Sümer, 2000)

As it can be seen from Table 7, it was determined that all the fit values of the scale are at desired level.

Mathematics Attitude Scale (MAS): It is the scale which is called as 'Your Thoughts about Mathematics' and developed by Nazlıçiçek and Erktin (2002). There are 20 items declaring positive and negative judgements about three dimensions related for showing perceived mathematical achievement level, perceived benefits of mathematics and the interest towards mathematics lessons in MAS. MAS is a likert type scale including always, often, sometimes, rarely and never and the Cronbach alpha reliability coefficient was found as " ,841".

Among the items of the scale, 3rd , 6th , 7th , 13th , 14th and 19th items are about the 'perceived achievement level in mathematics' and alpha reliability coefficient was found as " ,67"; 10th , 11th , 15th , 16th and 18th items are about 'the perceived benefits of the mathematics' and the alpha reliability coefficient was found as " ,59"; 1st , 2nd , 4th , 5th , 8th , 9th , 12th , 17th and 20th items of the scale are about 'the interest towards mathematics' and alpha reliability coefficient was found as " ,69".

Within the scope of Study II, first of for the reliability of the whole scale Cronbach Alpha, Spearman-Brown ve Guttman-Split half reliability analyses were conducted. In this sense, the data obtained from these analyses were presented in Table 8.

Table 8. Internal Consistency Coefficient of MAS

Reliability	r	p
Cronbach Alpha	,859	$p < ,05$
Spearman-Brown	,852	$p < ,05$
Guttman Split-Half	,851	$p < ,05$

When Table 8 is analyzed, Cronbach Alpha internal consistency coefficient is seen as " ,859"; Spearman-Brown internal consistency coefficient as " ,852" and Guttman Split-Half internal consistency coefficient as " ,851". In this sense, it can be said that the reliability coefficients of the scale are at desired level. After that, the reliability analyses regarding the sub-dimensions of the scale were conducted. In this sense, the perceived achievement level in mathematics dimension's alpha reliability coefficient was calculated as " ,804"; alpha coefficient of perceived benefits of mathematics dimension as " ,505"; the alpha coefficient of the interest towards mathematics dimension as " ,796". In this case, it can be said that the reliability coefficients regarding the sub-dimensions are at desired level. Later on, in order to test the validity of factorial structure of the scale, confirmatory factor analyses were conducted. The results of the confirmatory factor analyses of the scale which was used for determining the attitudes of students towards mathematical were given in Table 9.

Table 9. The Results of Comfirmatory Factor Analysis of MAS

Indexes	Value	Model Data Fit
N	341	
X^2	560,17	
df	167	
X^2/df	3,35	*Perfect fit* (Sümer, 2000)
NFI	0,90	*Good fit* (Hu, & Bentler, 1999; Sümer, 2000)
NNFI	0,92	*Good fit* (Hu, & Bentler, 1999; Sümer, 2000)
CFI	0,93	*Good fit* (Hu, & Bentler, 1999; Sümer, 2000)
GFI	0,86	*Sufficient fit* (Aydın, 2009)
AGFI	0,82	*Sufficient fit* (Aydın, 2009)
RMR	0,11	*Medium fit* (Kline, 2005)
SRMR	0,075	*Good fit* (Brown, 2006)
RMSEA	0,083	*Good fit* (Sümer, 2000)
PGFI	0,68	*Plain and simple* (Sümer, 2000)

As it can be seen from Table 9, it was determined that all the fit values of the scale are at desired level.

Data collection instruments were applied to study group between 03.10.2014 and 07.10.2014. Students were given 40 minutes to fill in the data collection instruments. As a result of the implementation, 370 scales were collected.

Data Analysis

First of all, all the collected data (n=370) were analyzed one by one by the researchers. As a result of the analysis, it was determined that some students answered scales with a single response (for example; they marked only strongly agree option) and some students filled only one of the scales. In this sense, 9 data collection instrument at 5th grade level, 6 at 6th grade level, and 2 at 7th grade level and 12 at 8th grade level, in total 29 data collection instruments were not evaluated. The necessary analyses were carried out with the data obtained from the remaining 341 data collection instruments.

In accordance with the aims of the study, it was decided to use correlation in data analyses operations. The correlation coefficient is used for finding and interpreting the amount of correlation and direction of correlation between two variables. Pearson correlation coefficient requires being continues in two variables and showing a dual normal distribution for the variables. If the variables are continuous but are not showing a normal distribution, then in order to explain the correlation in between Spearman Brown correlation coefficient is used (Büyüköztürk, 2012).

In this sense, whether or not the data obtained from both scales had a normal distribution was searched. The case of whether the obtained data is suitable to normal distribution is determined through the tests used in normality subject. As it is suggested to use Shapiro-Wilks test when the group size is smaller than 50 and to use Kolmogorov-Smirnov (K-S) test when the group size is bigger than 50 (Büyüköztürk, 2012; Büyüköztürk, Çokluk, & Köklü, 2010), Kolmogorov-Smirnov (K-S) test (N=341) was used for deciding the normality of the data in this study. Having p value bigger than the value of 0,05 is interpreted as the scores are not significantly (extremely) deviated from normal distribution at this significance level and they are suitable (Büyüköztürk, 2012). As a result of the analyses of the obtained data, it was determined that both DMULS values (p= ,002 < ,05) and MAS (p= ,01 < ,05) do not have normal distribution. Besides, as a result of the normality analysis regarding MAS's sub-dimensions, it was determined that the data of the both three dimensions do not have normal distribution (p= ,001 < ,05; p= ,000 < ,05; p= ,02 < ,05). In this regard, it was decided to use Spearman Brown correlation coefficient for explaining the correlation between variables.

In order to get the answers of the 3rd and 4th research questions within the scope of Study II, it is necessary to decide whether the obtained data will be analyzed by using parametric or non-parametric techniques. The normality analyses are needed in order to be able to take this decision. As a result of the conducted analyses, it was decided to analyze through non-parametric tests. While Mann-Whitney U test was used in analyses conducted according to gender variable, Kruskal-Wallis test was used in analyses conducted according to grade levels. According to Kruskal-Wallis test, it is necessary to determine the occurrence of the difference observed between groups by depending on significant differences between which groups. In this case, the source of the difference can be analyzed by implementing Mann-Whitney U test over dual combinations of the groups (Büyüköztürk, 2012). In this regard, in this study, the source of the difference occurred between groups according to the results of Kruskal-Wallis test was analyzed with Mann-Whitney U test. The significance level was accepted as " ,05" in all the statistical operations. SPSS 17.0 software was used in analysis operations.

Findings

The first sub problem of the study was determined as in the following. '*Is there a significant correlation between middle school students' mathematical understanding and their attitudes towards mathematics?*' The correlation between students' mathematical understanding and their attitudes towards mathematics was determined with Spearman-Brown correlation coefficient. The findings obtained from the conducted analysis were given in Table 10.

Table 10. The Correlation Between Middle School Students' Mathematical Understanding and their Attitudes towards Mathematics

Relationship	**r**	**p**	**N**	**Value**
DMULS/MAS	,708	,000	341	p < ,05

When Table 10 is analyzed, it is seen that there is a high level, positive and significant correlation between middle school students' mathematical understanding and their attitudes towards mathematics (r= ,708; p< ,05). Büyüköztürk (2012) states that having correlation coefficient between 0,70-1,00 as an absolute value indicates a high level correlation and having between 0,70-0,30 indicates a middle level correlation. According to this, it can be said that students whose mathematical understanding is high also have high attitudes towards mathematics.

The second sub problem of the study was determined as in the following. '*Is there a significant correlation between middle school students' mathematical understanding and sub-dimensions of mathematics attitude scale?*' Whether or not there is a correlation between them was determined with Spearman-Brown correlation coefficient. The findings obtained from the conducted analysis were given in Table 11.

Table 11: The Correlation between Middle School Students' Mathematical Understanding and Sub-dimensions of the Mathematics Attitudes Scale

Relationship	**r**	**p**	**N**	**Value**
DMULS/MAS I. Dimension	,674	,000	341	p < ,05
DMULS/MAS II. Dimension	,470	,000	341	p < ,05
DMULS/MAS III. Dimension	,571	,000	341	p < ,05

When Table 11 is analyzed, it is seen that there is a middle level, positive and significant correlation between middle school students' mathematical understanding and the sub-dimensions of mathematics scale (r_1= ,674; r_2= ,470; r_3= ,571; p <,05).

The third sub problem of the study was determined as in the following. '*Is there a significant difference between middle school students' mathematical understanding and their attitudes towards mathematics by gender?*' Whether or not there is a difference between students' mathematical understanding and their attitudes towards mathematics by gender was compared by using Mann-Whitney U test. The findings obtained from the conducted analysis were given in Table 12.

Table 12: Test Results Conducted for the Difference between the Students' Mathematical Understanding and Attitudes Scores by Gender

Mathematical Understanding & Gender	**N**	**Mean Rank**	**Sum of Rank**	**U**	**p**
Female	174	185,2	32225,0	12058,0	,007
Male	167	156,2	26086,0		
Attitude & Gender	**N**	**Mean Rank**	**Sum of Rank**	**U**	**p**
Female	174	177,7	30926,5	13356,5	,197
Male	167	163,9	27384,5		

When Table 12 is analyzed, it can be said that the mathematical understanding of the students showed a significant difference by gender (U=12058, 0; p= ,007 < ,05). When their mean ranks are considered, it is understood that the mathematical understanding of female students are higher than male students. When Table 12 is analyzed, it can be said that students' attitudes towards mathematics did not show a significant difference by gender (U=13356, 6; p= ,197 > ,05).

The fourth sub problem of the study was determined as in the following. '*Is there a significant difference between middle school students' mathematical understanding and their attitudes towards mathematics by grade levels?*' The obtained data was evaluated by using Kruskal-Wallis test. The findings obtained from the conducted analysis were given in Table 13.

Table 13: Test Results Regarding the Difference between Students' Mathematical Understanding and Attitudes Scores by Grade Levels

Mathematical Understanding & Grade	**N**	**Mean Rank**	**df**	X^2	**p**	**Significant Difference**
5	79	194,01	3	12,814	,005	5-7
6	86	185,17				5-8
7	93	161,88				6-8
8	83	144,64				
Attitude & Grade	**N**	**Mean Rank**	**df**	X^2	**p**	**Significant Difference**
5	79	178,16	3	15,355	,002	5-8
6	86	201,21				6-7
7	93	160,05				6-8
8	83	145,16				

When Table 13 is analyzed, it is seen that students' mathematical understanding significantly differed by grade levels (X^2 = 12,814; p= ,005 < ,05). When their mean ranks are considered, it is seen that 5th grade students have the highest mathematical understanding level and they are followed by 6th and 7th grade students. The occurrence of the significant difference between grade levels depending on significant differences between which groups was determined by conducting Mann-Whitney U test which were carried out over dual combinations of grade levels. According to the analysis results, it was appeared that 5th and 6th grade students' mathematical understanding is higher than 8th grade students and the differences are significant. It was also determined that the mathematical understanding of 5th grade students is higher than 7th grade students.

When Table 13 is analyzed, it is seen that the attitudes of students towards mathematics is significantly differentiated (X^2 = 15,355; p= ,002 < ,05). When mean ranks are considered, it is seen that 6th grade students have the highest attitude scores and they are followed by 5th and 7th grade students. The occurrence of the significant difference between grade levels depending on significant differences between which groups was determined by conducting Mann-Whitney U test which were carried out over dual combinations of grade levels. According to the analysis results, it was appeared that 5th and 6th grade students' attitude towards mathematics is higher than 8th grade students and the differences are significant. It was also determined that 6th grade students' attitude towards mathematics is higher than 7th grade students.

Discussion, Conclusion and Suggestion

Study I

The purpose of this study was to develop a scale for determining mathematical understandings of the students. With this purpose, first of all an item pool was created regarding mathematical understandings of the students and opinions of the experts were taken. After the necessary adjustments in the items, the scale was made ready for the application. After pilot/draft form was carried out with the study group, in order to determine the factor structure of the scale explanatory factor analysis, to test the construct validity confirmatory factor analysis and reliability analysis were conducted.

As a result of the explanatory factor analysis, it was seen that the scale composed of single factor. It was seen that factor loading values of the items differed between " ,552" and " ,689". The total variance explained by DMULS was 36,922%. Item-total correlation and item–remainder correlation values which specified the reliability coefficient of each item was seen as sufficient. Besides, it was determined that all the items in the scale had sufficient distinctive features at desired level. According to the results of CFA which was conducted for construct validity of DMULS concsited of 56 items, X^2/df rate of the scale was 2,06. This value shows that scale fits with the real data. When Table 3 is analyzed, it is seen that the other fit values are also in the accepted limits. In this sense, it can be said that DMULS is a useful and valid model.

As a result of the reliability analysis, Cronbach Alfa value of the scale was found as " ,969", Spearman-Brown value as " ,946" and Guttman Split Half value as " ,946". All the internal consistency coefficients were over 0,80 and this showed that the reliability of the scale was high. Having consistent results from the testing instrument in all the applications is another indicator of the reliability. For this reason, test-retest method was also used for DMULS. According to the related samples t-test results it was determined that there was not any significant difference between the average scores of DMULS as a result of the two applications. The fact that the correlation coefficient was high showed that there was a high relation between the scale scores obtained as a result of two applications. Since the data of this research were collected from middle school students it can be said that the scale is suitable for these groups. Whether this scale is suitable for the lower or upper level students or not depends on the results of the studies regarding this subject.

Study II

Also, the purpose of the study is to search for whether there is a correlation between middle school students' mathematical understanding and their attitudes towards mathematics and to present this mentioned correlation by analyzing it according to different variables. In this sense, it was appeared that there is a high level, positive and significant correlation between students' mathematical understand and their attitudes towards mathematics and there is a middle level, positive and significant correlation between students' mathematical understanding and the sub-dimensions of the mathematics attitude. In a study which was carried out by Dursun and Peker (2003) students stated that they had difficulties in understanding, comprehending and interpreting mathematics lesson. In addition to this, it is mentioned that students stay away from mathematical operations with the fear of making mistakes (Altun, 2005). It is thought that this is related with the attitude towards mathematics. It is known that the attitude towards mathematics is one of the most important variables which are explaining the mathematical achievements (Peker, & Mirasyedioğlu, 2003). It was found in many studies that there is positive correlation between achievement and attitude (Katrancı, 2009; Tapia, & Marsh, 2000; Yenilmez, & Özabacı, 2003). It was also determined in this study that students who have high attitudes towards mathematics also have high mathematical understandings. At this point, mathematical understanding of students can be discussed as an indicator that they can be successful in mathematics. In this regard, it can be said that this study is supporting the mentioned above studies. It is suggested to carry out a study which is considering the mathematical achievement scores of students in order to support this result obtained through by moving from the thought that mathematical understanding can be accepted as the mathematical achievements of students. In addition to analyzing whether students who have high mathematical understandings are successful in mathematics or not, the attitudes of these students should also be analyzed.

In accordance with the other purpose of this study, it was appeared that the mathematical understandings of the students showed a significant difference by gender whereas the attitudes of students towards mathematics did not show any significant difference by gender. In related literature, it was determined that there were not any studies for analyzing mathematical understanding according to various variables. At this point, the necessity of carrying out similar studies to this one has come into sight. In some other studies, it was shown that gender differences did not have any effect on the attitudes towards mathematics (Çelik, & Bindak, 2005; Ursini, & Sanchez, 2008). In addition to that there are some studies which are presenting the fact that female students have lower attitudes than male students (McGraw, Lubienski, & Strutchens, 2006; Pierce, Stacey, & Barkatsas, 2007; Yenilmez, & Özabacı, 2003). At this point, it is seen that a clear cut result can be obtained in studies analyzing the correlations between attitude and gender. In this study, it was appeared that the result obtained as the attitudes towards mathematics did not differ by gender was supported.

Finally it was appeared that both students' mathematical understanding and their attitudes towards mathematics differed significantly by grade levels. While 'as the grade levels of the students are getting increased, their mathematical understanding will also increase' is an expected result, according to the results of the study it was occurred in a way that as the grade levels of the students were getting increased their mathematical understanding were getting decreased. In this sense, it can be said that as the grade levels are getting increased, the mathematical understandings of the students are getting decreased. It is though that the reason of this can be the fact that mathematics subjects are getting more difficult and it is suggested to carry out a study emphasizing this point. In addition to this, it was occurred that after 6th grade, the attitude scores of the students were getting decreased. Similarly, it can be said that as the grade levels are getting increased, the attitude scores of students are getting decreased. Alkan, Bukova-Güzel and Elçi (2004) stated that there was a statically significant difference between students' grade levels and their attitudes towards mathematics. Yenilmez and Özabacı (2003) found that as the grade levels of students were getting increased, their attitude mean scores were getting decreased. It is seen also in this study that the result of 'as the grade levels are getting increased, the attitude towards mathematics is getting decreased' was supported. At this point, it is outstanding that both mathematical understanding and attitude are getting decreased according to grade levels. As the idea that this will also take the achievement together with them is important, it is thought that it necessary to focus on this point. It is thought that it will be beneficial to search these reasons with qualitative methods in the following studies.

References

Alkan, H., Bukova-Güzel, E., & Elçi, A. N. (2004, 6-9 July). Öğrencilerin matematiğe yönelik tutumlarında matematik öğretmenlerinin üstlendiği rollerin belirlenmesi. *XIII. Ulusal Eğitim Bilimleri Kurultayı*, Malatya: Eğitim Fakültesi İnönü Üniversitesi. Retrieved from http://www.pegem.net/dosyalar/dokuman/78653279.pdf

Altun, M. (2005). *Eğitim fakülteleri için ilköğretim öğretmenleri için: Matematik öğretimi*. Bursa: Alfa Yayıncılık.

Altun, M. (2008). *Matematik öğretimi (ilköğretim ikinci kademe 6, 7 ve 8. sınıflarda)*. Ankara: Aktüel Yayınları.

Argat, A. (2012). *Pirie-Kieren dinamik modeli ile öğrencilerde matematiksel anlamanın gelişiminin incelenmesi* (Unpublished master's thesis). University of Marmara Education Science Institute: İstanbul.

Arslan, E. (2013). *Ortaokul öğrencilerinin "Pirie ve Kieren modeli"ne göre matematiksel anlama seviyelerinin belirlenmesi* (Unpublished master's thesis). University of Erzincan, Institute of Naturel and Applied Science: Erzincan.

Ashlock, R. B. (2001). *Error patterns in computation: Using error patterns to improve instruction.* Columbus, OH: Merrill Prentice Hall.

Aydın, F. (2009). *İşbirlikli öğrenme yönteminin 10. Sınıf coğrafya dersinde başarıya, tutuma ve motivasyona etkileri* (Unpublished doctoral thesis). University of Gazi, Institute of Educational Science, Ankara.

Bal, A. P. (2006). İlköğretim beşinci sınıf öğrencilerinin matematiksel kavrama ve işlem becerileri arasındaki farkın bazı değişkenler açısından değerlendirilmesi. *Çukurova Üniversitesi Eğitim Fakültesi Dergisi, 3*(32), 13-23.

Balcı, A. (2009). *Sosyal bilimlerde araştırma: Yöntem, teknik ve ilkeler*. Ankara: Pegem Akademi.

Barmby, P., Harries, T., Higgins, S., & Suggate, J. (2007, 8-13 July). How can we assess mathematical understanding? Woo, J. H., Lew, H. C., Park, K. S., & Seo, D. Y. (Eds.). Proceedings of the *31st Conference of the International Group for the Psychology of Mathematics Education*, *2*, (pp. 41-48). Seoul: PME.

Baykul, Y. (2009). *İlköğretim matematik öğretimi (1-5. sınıflar)*. Ankara: Pegem Akademi.

Bike-Kalkan, D. (2014). *Sekizinci sınıf öğrencilerinin kavramsal anlama ve cebirsel muhakeme yapıları* (Unpublished master's thesis). University of Anadolu, Institute of Educational Science: Eskişehir.

Boylu, A. (2010). How understanding makes knowledge valuable. *Canadian Journal of Philosophy, 40*(4), 591-609. doi:10.1353/cjp.2010.0024

Brown, T. A. (2006). *Confirmatory factor analysis for applied research*. NY: Guilford Publications, Inc.

Büyüköztürk, Ş. (2012). *Sosyal bilimler için veri analizi el kitabı*. Ankara: Pegem Akademi.

Büyüköztürk, Ş., Çokluk, Ö., & Köklü, N. (2010). *Sosyal bilimler için istatistik*. Ankara: Pegem Akademi.

Büyüköztürk, Ş., Kılıç-Çakmak, E., Akgün, Ö. E., Karadeniz, Ş., & Demirel, F. (2011). *Bilimsel araştırma yöntemleri*. Ankara: Pegem Akademi.

Cavey, L. O. (2002, 26-29 October). Growth in the mathematical understanding while learning how to teach: A theoretical perspective. Proceedings of the *Annual Meeting (of the) North American Chapter of the International Group for the Psychology of Mathematics Education*. Athens, GA.

Çanakçı, O. (2008). *Matematik problemi çözme tutum ölçeğinin geliştirilmesi ve değerlendirilmesi* (Unpublished doctoral thesis). University of Marmara, Institute of Educational Science: İstanbul.

Çelik, H. C., & Bindak, R. (2005). Sınıf öğretmenliği bölümü öğrencilerinin matematiğe yönelik tutumlarının çeşitli değişkenlere göre incelenmesi. *Kastamonu Eğitim Dergisi, 13*(2), 427-436.

Çokluk, Ö., Şekercioğlu, G., & Büyüköztürk, Ş. (2010). *Sosyal bilimler için çok değişkenli istatistik SPSS ve LISREL uygulamaları*. Ankara: Pegem Akademi.

Dursun, Ş., & Peker, M. (2003). İlköğretim altıncı sınıf öğrencilerinin matematik dersinde karşılaştıkları sorunlar. *Cumhuriyet Üniversitesi Sosyal Bilimler Dergisi, 27*(1), 135-142.

Ghazali, N. H. C., & Zakaria, E. (2011). Students' procedural and conceptual understanding of mathematics. *Australian Journal of Basic and Applied Sciences, 5*(7), 684-691.

Goldin, G. A. (2002). Representation in mathematics learning and problem solving. In L. D. English, (ed.). *Handbook of international research in mathematics education* (pp. 197-218). London, England, Lawrence Erlbaum Associates.

Gülkılık, H. (2013). *Matematiksel anlamada temsillerin rolü: Sanal ve fiziksel manipülatifler* (Unpublished doctoral thesis). University of Gazi, Institute of Educational Science, Ankara.

Hasenbank, J. F. (2006). *The effects of framework procedural understanding on college algebra students' procedural skill and understanding* (Unpublished doctoral dissertation). Montana State University, Montana.

Hu, L., & Bentler, P. M. (1999). Cutoff criteria for fit indexes in covariance structure analysis: Conventional criteri versus new alternatives. *Structural Equation Modeling, 6*, 1-55.

Joffrion, H. K. (2005). *Conceptual and procedural understanding of algebra concepts in the middle grades* (Unpublished master thesis). Office of Graduate Studies of Texas A&M University, Texas.

Jung, I. (2002). *Student representation and understanding of geometric transformations with technology experience* (Unpublished doctoral dissertation). University of Georgia, Georgia, USA.

Karasar, N. (2003). *Bilimsel araştırma yöntemi - kavramlar, ilkeler ve teknikler*. Ankara: Nobel Yayınları.

Kardeş-Birinci, D., Delice, A., & Aydın, E. (2013). Anlamayı anlamak: Matematik eğitimi lisansüstü öğrencilerinin lineer cebir kavramlarını anlamalarının incelenmesi. *Sakarya Üniversitesi Eğitim Bilimleri Enstitüsü Yayınları, 6*, 55-60.

Katrancı, Y. (2009, 1-3 October). Cinsiyet, yaşam standardı ve matematik başarısı ile matematiğe yönelik tutum arasındaki ilişki. *XVIII. Ulusal Eğitim Bilimleri Kurultayı*, Ege Üniversitesi: İzmir.

Kayış, A. (2009). Güvenirlik analizi. In Ş. Kalaycı (Ed.). *SPSS uygulamalı çok değişkenli istatistik teknikleri* (pp. 403-419). Ankara: Asil Yayın Dağıtım.

Kline, R. B. (2005). *Principles and practice of structural equation modeling*. NY: Guilford Publications, Inc.

Martin, L. C. (2008). Folding back and the dynamical growth of mathematical understanding: Elaborating the Pirie-Kieren theory. *The Journal of Mathematical Behavior, 27*, 64-85. doi:10.1016/j.jmathb.2008.04.001

Martin, L. C., & Pirie, S. (2003). Making, images and noticing properties: The role of graphing software in mathematical generalization. *Mathematics Education Research Journal, 15*(2), 171-186. doi:10.1007/BF03217377

Martin, L. C., & Towers, J. (2009). Improvisational coactions and the growth of collective mathematical understanding. *Research in Mathematics Education*, *11*(1), 1-19. doi:10.1080/14794800902732191

McGraw, R., Lubienski, S., & Strutchens, M. E. (2006). A closer look at gender in NAEP mathematics achievement and affect data: Intersections with achievement, race/ethnicity, and socioeconomic status. *Journal for Research in Mathematics Education, 37*(2), 129-150. doi:10.2307/30034845

Meagher, M. (2005). *The processes of learning in a computer algebra system (CAS) environment for college students learning calculus* (Unpublished doctoral dissertation). The Ohio State University, USA.

Milligan, A., & Wood, B. (2010). Conceptual understandings as transition points: Making sense of a complex social world. *Journal of Curriculum Studies, 42*(4), 487-501. doi:10.1080/00220270903494287

Nazlıçiçek, N., & Erktin, E. (2002). İlköğretim matematik öğretmenleri için kısaltılmış matematik tutum ölçeği. In *V. Ulusal Fen Bilimleri ve Matematik Eğitimi Kongresi Bildiri Kitapçığı*, (pp. 860-865). Ankara: Midde East Technical University.

Patton, M. Q. (1990). *Qualitative evaluation and research methods*. London: Sage Publications.

Peker, M., & Mirasyedioğlu, Ş. (2003). Lise 2. sınıf öğrencilerinin matematik dersine yönelik tutumları ve başarıları arasındaki ilişki. *Pamukkale Üniversitesi Eğitim Fakültesi Dergisi, 14*(2), 157-166.

Pierce, R., Stacey, K., & Barkatsas, A. (2007). A scale for monitoring students' attitudes to learning mathematics with technology. *Computers & Education, 48*(2), 285-300.

Pirie, S., & Kieren, T. (1989). A recursive theory of mathematical understanding. *For the Learning of Mathematics, 9*(3), 7-11.

Pirie, S., & Kieren, T. (1992). Creating constructivist environments and constructing creative mathematics. *Educational Studies in Mathematics, 23*(5), 505-528.

Pirie, S., & Kieren, T. (1994). Growth in mathematical understanding: How can we characterize it and how can we represent it? *Educational Studies in Mathematics, 26*(2/3), 165-190.

Ravid, R. (1994). *Practical statistics for educators*. New York: University Press in America.

Skemp, R. R. (1976). Relational understanding and instrumental understanding. *Mathematics Teaching, 77*, 20-26.

Skemp, R. R. (1987). *The psychology of learning mathematics*. Hillsade, NJ: Erlbaum Associates.

Smith, K. B. (1996). Guided discovery, visualization, and technology applied to the new curriculum for secondary mathematics. *Journal of Computers in Mathematics and Science Teaching, 15*(4), 383-399.

Sümer, N. (2000). Yapısal eşitlik modelleri. *Türk Psikoloji Yazıları, 3*(6), 49-74.

Şencan, H. (2005). *Sosyal ve davranışsal ölçümlerde güvenilirlik ve geçerlik*. Ankara: Seçkin Yayınları.

Tabachnick, B. G., & Fidell, L. S. (2007). *Using multivariate statistics*. Boston: Allyn & Bacon.

Tapia, M., & Marsh, G. E. (2000). *Effect of gender, achievement in mathematics, and ethnicity on attitudes toward mathematics.* Annual Meeting of the Mid-South Educational Research Association, Bowling Green, KY, USA.

Tavşancıl, E. (2010). *Tutumların ölçülmesi ve SPSS ile veri analizi*. Ankara: Nobel Yayıncılık.

Thom, J. S., & Pirie, S. E. B. (2006). Looking at the complexity of two young children's understanding of number. *Journal of Mathematical Behavior, 25*, 185-195. doi:10.1016/j.jmathb.2006.09.004

Ursini, S., & Sanchez, E. G. (2008). Gender, technolgy and attitude towards mathematics. *Mathematics Education, 40*(5), 559-577. doi:10.1007/s11858-008-0120-1

Usiskin, Z. (2012, 8-15 July). What does it mean to understand some mathematics? *12th International Congress on Mathematical Education*. Seoul: Korea. Retrieved from http://www.icme12.org/upload/submission/1881_F.pdf

Veneziano, L., & Hooper, J. (1997). A method for quantifying content validity of health-related questionnaires. *American Journal of Health Behavior, 21*(1), 67-70.

Von Glasersfeld, E. (1987). Learning as a constructivist activity. In C. Janvier (Ed.), *Problems of representation in the learning and teaching of mathematics* (pp. 3-18). Hillsade, NJ: Lawrence Erlbaum.

Yenilmez, K., & Özabacı, N. Ş. (2003). Yatılı öğretmen okulu öğrencilerinin matematik ile ilgili tutumları ve matematik kaygı düzeyleri arasındaki ilişki üzerine bir araştırma. *Pamukkale Üniversitesi Eğitim Fakültesi Dergisi, 14*, 132-146.

Appendix 1.

The last form of the Determining the Mathematical Understanding Levels Scale (DMULS) was given in paper's Turkish version (pp. 122-123)