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An Investigation into Sociomathematical Norms in a Technology and Inquiry Based Classroom for Teaching Circle Properties

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Abstract

This article aims to document the discovery of sociomathematical norms by prospective teachers in a technology and inquiry based classroom, and the role played by the instructor in this process. The findings are obtained from the dialogs between the prospective teachers and the researcher as well as from classroom communications observed during a 5-week long instructional period that was focused on teaching circle properties. After the transcription of these communications, the repeating patterns of explanations, interpretations, proofs, and argumentations are extracted which are then classified as sociomathematical norms based on the existing theoretical frameworks. Special emphasis was put on three norms that relate to using technology. These norms were (1) inquiring about the effects of a change made in a question or a solution; (2) reaching conclusions by using the properties of the tools in the dynamic software; and (3) dynamically verifying a solution or a hypothesis. It is hoped that the findings of this study can help other researchers, teachers, teacher candidates, and instructors that educate teacher candidates who desire to create inquiry based teaching and learning communities that have established sociomathematical norms.

Keywords

Social norms

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Introduction

A traditional teaching method is the IRE script where the teacher initiates, the students respond, and the teacher evaluates (Mehan, 1979). This type of teaching is still the norm in most classrooms (Black & Wiliam, 2009). However, the recent research highlights the importance of a social classroom environment for effective teaching and learning. Studies suggest that learning is a product of social interactions within the members of a classroom community as well as individual reasoning, and therefore the environments which promote social interactions positively affect the process of learning (Cobb, Stephan, McClain, & Gravemeijer, 2011; Stephan & Akyuz, 2012; Stephan, Bowers, Cobb, & Gravemeijer, 2003).

In social classrooms students solve problems through inquiry. Inquiry is the process of wondering about an unknown subject or a problem, investigating, and attempting to answer it through cooperation with others (Wells, 1999). Chapman (2011) considers inquiry as a tool which can help students learn mathematics. She defines an inquiry-based environment as student-centered, rich in communication and cooperation, and based on research and asking questions. Despite the significant role of the teacher in creating inquiry-based environments (Johnson, 2013), it is observed that they face considerable difficulties in establishing and maintaining such an environment (Stein, Engle, Smith & Hughes, 2008). An important reason for this difficulty is suggested as the need for the teacher to effectively deal with and coordinate discussions that may ensue from a potentially large number of

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solutions that may be proposed within the classroom (Leinhardt & Steele, 2005). Teachers' lack of sufficient concept knowledge accentuates this problem. Furthermore, teachers' perception about mathematics that it is a set of rules and procedures and their belief that mathematics can only be learned with memorization rather than knowledge construction hinders establishing inquiry-based environments (Handal, 2003).

Different ways of behaviors that students adopt in such social classrooms are generally termed as social norms. (Cobb, Yackel & Wood, 1992). They define what is expected from the student in classroom discussions and what are appropriate or inappropriate ways of behaving. Among some examples of social norms are explaining a proposed answer, sharing alternative solution strategies, and expressing disagreement with others' solutions politely (Yackel & Cobb, 1996, Özmantar et al., 2009). Social norms are established through the joint effort of the teacher and the students and it is expected that each classroom community may have its own set of social norms.

Another type of norms that are specific to mathematics classrooms are called sociomathematical norms (Yackel & Cobb, 1996). These norms define the normative aspects of mathematical argumentations. For example, differentiating what is a mathematically different, elegant, or efficient solution can be considered among sociomathematical norms (Yackel, 2002). To clarify, in any subject matter a different solution/answer than those already provided can be given. This constitutes a social norm. However, proposing a mathematically different solution belongs to the realm of sociomathematical norms (Yackel, Rasmussen, & King, 2000). Sociomathematical norms are formed as a result of individuals' beliefs, values, and opinions related to mathematics. However, this relationship is reflexive in that in a classroom where sociomathematical norms are promoted individuals' beliefs, values, and opinions toward mathematics may change (Bowers, Cobb & McClain, 1999). Similar to social norms, it is expected that each classroom microculture may develop its own set of sociomathematical norms (Kazemi & Stipek, 2001).

Norms are typically identified based on the extent to which they are exercised within the classroom. Sfard (2008)'s theoretical perspective can be used for this purpose. According to this framework, for both social and sociomathematical norms to be counted as such they must be adopted by most of the classroom members and they must exhibit themselves clearly throughout the classroom discussions. As such, they are different from the rules or demands of the teacher; they must be internalized by the students themselves.

Recently, due to the increased use of technology in mathematics education, an important question that arose is how to effectively integrate technology in inquiry-based social classrooms. The research indicates that the effective use of technology may support inquiry-based education (Goos, Galbraith, Renshaw & Geiger, 2003; Hahkiöniemi, 2013). However, it is observed that teachers face difficulties for integrating technology into inquiry-based education (Drijvers et al., 2010). Beliefs of some teachers that mathematics is a set of procedures that must be memorized (Handal, 2003) or that memorization is essential in fulfilling the curriculum (Obora & Sloan, 2009) are seen as important obstacles for these teachers integrating technology and inquiry-based education. Another reason which impacts this integration is the lack of technological pedagogical content knowledge for many teachers (Mishra & Koehler, 2006). In order for the teachers to use technology effectively their content knowledge, pedagogical knowledge, and technological knowledge must be strong together and lesson plans must be adapted accordingly (Bowes & Stephen, 2011). Zbiek and Holllebrands (2008) highlight the importance of choosing the right problems in order to engage students in effective classroom discussions using technology. They argue that if right problems are chosen, teachers can support sociomathematical norms by using technology by asking students to share their solutions in a way that is accessible to all members of the classroom community. Despite these findings, the number of studies that investigate the social and sociomathematical norms in technology enriched classrooms is limited (Herskowitz & Schwarz, 1999; Pierce & Stacey, 2001). In particular, there is a lack of studies that investigate the sociomathematical norms developed by preservice teachers in an inquiry-based classroom that uses technology. Studies involving preservice teachers are especially important in that they will be more likely to be motivated to establish social and sociomathematical norms in their future classrooms.

Among many types of technology, dynamic geometry environments (DGEs) or software (DGS) open a new perspective in teaching and learning geometry (Healy & Hoyles, 2002; Straesser, 2002). DGEs allow users to create geometrical constructs such as lines, polygons, and circles and make interactive modifications on them. Among these modifications dragging, which is the process of modifying the shapes by moving their keypoints, has been the focus of many studies (Hölzl, 2001; Sinclair, 2004). These studies reveal that dragging is a pedagogical tool that facilitates understanding of mathematical reasoning, particularly in the process of generating conjectures, checking the validity of new problem situations, and generalizing the problems. Other types of interactive modifications include transformations such as translation, rotation and reflection (Abu Bakar, Ayub, Fauzi, & Ahmad Tarmizi, 2010). It has been shown that DGEs afford rich learning opportunities for proving conjectures as well (Laborde, 2000). DGEs are becoming more widely used thanks to open initiatives such as Geogebra[2] which are free and support numerous languages.

The main purpose of this study is to document the development of sociomathematical norms in a preservice teacher education course (course name: Exploring Geometry with Dynamic Geometry Applications) that uses inquiry-based teaching and learning in a technology-supported environment. More specifically, this study seeks the answer of the following research question: What are the technology related sociomathematical norms that are developed within an inquiry-based teaching and learning environment that uses dynamic geometry software for teaching circle properties to junior and senior level preservice mathematics teachers? Given the increased importance attributed to using technology (and especially dynamic geometry) in mathematics education and modern teaching approaches that utilize inquiry, it is hoped that the findings can help other researchers, teachers, and teacher candidates establish similar environments in their classrooms. This study is one of the first studies which aim to identify sociomathematical norms during the instruction of a course using dynamic geometry.

Method

Participants

This study includes ten junior and senior level college students from a large public university in Ankara, Turkey. Data is collected during an elective class which had the objective of teaching prospective teachers how to teach geometry effectively using DGS. All class sessions were conducted in a computer lab. Among the 10 students, 2 were senior and 8 were junior level. Their Grade-Point Averages (GPAs) were evenly distributed suggesting that they had different achievement levels.

Data Collection

The classroom sessions were conducted twice a week with each meeting two hours (4-hour per week). The data was collected during the 5-week part of the course that focused on the circle topic. All classroom sessions were video-taped. The research team comprised of a masters and two doctoral students along with the researcher met after every class and these meetings were also recorded. The students' homework assignments during this 5-week period were collected. These assignments allowed the research team to assess students' development and make necessary modifications in the instructional activities. Finally, the presentations made by each student at the end of the topic were video-taped. In these presentations, the students designed activities about the circle topic and asked their friends to solve them (i.e. they took the role of the teacher).

[2] http://www.geogebra.org

Course Content

Before the beginning of the instruction, the researcher prepared the content of the course as a series of activities according to the realistic mathematics education (RME) theory (Gravemeijer, 1994). The key aspect of the RME theory is to teach mathematics by connecting it with students' real-life experiences. Therefore, RME involves mathematization of a real-life concept. However, this does not mean that the concept must be based on the real-world but rather that students must be able to image it (therefore an imaginary science-fiction scenario might very well be used for RME). The current study began with an archeological excursion scenario but used other real-life concepts such as the solar eclipse. The initial activities are reviewed by the research team comprised of the researcher and three assistants who met after every class session and made changes based on the performance of the students. In the preparation of these activities, the researcher first partitioned the curriculum into sub-topics according to their order. Then for each sub-topic several RME based activities were created resulting in a total of 16 activities. These activities involved various circle tasks such as determining the relationship between an inscribed and central angle, understanding properties of a cyclic quadrilateral, locating the center of a circle, determining whether a given set of points lie on a circle, finding the number pi, computing the distance covered by a circle as it rolls on a flat surface, understanding the relationship between the number of rotations and the radii of various circles, computing areas of sectors, constructing tangent and secant lines, solving Regiomontanus' angle maximization problem, finding the trajectory of the midpoint of a ladder as slides down a vertical wall, determining where two circles that are rotating toward each other intersect, computing the shadow of a circle on another circle as in the case of a solar eclipse, and assessing when the ratio of a chord to the corresponding arc becomes the maximum. In post-class discussions sometimes these activities were revised, reordered, some of them were removed or new activities were added. Most activities were created by the researcher based on the literature and relevant examples from textbooks (Johnston-Wilder & Mason, 2005). In the preparation of these activities, special care was placed on them having multiple solutions which can be explored using DGS. Some activities were directly created on the DGS while others were created on the paper and students were expected to construct them during the class.

For a month before the beginning of the circle topic, the researcher tutored students about Geogebra through several activities that included various geometrical topics such as triangles, quadrilaterals, and transformations. These activities not only helped students to become competent with the software but also helped establishing classroom social norms. These norms included (a) openly expressing disagreement when necessary; (b) teacher's repetition of students' solutions to make them clear for everybody; (c) asking students that have different solutions to share their solutions; and (d) students' recreation of different solutions proposed by their friends on their own computer. The establishment of these norms allowed the inquiry-based education to become more effective.

Data Analysis

In the first phase of the analysis, the video and voice recordings were transcribed and coded. The researcher's statements were coded using **T** and an increasing number such as **T1**, **T2**, **T3**, etc. Students' statements were coded using letters **B** through **K** (for a total of 10 students) followed by successive integers. The assistant's expressions were coded by **A** and students' joint responses were coded by **Z** again followed by numbers.

After the coding process, the transcripts were analyzed to find repeating patterns of student behaviors. Sfard (2008)'s aforementioned theoretical perspective was used in this process. In order to identify norms, the candidate behaviors were put under investigation to ascertain whether they fit into Sfard's criterion. For clarity, this process will be briefly explained with an example. During the discussion of an activity which asked the students to explore the relationship between an inscribed angle and the corresponding central angle, one of the students offered the following explanation:

E1: You can also use the property that an exterior angle is the sum of the two interior angles.

E1 made this response after hearing a different explanation made by **J1**. This observation made the researcher suspect that proposing an alternative mathematical solution may be a sociomathematical norm. Later in another activity the following student response was observed:

K11: Can we do this? Instead of drawing the tangents from the left can we draw them from the right?

This proposal was again made after **J11** concluded his explanation about how to find the center of a circle using tangent lines. This strengthened the suspicion that proposing an alternative solution may indeed be a sociomathematical norm. Subsequent dialogues confirmed this suspicion. For instance, one student offered the following alternative solution during an activity which asked to find the tangent point on a circle:

D7: Wouldn't it have worked as well if we randomly drew a line and measured if the angle is 90 degrees?

In the subsequent activities several other examples of proposing alternative solutions were exhibited by other students as well. This led the researcher to conclude that the observed behavior of autonomously proposing an alternative mathematical solution had become an established sociomathematical norm. It was considered sociomathematical rather than social because the alternative solution must have been mathematically different.

Contrary to the above example, in many cases an initially suspected behavior was discounted as a norm due to not being sufficiently prevalent. For example, one of the candidate norms, "expressing that a mathematical rule must be known by the classroom members before being used", was rejected as a sociomathematical norm as it was only observed to be performed once by a specific student (as such it was not adopted by the classroom community at large).

To distinguish sociomathematical norms from social norms, the degree to which it involves mathematics (i.e. specific to a mathematics classroom) was used as the decision criterion. To this end, whether such a statement makes sense in a non-mathematics classroom was investigated. For example, one of the investigated norms was "sharing a solution/answer without being certain". This behavior was exhibited several times throughout the instructional sequence. Some examples are given below:

J17: I have an idea but I couldn't do it [starts explaining]

G20: I've thought of a way but I guess it doesn't work [shares the idea]

C16: I'm not exactly sure and would be happy if my friends help [shares the solution]

An investigation of these statements reveals that they are not specific to mathematics. Such behaviors may well be observed in history or in physics classrooms as well. Therefore, this norm was classified as a social norm rather than sociomathematical norm. To summarize, in order to identify the sociomathematical norms discussed in the next section the following three-stage process has been used:

1. Evaluate whether a suspected behavior is exercised `sufficient` number of times within the classroom. Here, there is no rule for how many times is sufficient but it must be exercised multiple times by multiple students throughout the instruction.
2. Evaluate whether the norm involves mathematics. For this purpose, answer the question of whether this norm may have been exhibited in a non-mathematics classroom. If the norm is not specific to mathematics, classify it as a social norm. Otherwise it is a sociomathematical norm.
3. Evaluate whether the norm involves technology. In other words, assess whether the fact that this course was taught using technology may have played a role in the development of this norm. If so, identify this norm as a technology related sociomathematical norm. Otherwise identify it as a general sociomathematical norm.

Findings

In this section, the sociomathematical norms that were found by using the aforementioned theoretical perspective during a 5-week long circle topic are shared. However, the focus is put on three specific norms that were related to using technology as this was a distinguishing aspect of the current study.

Sociomathematical Norm 1: Inquiring about the Effects of a Change Made in a Question or a Solution

This norm may include mathematical inquiries such as "what would have happened if the given circle had double the radius" or "what if we were given two points instead of three" as well as technology related ones such as "would the results have changed if we dynamically moved the position of the circle". The importance of this norm comes from the fact that it shows that students not only solve the current question but also want to explore alternative situations to gain a deeper understanding of a problem or a solution. One of the dialogues that evidence the establishment of this norm was seen during the third activity of the first class. This activity is shown in Figure 1.

F2: First, I found the intersection of the line and the circle. Next, as the line that passes through the center and perpendicular to this chord divides this chord into two equal parts, I found the midpoint of this chord [calls this **T**]. Finally, I draw a perpendicular line through **T** (Figure 1 (b) and (c)).

K6: But it disappears if we move the line.

F3: It is visible when you move it on top of the circle.

K7: What if we move the circle itself?

F4: It still doesn't matter, it divides into two.

Z4: Wherever we put the circle it divides it into two equal parts.

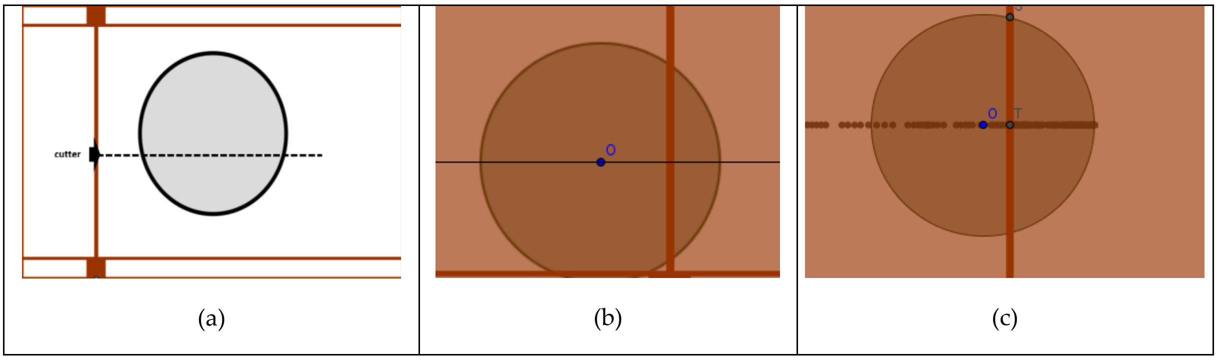

Figure 1. This activity asks where the cutter should be positioned so that it cuts the circle into two equal parts. The bar on which the cutter is placed can move left and right. The cutter can move up and down on this bar.

Here one of the students (**F2**) explains that in order to find the correct location of the cutter, she first finds the intersection of a vertical line and the circle (Figure 1 (b)). Next, she finds the midpoint of the chord created by this intersection (calls this midpoint **T**). She argues that if a perpendicular line is drawn to this chord through **T** it will divide the circle into two equal parts. However, **K6** argues that these lines are lost if the initial vertical line is moved sideways (this is because when the vertical line no longer intersects the circle the chord and all dependant drawings vanish). **F3** responds by saying that this vertical line must intersect the circle. Next, **K7** inquires what would have happened if the circle was moved instead of the vertical line. **F4** and **Z4** respond by saying (and showing) that regardless of the position of the circle a line perpendicular to the chord and that passes through its midpoint will always cut the circle into two equal parts. In this dialogue the statements that support the norm in question were found to be the inquiries made by **K6** and **K7** about what would have happened if the configuration were to change.

Another example that supports the same norm was observed during the second class – fourth activity (Figure 2). This activity asks whether the given points (symbolizing archeological excavation sites) reside on a circle or not. The following dialogue was observed during the solution:

H4: I took two points that are outside. I drew tangents from these points and found their intersection with the arcs. If I draw perpendicular lines from these points their intersection gives me the center.

E10: If these were not the arcs of a circle you wouldn't be able to draw the tangent, right?

T40: Yes. How else can we solve without connecting the points?

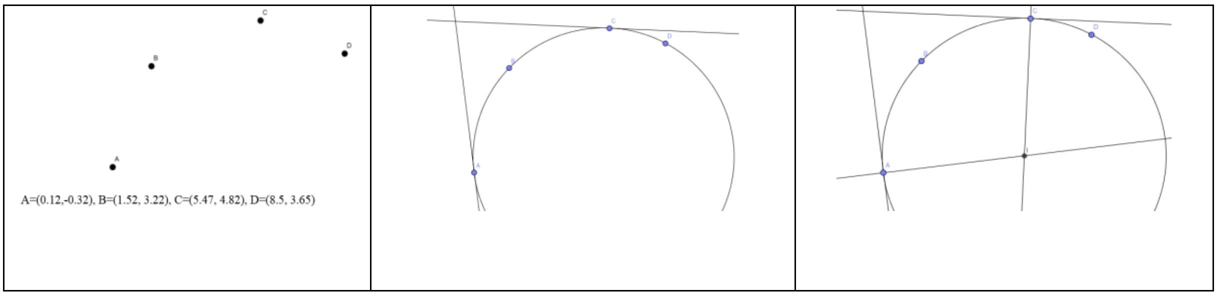

Figure 2. This activity asks whether the given four points symbolizing archeological excavation sites reside on a circle and if they do what should be the center of the circle.

Here, **H4** first connects the dots assuming that they lie on a circle and then draws two tangent lines from a point outside of the circle. He then argues that the intersection of the perpendicular lines that are drawn from the tangent points would give the center of this circle. However, E10, by asking that whether the same conclusion could be drawn had the points did not reside on a circle, questions about the effects of a configuration change on the proposed solution.

The second part of the same problem asks about how many points at minimum are required to ascertain whether the given points lie on a circle. After a period of individual activity and some discussions the students reached the conclusion that 3 points at minimum would define a unique circle and explained this algebraically (**J25:** Circle equation has three unknowns. We need three constraints to determine these three unknowns). The teacher then asked about how else this could be explained. The following excerpt is taken from the ensuing dialogue:

T63: Is there a different way?

H11: Let's consider a point that is closest to the given points. For instance, if we had two points this point can be on the right or on the left.

T64: Did we understand this? He is saying if we are given two points I can find multiple points that are at the same distance to them. But, if I have three points there is only one such point. How do we know three is enough?

H12: Three is minimum; it can be four or five as well.

Here **H11** argues that if we were given two points instead of three the circle that passes through them can be to the left or to the right. The teacher then asks what is special about three and **H12** replies that three is the minimum number but it would not hurt to have more points. As it can be seen from these dialogues, the statements **H11** and **H12** first inquire and then answer about what might happen if the configuration was different.

Throughout the study, there were many other examples of inquiries about what could happen if a change was introduced to the question, solution, or the configuration in general. Therefore, based on the adopted theoretical framework, this repeating behavior was considered as a sociomathematical norm. The investigation of multiple situations and effects of changes made are considered as important as it shows that students were not merely using what is given to them but they were actively exploring possible variations. This allowed them to reach more general conclusions from a specific problem.

It can be argued that students' development of this norm may be effected by the teacher's insistence for finding different solutions. Some examples are visible in the above transcripts. For example, in T40 the teacher asks about how the problem could be solved without connecting the points (thus making a change in the proposed solution). In **T63**, the teacher asks for a non-algebraic solution after a student explains his algebraic approach. In **T64**, the teacher questions what makes students believe that 3 points are enough to uniquely define a circle. In general, the teacher had demonstrated a similar behavior throughout the entire semester. It is likely that this approach made inquiring about variations in a question or a solution to be a very natural response which became embraced by the classroom community itself. Due to the mathematical nature of these inquiries, this behavior was considered as the first identified sociomathematical norm. Furthermore, this norm is attributed to technology as investigating different scenarios was facilitated by the use of the DGE.

Sociomathematical Norm 2: Reaching Conclusions by Using the Properties of the Tools in the Dynamic Software

This norm pertains to the behavior of solving questions by using the properties of the tools provided by the dynamic software that would otherwise be difficult to do on the paper. Using the tools of the dynamic software for solving questions was one of the most commonly exhibited behaviors by the students and hence identified as a norm. This norm exhibited itself in two forms. In the first form, students directly used a tool to compute an answer but did not reason about why or how the tool works. For instance, by using the "tangent tool" students could draw a tangent line to a circle from a point outside of the circle. In the second form, students used various tools to indirectly compute an answer. For instance, instead of using a tangent tool, students first made a distance computation and drew another circle whose diameter matched the computed distance. The tangent point was computed as the intersection of the two circles. This second form was considered as a more elaborate and preferred solution. In general, the researcher tried to discourage the first form of this norm as it did not contribute to mathematical understanding.

An example to this norm is given below. Here, the activity asks that where the shadow of a clock that sits on a horizontal plane would fall due to a light source whose position can be dynamically changed (Figure 3). The dialogue below shows the dynamic geometry solutions proposed by the students when the light source is at position A:

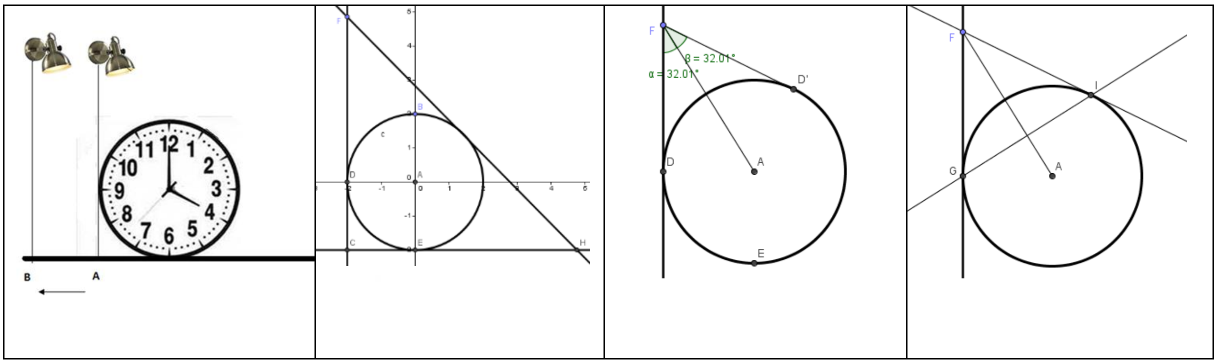

Figure 3. This activity asks where the shadow of the clock would fall with respect to a light source whose position can be changed from A to B.

G11: First I drew a circle. Then I drew a tangent to this circle. I took this point as the tangent may pass through both from the bottom and here (point F in Figure 3). Finally, I intersected these with the x-axis on which the clock sits.

T126: How else can I do it without using the property of the tangent tool?

F20: According to the tangent's property, a line segment through the center must be the angular bisector.

T130: Does everybody remember? Is there such a property?

Z17: Yes.

F21: Therefore I first found the angle [calls this alpha]. If I rotate point D around F by 2*alpha, I can find the other arm of the angle (Figure 3).

T131: How else can I do it?

F22: I can use symmetry and then combine.

[Here some students that do not understand this proposal ask for clarification and **F** explains in more detail.]

T133: Ok, let's look at your solution. If there are other proposals we can discuss later.

H22: I first drew the circle and then put the light. I want to take integer coordinates. First, I drew a line segment that connects the light with the center of the circle. Then I drew a line perpendicular from the light down to the circle and found the point of intersection as **G** [the tangent point]. Next, I drew a line perpendicular from **G** to the first line segment through the center. I formed an isosceles triangle [which gives the second arm].

In this question, most students observe that the shadow of the clock would fall between the intersection points of the tangent lines from the light and the x-axis. To find these points, **G11** proposes to use the tangent tool in the software. The teacher then asks how this can be solved without using the tangent tool. F20 argues that based on a rule that she remembers the line segment that connects an outside point with the center of the circle becomes the angular bisector of the angle formed by two tangents from that point. Therefore she first connects the light position with the center of the circle. Then she measures the angle between this line and the tangent line that goes down vertically from the light (calls this angle alpha). Then by rotating this tangent line around the light position by twice this angle she obtains the second tangent. The teacher then inquires about an alternative solution to which **F22** replies by saying she could also use symmetry. The ensuing discussion reveals that by symmetry what she meant was finding the reflection of the first tangent with respect to the line that connects the light to the center of the circle. Finally, the teacher asks **H** to share his solution. **H22** explain that if he draws a line perpendicular from point **G** to the segment **FA**, the intersection of this line with the circle gives the second tangent's intersection point I (see Figure 3). He bases this argument on the fact that the FGI triangle is isosceles.

As can be seen in this example, the students used the properties of four different tools in solving this problem: drawing a tangent from a point to the circle (**G11**), rotating a line around a point by a specified angle (**G21**), reflection transformation (**F22**), and drawing a perpendicular line from a point to a segment (**H22**). It can be argued that the first of these four solutions is more rudimentary compared to the other three as it simply involves using a tool without basing it on any mathematical property. The other solutions are more sophisticated in that although the tools afforded by the software are used they require understanding of several different mathematical ideas. As this example demonstrates, different ways of using the software tools may give clues about the level of students' mathematical understanding (**F** and **H** appear to have a better grasp of the tools as well as the geometric rules). The appearance of these sophisticated solutions seems to be caused by the teacher's repeated inquiry for alternative solutions (**T126**, **T131**, **T133**).

Another example of this norm can be seen in the excerpt below. This activity asks what type of trajectory would the midpoint of the ladder make as it slides left on the ground (Figure 4).

J38: First, I found the midpoint of BC. Then I drew a segment that connects this point with the origin. When I trace point F it looks like the arc of a circle. I rotated the AF segment. By connecting the rotation angle to a slider I covered many different values.

T152: Let's do it.

J39: I rotate the segment by alpha. It gives a circle.

[At this point students talk for a while]

T154: Ok, why this gives us a circle?

K40: In any right triangle we can find the median from the midpoint. Here AF is the median. In a right triangle, the median is equal to the parts it divides. And because these parts remain constant so does the median giving us a circle.

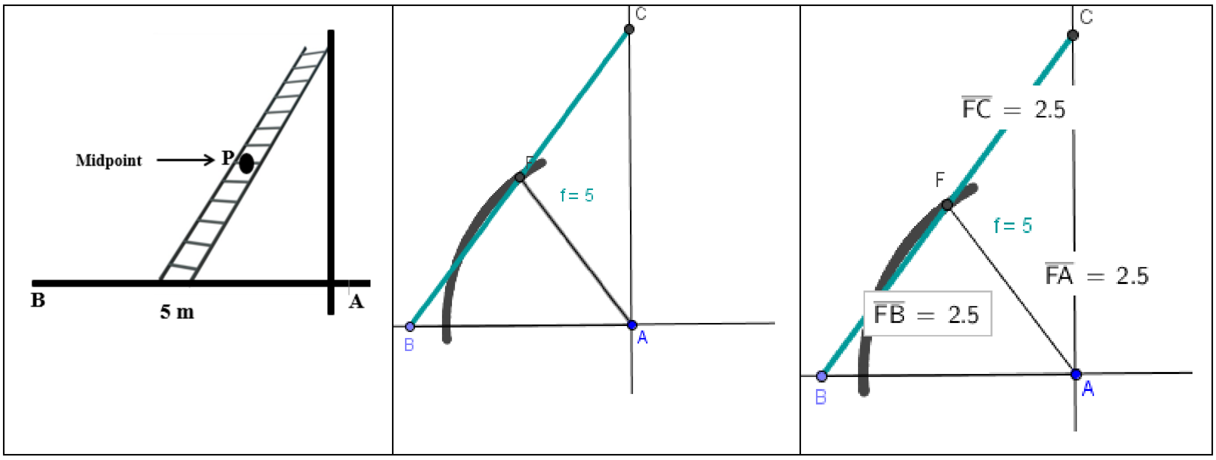

Figure 4. This activity asks what type of trajectory would the midpoint of the ladder make as it slides left on the ground

In the solution of this question, **J38** first finds the midpoint of the ladder (point F) by using the midpoint tool in the software. Next, he connects this point with the origin. When he traces this point, he observes that the resulting trace resembles a circle. Here it can be seen that the student was initially uncertain about the trajectory but he gained confidence by using the trace tool in the software. After this, the teacher asks about why the resulting shape could be a circle. **K40** argues that in a right triangle the median is equal to the parts that it divides and because these parts remain constant regardless of the position of the ladder so does the median. Hence this median can be considered as the radius of a circle centered at A.

An important point that is constantly observed in these examples (and other examples supporting this norm) is the teacher's inquiry about why a solution developed by using a tool is correct. This compelled student to go beyond a tool based solution and investigate the mathematical principles that underlie it.

Sociomathematical Norm 3: Dynamically Verifying a Solution or a Hypothesis

This norm involves dynamically changing a figure to verify whether a hypothesis is correct or not. The students exhibited this norm to either illustrate that a solution that they are confident about works for the general case or to build confidence about a less certain hypothesis. The excerpt below is taken from a session where this norm was observed. Firstly, **K1** makes the following observation for a question which asks the relationship between an inscribed angle (angle made by two chords) and a central angle (Figure 5):

K1: First let's draw an isosceles triangle. Next, because these angles are equal so are the corresponding arcs. I can show this dynamically changing the position of the points.

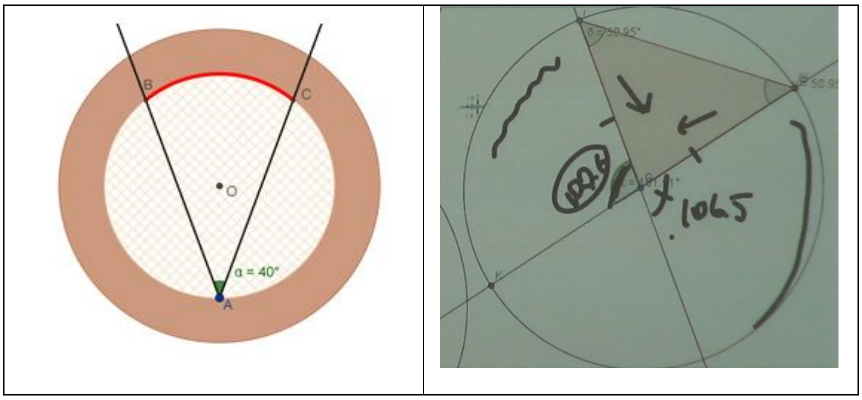

Figure 5. This activity asks how much the field of view angle of a camera should change when a photographer who was originally at point A and capturing his friends on the chord BC moves to point O (center of the circle).

In this solution, to show that a central angle is twice the size of an inscribed angle that sees the same arc, **K1** first draws an isosceles triangle. Next, she measures the exterior angle of this triangle to the left of the center of the circle and argues that the angle to the right is equal to this angle. From here she observes that the sum of the two interior angles is equal to the opposite exterior angle (and because the interior angles are the same for a circle, they become half of the exterior angle). Finally, she drags the points dynamically to show that this relationship holds for different isosceles triangles.

In other activity that was discussed earlier, the question asks how to cut a given circle into two equal parts (Figure 1). A possible solution proposed to draw two tangents to the circle from a point on the bar and intersect perpendiculars drawn from the tangent points to find the center of the circle. The following dialogue is taken from the subsequent discussion:

H2: From this intersection point let's draw a perpendicular to the cutter and find their intersection.

E8: But then we didn't use our other drawings.

H3: We used them to find the center.

J16: Ok, now it works.

E9: Move the circle.

Here, **H2** proposes to move the cutter to the point which is at the intersection of a vertical line drawn from the center of the circle to the bar on which the cutter sits. E8 then argues that the other drawings (tangents) are not used. **H3** respond by saying that they are used to find the center of the circle. Finally, **E9** asks to move the circle dynamically to check that whether the proposed solution still works for arbitrary positions of the circle. In doing so, she provides an example of this norm.

Another example that can be given to this norm was observed in the ladder activity discussed earlier (Figure 4). Here, **J39** had argued that the midpoint of the latter follows a circular trajectory by using the trace tool. In the dialogue below, **G23** verifies this by dynamically dragging the bottom point of the ladder:

T155: Did you see that it is a circle without tracing?

Z21: Yes

T156: How?

G23: The length of AF does not change. As we move point **B** it remains 2.5. Then it must be radius and it is a circle.

F29: I think this is where this proof comes from [that median of the hypotenuse is equal to the parts it divides].

H29: Indeed, it is like the proof of this property.

Here, the teacher asks the students how they can conclude that the trajectory is a circle without using the trace tool. **G23** respond that as she moves the bottom point of the ladder (point **B**), the length of the **AF** segment does not change and therefore this segment must be the radius and the followed trajectory the arc of a circle. The following observations made by **F29** and **H29** are interesting in that they show how the students realized where a property comes from. As the lengths of **BF**, **FC**, and **FA** segments remain constant as the point **B** is moved, they observe that the median drawn to the hypotenuse is always equal to the bisected parts; a well-known property of right triangles.

An important point in this norm is that the teacher must stress that it is not enough to solve a question by dynamically changing the figure. Dynamic verification should only be used to develop an intuition and it should only help to discover the underlying mathematical principles.

In this section, the focus was put on sociomathematical norms that involve technology. However, other social and sociomathematical norms were observed in the current study as well. Due to space constraints, it is not possible to discuss them at length but they are listed below for the sake of completeness:

- Proposing an alternative mathematical solution
- Sharing a solution/answer without being certain
- Supporting a peer's solution
- Asking for clarification for a peer's solution

The latter three of these norms may be considered as social norms as they may be employed in non-mathematics classrooms as well. The first one is considered sociomathematical as it involves the notion of mathematical difference.

Discussion and Conclusion

This study focused on the identification of sociomathematical norms in a mathematics classroom that used technology during the teaching of the circle topic. Special emphasis was given on the norms that involve technology. With respect to existing literature, it can be argued that these three norms that are relevant to the combination of technology and mathematics are first identified by the current study. These new norms suggest that in mathematics classrooms that integrate technology a new type of norms that may be termed as "techno-sociomathematical norms" may be postulated in addition to existing social and sociomathematical norms.

Given that technology becomes more widely adopted in mathematics education (and in every other field), understanding such norms may shed light on in which ways students use technology. For example, the second norm identified in this study, namely "reaching conclusions by using the properties of dynamic software" gives clues about how students actually use technology when solving a problem. According to this, if students are not carefully guided by the teachers they tend to solve questions by merely relying on the tools that exist in the software without understanding the underlying mathematical ideas. Another norm that requires careful observation appears to be the third norm which is "dynamically verifying a solution or a hypothesis". Similar to the second norm, students seem to have a premature tendency to show that a solution works by simply dragging the figure and without understanding why it works. Transforming these norms into valuable habits seems to depend on the teacher. The teacher should not accept such explanations as sufficient but should prompt students to use these as intuition-builders for understanding underlying ideas. To this end, the role played by the teacher in the current study can be linked to instrumental orchestration which states that the teacher has an important guiding role in order for the students to develop correct strategies for using a tool (Trouche, 2004).

What other responsibilities does the teacher have in order to develop effective norms? Firstly, it must be understood that norms are different from the demands of the teacher from the students (Levenson, Tirosh & Tsamir, 2009). For establishing effective norms it is not enough to demand them or leave students on their own to discover them (Tatsis & Koleza, 2008). In inquiry-based settings, teachers have an important coordinating role to help students discuss different solutions effectively and clarify them to reach conclusions. If this is not done, that is if the questions that students may have are not satisfactorily answered, students may have difficulties in comprehending the subjects (Sanchez & Garcia, 2014). Existing studies show that the same norm may get associated with a different norm by different teachers. For instance in one study, while both classrooms established the norm of explaining solutions, one teacher focused on other students repeating a proposed solution and another teacher focused on relating a proposed solution with other solutions (Lopez & Allel, 2007). This highlights the importance of the guiding role that the teacher has in establishing different norms.

Educational programs define mathematics learning environments as one where students can inquire, communicate, think critically, and openly share their possibly different ideas (MEB, 2013). Such an environment is only possible through norms that support this environment. Although norms may change from classroom to classroom, existing research suggest that some norms that promote inquiry-based environments may be common across classrooms. Identification, sharing, and demonstration of these norms are important for other teachers, teacher candidates, and educators who may desire to create similar environment in their own classrooms. To serve this purpose, in this study several sociomathematical norms that were established during a 5-week long instruction period that is integrated with technology are identified and discussed with examples. Among the discovered norms, it was observed that some norms appear to be specific to combining technology with mathematics and their treatment under a novel theoretical categorization and framework is identified as a future study. Additionally, it was observed that these norms are not necessarily useful on their own and can turn into useful behaviors only under the correct orientation of the teacher. It is hoped that these findings will help teachers, teacher candidates, and educators in general to establish or provide assistance in establishing similar norms in other classrooms.

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