Education and Science
Vol 48 (2023) No 216 39-66
Original Article

An Investigation of 4th Grade Students' Statistical Thinking

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Abstract

The purpose of the current study is to investigate 4th grade students' statistical thinking. The research was conducted on 187 fourth grade students and the students were asked to work on tasks developed on the basis of four different contexts. In the study, the qualitative survey research design was adopted. The collected data were analyzed on the basis of the framework of statistical thinking levels for primary school students. The findings of the study revealed that the students' levels of statistical thinking are higher in the constructs of describing, representing, analyzing and interpreting data than in the constructs of organizing and reducing data. The students were found to be most successful in reading the data related to the construct of describing data, followed by the evaluation of similarities/differences of data and graphs. The students were observed to have difficulty in interpreting the concepts of average, distribution and variation considered to be related to the construct of organizing and reducing data. In relation to the construct of representing data, the students were found to be more successful in completing an uncompleted graph than representing data with different types of representation. In relation to the construct of analyzing and interpreting data, the students were found to be more successful in reading between the data than in reading beyond the data.

Keywords

Statistical thinking
4th grade students
Data handling
Graphs
Average
Distribution
Variation

Article Info

Received: 06.24.2022
Accepted: 06.19.2023
Published Online: 10.24.2023

DOI: 10.15390/EB.2023.11986

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Introduction

In today's world, the presence of quantitative information in all areas of life has been emphasized (Ben-Zvi, 2000). Being able to critically evaluate data-based claims and present evidence-based arguments are important skills that all people should have (Burrill & Ben-Zvi, 2019). In our age, the critical role of this information in our lives requires being able to read this information correctly, analyze it and make decisions on the basis of it (National Council of Teachers of Mathematics [NCTM], 2000). Statistical thinking can be defined as the ability to see statistical research holistically as well as understanding how and why statistical studies are done (Chance, 2002). High statistical thinking individuals can see the connections between statistical ideas and solve statistical problems with different methods (Carver et al., 2016). This situation requires individuals to have knowledge and skills of statistics. This awareness emphasizes giving more importance to statistics education and helping students gain statistical thinking skills at every grade level (Bargagliotti et al.,2020; Franklin et al., 2007; Ministry of National Education [MoNE], 2018). This emphasis includes a broad perspective that considers the process of doing statistics as a whole, rather than a narrow perspective that considers just drawing graphs or calculating measures of central tendency as enough for students (Jones et al., 2000; Kinnear, 2013; Leavy & Hourigan, 2018; Shaughnessy, Garfield, & Greer, 1996). Students' having statistical thinking skill has a great role in understanding this process. Imparting the statistical thinking skill to students and thus preparing them for real world are stated to be an important stage of the teaching process (Carver et al., 2016; Franklin et al., 2007) because the increasing need for the results of statistical studies in the society makes the statistical thinking skill one of the important determinants in the development of countries (Franklin et al., 2007; Groth, 2006). Therefore, mathematics teachers and curricula should encourage students to think statistically. (Franklin et al., 2015; Wild, Utts, & Horton, 2018). For this encouragement to be successful, it is necessary to understand students' statistical thinking (Groth, 2006).

In Turkey, at the primary school level, statistical knowledge and skills are addressed within the "data handling" learning domain and knowledge and skills related to this learning domain are included at every grade level of primary education (MoNE, 2018). In pre-school education (although not compulsory in Turkey), which forms the basis of primary school, it is observed that knowledge and skills related to statistics (e.g., classification, creating graphics) are addressed at a very basic level (MoNE, 2013). In the primary school mathematics curriculum, it is emphasized that data teaching should be structured by considering the components of creating researchable questions, collecting data, processing and analyzing data and interpreting the results (MoNE, 2018). In the first grade, the focus is on reading simple tables with at most two data groups, while in the second grade, the focus is on collecting data for a given research question, representing and interpreting the data with tables and picture graphs, preparing a frequency table and tree diagram and reading the picture graph. In the third grade, it is aimed that students work with more data groups, read and interpret simple tables with at most three data groups and organize the data they get from the table. In the fourth grade, it is aimed that students evaluate and create the bar chart and use different representations of the data in the process of solving problems related to daily life (MoNE, 2018). However, it has been observed that statistical concepts such as distribution, average and variation, which are described as big ideas in statistics, are either not included in the curriculum or they are implicitly included (Batur, Özmen, Topan, Akoğlu, & Güven, 2021; Frischemeier, Kazak, Leavy, Meletiou-Mavrotheris, & Paparistodemou, 2022).

The existing research shows that the main focus in the early years of education is on students' creating and reading simple data representations, rather than their questioning data that contains original and meaningful context (Fielding-Wells, 2018). However, what is emphasized should be that students are enabled to take the process of doing statistics as a whole and experience the processes of defining, organizing and reducing and interpreting data. This suggests that more research is needed in primary school classrooms where there is a tendency to focus on graphs (Jones et al., 2001; Shaughnessy et al., 1996; Frischemeier, 2018). However, it has been shown that most of the studies on students' statistical knowledge have focused on university students, some on middle school students and very few on primary school students in the last decade (Eichler & Zapata-Cardona, 2016). It can be said that the basis of the studies carried out on the statistical thinking of middle school students is formed by the studies on primary school students (Mooney, 2002). The results of the studies conducted on middle school students showed that while students showed higher statistical thinking levels in describing data construct, they showed lower levels of statistical thinking in other constructs (organizing and reducing data, representing data, analyzing and interpreting data) (Akkaş, 2009; Koparan & Güven, 2013, 2014; McGatha, Cobb, & McClain, 2002; Mooney, 2002). Altaylar and Kazak (2021), on the other hand, in their study carried out to reveal the effect of realistic mathematics education on the statistical thinking levels of 6th grade students. The findings revealed that both the experimental and control group students' describing data, representing data and analyzing and interpreting data constructs were concentrated at Level 1 before the instruction. Students were found to be mostly at Level 3 in organizing and reducing data construct.

The existing research on primary school students' statistical thinking has focused on a variety of topics. In addition to the studies in which students' knowledge and skills about graphs were examined (Estrella, 2018; Pereira- Mendoza & Mellor, 1991), how students perceived informal concepts was also investigated. For example, there are studies on central tendency and dispersion measures (Konold & Higgins, 2003; Mokros & Russell, 1995; Petrosino, Lehrer, & Schauble, 2003; Strauss & Bichler, 1988; Watson & Moritz, 2000), sampling (Abrahamson, 2012; Makar, Fielding-Wells, & Allmond, 2011), statistical reasoning (Leavy & Hourigan, 2018; Lopes & Cox, 2018) and inferring meaning from data (Doerr, Delmas, & Makar, 2017; Fielding-Wells, 2018; Paparistodemou & Meletiou-Mavrotheris, 2008). However, there are very few studies (Jones et al., 2000, 2001) that include the whole statistical process. In their study, Jones et al. (2000) aimed to reveal and validate a framework to be used in the analysis of the statistical thinking of primary school students. To this end, interviews were conducted with 20 students from 1st to 5th grade. The results obtained confirmed that primary school students experience difficulties similar to the ones experienced by middle school students. It was found that the students' statistical thinking related to describing data is higher than their statistical thinking related to other constructs (Jones et al., 2000). Jones et al. (2001) designed and evaluated a teaching experiment for second graders. After the teaching experiment, it was revealed that the students' experiences with different types of data reduced their own unique definitions. However, the students had more difficulties while working on categorical data than on numerical data. It was observed that the use of technology in the teaching experiment process had a positive effect on the students' thoughts about the organization and representation of data. The students were able to think conceptually about the concepts of centre and dispersion. In this process, the students' contextual knowledge played a key role. When the statistical thinking of the students was evaluated holistically, it was revealed that at least 84% of the students displayed Level 2 or better thinking in all the constructs after the teaching experiment.

Studies (Jones et al., 2000) draw attention to the need for more research on the statistical thinking levels of larger samples of primary school students from different cultures and linguistic backgrounds. Jones et al. (2000) focused on all the components of statistical thinking, but they worked on only 20 students including students from each grade level of primary education (from 1st to 5th grade), indicating that there is a need for more detailed information about the statistical thinking of larger samples of primary school students. Since the whole process is examined in the current study, it can be thought that it will contribute to the literature in this context because it is argued that the knowledge and skills related to statistics should be emphasized from an early age on a broad perspective that includes describing, organizing, representing, and interpreting data instead of focusing only on drawing graphs (Jones et al., 2001; Shaughnessy et al., 1996). Another point that researchers (e.g., Jones et al., 2001) draw attention to is that students' statistical thinking is a support in designing instructional activities or hypothetical learning trajectories related to the teaching process. At this point, it can be thought that the findings obtained from the current study will help teachers in the process of developing appropriate instructional activities. In addition, it is thought that it is important to reveal what students' statistical thinking levels are because given that one of the main purposes of teaching is to equip students with statistical thinking skills (Carver et al., 2016; Franklin et al., 2007), it can be thought that revealing students' statistical thinking can provide important insights for the teacher in terms of shaping the teaching process.

In addition, in the current study, it is aimed to observe what students know informally about various concepts (e.g. average, variation, and distribution) as well as some concepts they have learned formally. What is meant by informal is what students know and what understandings they have about the concepts (e.g., average, variation and distribution) that are not included in the curriculum because these concepts are the concepts at the centre of statistics, but they are seen to be very difficult concepts for students (Garfield & Ben-Zvi, 2005; Paparistodemou & Meletiou-Mavrotheris, 2008). Also, students' informal understanding of statistical concepts is seen to be an important factor affecting their statistical thinking because it is argued that it will be beneficial for their formal education to reveal how they perceive the concepts informally before they see them formally (Makar, Bakker, & Ben-Zvi, 2011) because these concepts are the basis for the development of statistical thinking and allow us to see the world from a statistical point of view (Campos, Wodewotzki, & Jacobini, 2011). Indeed, Statistical Reasoning, Thinking, and Literacy (SRTL) international research forum have drawn attention to the need to focus on what students know about their informal inferences (SRTL, 2017). Another point emphasized is that considering students' informal inferences in the learning process supports students to develop rich conceptions of statistical ideas (English & Watson, 2015; Makar, 2014, 2016; Watson & English, 2015). Revealing what students know informally before they learn the concepts formally can support the shaping of formal teaching. In addition, the results obtained can support both practitioners and curriculum developers. Knowing what students know at each level of statistical thinking constructs and how students' statistical thinking levels differ can guide both teachers who structure the teaching process and researchers working in this field. If teachers are aware of their students' statistical thinking levels, they can be aware of the difficulties they will encounter in advance, and they can plan their lessons and choose appropriate teaching methods in such a way as to help students cope with these difficulties and they consider these levels in the evaluation. The aim of this study is to investigate how primary school 4th grade students make meaning of some concepts (variation, distribution, average) informally as well as of the statistical concepts they have learned formally, and in this way, to examine their statistical thinking. Thus, answers to the following research question were sought:

What is the fourth grade students' level of statistical thinking regarding the construct of i) describing, ii) organizing and reducing, iii) representing, iv) analyzing and interpreting data?

Theoretical Background

Statistical thinking has been the focus of attention of many researchers and what it includes has been investigated widely (Ben-Zvi & Friedlander, 1997; Groth, 2003; Hoerl & Snee, 2001; Jones et al., 2000; Mooney, 2002; Wild & Pfannkuch, 1999). Wild and Pfannkuch (1999) created a general perspective on statistical thinking and presented a model consisting of four different dimensions (the investigative cycle, types of thinking, the interrogative cycle, dispositions). Another striking point in this model is that it aims to reveal what individuals think about each dimension at the same time (Wild & Pfannkuch, 1999). On the other hand, Ben-Zvi and Friedlander (1997) tried to reveal which statistical thinking processes 13–15-year-old students go through in teaching processes and they defined a model consisting of four levels. These levels are described as follows; Level 0: Uncritical thinking, Level 1: Meaningful use of a representation, Level 2: Meaningful handling of multiple representations: developing metacognitive abilities, Level 3: Creative thinking (Ben-Zvi & Friedlander, 1997). Hoerl and Snee (2001) examined the statistical thinking processes of individuals by creating a checklist based on two main models (statistical thinking and evaluation) and two main strategies (problem solving and process development).

Since the models proposed by Wild and Pfannkuch (1999) and Hoerl and Snee (2001) show different types of thinking involved in statistical thinking, they accept that several different models have become functional for statistical thinking (Pfannkuch & Wild, 2002). In addition, the model created by Hoerl and Snee (2001) mostly focuses on statistical thinking in the field of business (Pfannkuch & Wild, 2002). Although the model proposed by Wild and Pfannkuch (1999) provides a general perspective on statistical thinking, it is noted that the dimensions and strategies in these models should be further elaborated (Pfannkuch & Wild, 2002). Although Ben-Zvi and Friedlander (1997) define levels to characterize students' statistical thinking, it is stated that more research needs to be conducted on whether students' determined developmental stages are hierarchical and whether students pass through each stage linearly (Pfannkuch & Wild, 2002).

Jones et al. (2000) focused on the statistical thinking of young children and constructed a model based on the four levels of thinking (idiosyncratic, transitional, quantitative, analytical) from the solo taxonomy (Biggs & Collis, 1982) and four basic constructs (describing data, organizing and reducing data, representing data, analyzing and interpreting data). Jones et al. (2000) used this model to reveal how statistical thinking of children from 1st to 5th grades was and validated this model. This model can serve as an indicator of what the teacher should follow while planning his/her learning processes, as well as informing the teacher about the level of statistical thinking of his/her students (Pfannkuch & Wild, 2002). Indeed, it can be said that the foundations of the model used in many studies examining students' statistical thinking at different grade levels (e.g., Groth, 2003; Mooney, 2002) are derived from the model proposed by Jones et al. (2000). Mooney (2002) analyzed the statistical thinking of middle school students in depth, based on the framework developed by Jones et al. (2000) for primary school students. He conducted interviews with middle school 6th, 7th and 8th grade students. Based on the data obtained, he both revealed the statistical thinking of middle school students and validated the created framework. This framework is composed of 'describing data', 'organizing and reducing data', 'representing data' and 'analyzing and interpreting data' constructs. Groth (2003) characterized the statistical thinking of high school students based on similar components.

Although some models (e.g., Ben-Zvi & Friedlander, 1997; Hoerl & Snee, 2001; Wild & Pfannkuch, 1999) are observed to offer important insights into statistical thinking, it is emphasized that more research is needed to make these models functional (Pfannkuch & Wild, 2002). In this context, it can be said that Jones et al. (2000) differ from other statistical thinking models because they presented detailed indicators about constructs and levels of statistical thinking. In addition, studies (e.g. Groth, 2003; Mooney, 2002) have been conducted to validate these indicators both at primary school and at different grade levels (e.g., middle, high). These points make us think that this model will allow examining the statistical thinking of students in more detail. Moreover, Jones et al. (2000) focused on primary students. Since the current study aims to examine the statistical thinking of primary school students in detail, Jones et al. (2000) model was used. Jones et al. (2000) worked on students of 1st-5th grades and evaluated the statistical thinking of students from two perspectives. First, they divided statistical thinking into four constructs: describing data, organizing and reducing data, representing data, analyzing and interpreting data. They analyzed each construct in terms of four levels of thinking, which they named idiosyncratic, transitional, quantitative, and analytical. Each construct and each thinking level is explained below.

Statistical thinking constructs

Jones et al. (2000) stated that describing data construct includes reading data representations, showing awareness of basic rules of forming graphs (e.g. title, axis labels), being aware of the representation of the same data with different representations, and evaluating the representation of the same data with different types of representation. They also stated that students can be asked questions like "What does this picture tell you?, Do you think these pictures represent the same data?" to reveal their knowledge and skills related to this construct (Jones et al., 2000; p. 274).

Organizing and Reducing Data includes grouping and ordering data, recognizing that information may be "lost" in rearranging the data, explaining the data in terms of representativeness or typicality, and evaluating the data in terms of distribution. Questions such as "How would you organize this data in another way?, What is the average number of friends who came to visit?, Which of these sets of scores have the greatest spread, or do they have the same spread?" can be used to measure this construct (Jones et al., 2000; p. 275).

Representing Data construct includes completing a partially constructed data representation and being able to represent a data set with different types of representation. Questions such as "Complete this graph, How would you organize and present this data in another way?" can be asked to elicit knowledge and skills related to this construct (Jones et al., 2000; p.276).

Analyzing and Interpreting Data construct as comparing and combining (reading between the data) and making predictions on the basis of the data (reading beyond the data). Questions such as "Which day had the highest number of visitors? (compare), How many friends came to visit during the week? (combine), About how many friends would you expect to visit during the next 4 week? (predict)" to uncover knowledge and skills related to this construct (Jones et al., 2000; p.277).

Statistical thinking levels

Jones et al. (2000) defined statistical thinking at four levels and expressed these levels as idiosyncratic (level 1), transitional (level 2), quantitative (level 3), analytical (level 4). In the table below, besides the definitions of these levels, sample student answers for each level are given.

Table 1. Statistical thinking levels

Level	**Definition**	**Sample situation**	**Sample student answer**
Level 1 idiosyncratic	Students tend to make explanations that are unrelated to the data. Students' comments are mostly based on their personal experiences	How many friends do you expect to come to Sam's house each week during summer vacation?	I expect four people to come because four friends of mine visited me in summer (Jones et al., 2000; p. 293).
Level 2 transitional	Students try to make inferences using quantitative thinking. However, these inferences may be incomplete and hesitant	What is the average number of friends that came over to visit Sam each day?	Between 7 and 0. It's somewhere there, but I don't know (Jones et al., 2000; p. 293).
Level 3 quantitative	Students can use quantitative thinking effectively. While they can approach problem situations from different perspectives, they are successful in developing their own solution strategies.	What is the average number of friends that came over to visit Sam each day?	About 3 or 4. This one has 3, this has 4, this has 7. So if you take 3 away from that [the 7] and give it to the day with 0, you have about 4 (Jones et al., 2000; p. 297).
Level 4 analytical	Students have the multidimensional thinking skill. While making inferences from the data, students at this level use their contextual knowledge as well as quantitative thinking.	When they were asked whether bar graph and line plot representations consisting of the same data represent the same data	Well, like on Tuesday, no one came; on Saturday, 7 came on both of these graphs; they are all the same number [of friends] that come over on the other days. And they are all friends that came. (Jones et al., 2000; p. 299).

As seen in Table 1, while the answers of students in level 1 thinking are based on their personal experiences, students in level 2 thinking try to explain their inferences using quantitative thinking. However, these comments may be incomplete and hesitant although the student cannot give a specific value about the average, it can be said that he/she is aware that the value is in the range of 0-7). Students in Level 3 can use quantitative thinking effectively (e.g., although he/she could not say anything clearly, he/she structured his/her comments by using the balance point meaning of arithmetic mean). Students who are in level 4 thinking have multidimensional thinking skills and can use contextual knowledge as well as quantitative thinking when they make inferences from data (e.g., students were able to evaluate two different representations by associating their quantitative thinking with their contextual knowledge).

Method

Design of the study

In the current study, it aimed to examine the thinking levels of 4th grade students regarding various constructs of statistical thinking (describing data, organizing and reducing data, representing data, analyzing and interpreting data). Therefore, the survey research design was employed. This design allows questioning and explaining the focused characteristics of a population from various aspects (Fraenkel & Wallen, 2006). In addition, the qualitative survey research design was adopted as it aimed to analyze how students' responses are distributed (Jansen, 2010).

Participants and context of the study

A total of 187 fourth grade students participated in the current study. The reasons for working on 4th grade students were stated under several headings. First of all, students at this grade level received formal training on some statistical concepts. In addition, they might have had some daily life experiences related to statistical concepts that they have not seen formally. Moreover, the thinking processes of 4th grade students have an important role in structuring the teaching processes of middle school students regarding statistical concepts.

Of the participating students, 97 are females (52%), 90 are males (48%). The participants were selected by means of the convenience sampling method (Fraenkel & Wallen, 2006) from three different state schools located in Karaman in the Central Anatolian Region of Turkey. The socio-economic levels of these schools were determined to be medium, close to each other. In addition to representing the data with a tally or frequency table, these students also learned about reading these tables during the instructional process. They have also learned about collecting data by asking questions about a problem or a subject, representing these collected data with picture and bar graphs and making conversions between these representations (MoNE, 2018). However, it has been mentioned that the concepts of average, distribution and variation are not included in the primary school mathematics curriculum. In this study, besides the questions about using the bar graph, picture graph and table, questions about how they perceived the concepts of average, distribution and variation informally were also included.

Data collection tool

In order to reveal the statistical thinking of 4th grade students, questions were developed on the basis of the study by Jones et al. (2000). In addition, formal concepts and informal concepts that students learned were examined and the curriculum was used in this regard. The first and second questions were formulated based on the context of "Sam's friends" from the study by Jones et al. (2000). In the context of Sam's friends, they were asked to evaluate the number of friends Sam visited during one week, based on different representations. In this context, line plot and bar graph were created and they were expected to make explanations on the data in these two graphs. In the current study, the "number of pages read" context, which includes the number of pages that Ali read for five days, was created. Since the line plot is not taught in the primary school mathematics curriculum, the picture graph and bar graph representations they saw in the curriculum were included and the questions were formed within this framework.

The context used in the third question, "materials in pen holder" was also created on the basis of "The Beanie Baby" in the study by Jones et al. (2000). This context is related to the dolls and their types in the doll collection of students. In addition, in this context, the conversion of these data to bar graph was also included. In the current study, it was thought that the context of "The Beanie Baby" would not be appropriate since the study would be conducted on 4th grade students and the context of "materials in pen holder" was created, which deals with the materials in the penholders of students. In this context, it was aimed to describe the data and test their knowledge of the concept of average.

The fourth question, "loved fruits", was created to focus on completing the bar graph based on the data provided in the table created by the researcher and display of the data with different representations of the data.

The last question, "amount of pocket money", was created based on the context of "The beanbag game" from the study by Jones et al. (2000). In this context, the results of the game played by two students are discussed. It was thought that this context would not be suitable for the students in the current study, and it was changed as the amount of pocket money received by two students.

The questions were prepared within the contexts of "number of pages read", "materials in pen holder", "loved fruits" and "amount of pocket money". Six questions are related to the construct of describing data, six questions are related to the construct of organizing and reducing data, two questions are related to the construct of representing data, and four questions are related to the construct of analyzing and interpreting data. Each question is labelled with its associated construct. Contexts and questions are presented in the section from Figure 1 to Figure 4.

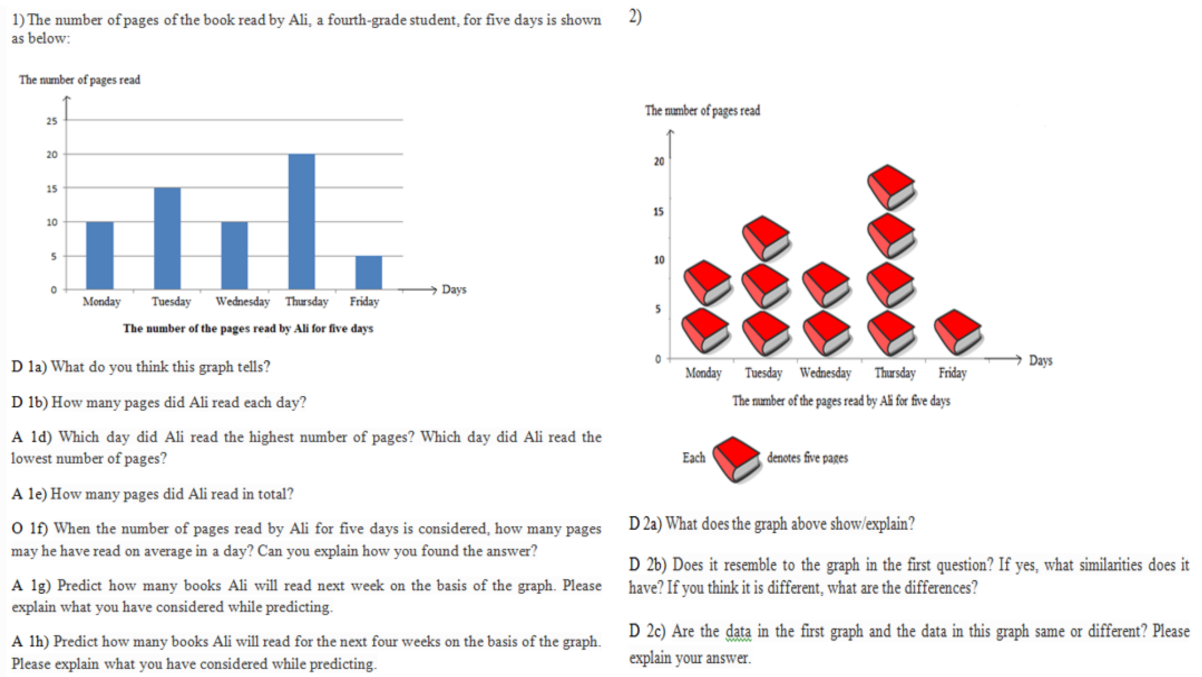


Figure 1. The number of pages read (D: describing data, O: organizing and reducing data, A: analyzing and interpreting data)

In the context of "the number of pages read" presented in two different types of representation, the questions 1a, 1b, 2a, 2b and 2c aim to reveal the knowledge and skills of students regarding the construct of describing data, the questions 1d, 1e, 1g and 1h aim to reveal their knowledge and skills regarding the construct of analyzing and interpreting data, the question 1f aims to reveal their knowledge and skills regarding the construct of organizing and reducing data (Figure 1).

3) Below are shown the materials in the pen holders of four students

There are 1 red pen, 3 pencils and 1glue in Ahmet's pen holder
There are 2 red pens, 3 rubbers, 2 glues in Ayşe's pen holder
There is no material in Veli's pen holder
There are 1 glue, 1 rubber, 1 red pen and 1 pencil in Nazlı's pen holder.

D 3a) How many materials does each student have in their pen holders?

O 3b) Assume that you are a teacher responsible for equally sharing these materials among the students. How do you do this?

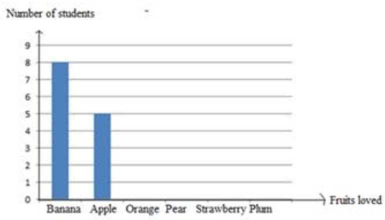
Figure 2. Materials in the pen holder (D: describing data, O: organizing and reducing data).

Within the context of the materials in the pen holder, the question 3a aims to reveal knowledge and skills related to the construct of describing data and the question 3b aims to reveal the construct of organizing and reducing data (Figure 2).

4) The teacher Melis has started to draw a graph by using the data given in the table below.

Tablo: 4/A sınıfındaki öğrencilerin sevdiği meyveler

Fruits loved	Number of students
Banana	8
Apple	5
Orange	3
Pear	2
Strawberry	7
Plum	4



Graph: 4/A fruits loved by the students in the class

But while she was drawing the graph, her pencil broke and she could not continue drawing.

R 4a) Can you complete the graph the teacher Melis has started to draw?

R 4b) How can you organize and present the data regarding the fruits loved by the students in a different way?

Figure 3. Fruits loved (R: representing data)

5) Ahmet and Veli get pocket money from their fathers before going to the school every day. The amounts of the pocket money they get for a week are given below:

Day	Ahmet	Veli
Monday	3 TL	3 TL
Tuesday	5 TL	0 TL
Wednesday	3 TL	5 TL
Thursday	3 TL	2 TL
Friday	1 TL	10 TL

O 5a) How are the amounts of the pocket money Ahmet and Veli get distributed on the basis of the table given?

O 5b) If you make an evaluation on the basis of the amounts of the pocket money Ahmet gets, can you say how many TLs he gets daily on average? Explain how you found your answer.

O 5c) If you make an evaluation on the basis of the amounts of the pocket money Veli gets, can you say how many TLs he gets daily on average? Explain how you found your answer.

O 5d) On the basis of the amounts of the pocket money Ahmet and Veli get, can you say whose amounts of pocket money varied more?

Figure 4. Amounts of pocket money (O:organizing and reducing data)

Within the context of the "loved fruits", the questions aim to reveal students' knowledge and skills regarding the construct of representing data (Figure 3). All the questions asked within the context of the amounts of pocket money aim to reveal students' knowledge and skills regarding organizing and reducing (Figure 4). In Table 2, the questions in the context of each construct are included and detailed.

Table 2. Questions prepared according to the constructs of statistical thinking

Constructs of statistical thinking	Questions
Describing data	1a, 1b,2a,2b,2c,3a
Organizing and reducing data	1f,3b,5a,5b,5c,5d
Representing data	4a,4b
Analyzing and interpreting data	1d,1e,1g,1h

With the questions 1a and 2a, from among the questions prepared regarding the construct of describing data expressed in Table 2, it was aimed to question what the graphics in different types of representation (bar and picture graph) tell. With the questions 2b and 2c, the students were asked to evaluate different representations of the same data in the same context. With the questions 3a and 1b, the students were asked to read the data (raw data and bar graph respectively). Among the questions prepared in relation to the construct of organizing and reducing data, the questions 1f, 3b, 5b and 5c aimed to investigate how students perceived the concept of average in different contexts and with different types of representation. With the question 5a, it was focused on how the data were distributed and with the question 5d, it was focused on how the data varied. With the question 4a, one of the questions regarding the construct of representing data, it was aimed to reveal students' knowledge and skills about completing a graph and with the question 4b, it was aimed to reveal students' knowledge and skills regarding the display of data by means of different forms of representation. With the questions related to the construct of analyzing and interpreting data, it was aimed to reveal students' knowledge and skills regarding comparing (1d) and combining (1e) and predicting (1g, 1h).

In order to determine the suitability of the data collection tool (statistical thinking questions) for the purpose of the study, two mathematics educators working in this field were consulted. The mathematics educators were asked to analyze and interpret the questions in terms of their consistency with the objectives and focused constructs, clarity and comprehensibility. In the first version of the data collection tool, 0 TL was not included in the amounts of daily pocket money received by Veli in the fifth question. The experts stated that adding 0 to the data set will give the opportunity to better observe how the students' informal knowledge of distribution, average and variation concepts is. Based on this suggestion, 0 TL was added to the data set regarding the amount of pocket money received by Veli. After making the necessary revisions in line with the expert opinions, a pilot study was conducted with 30 4th grade students in a school. Students were expected to solve these questions in 20-30 minutes. However, in the pilot study, it was observed that some students solved them in 35-40 minutes. Thus, it was decided to change the time period given for students to solve them. The data collection tool was finalized in line with the data obtained from these students and the analyses made. Then, after the mathematics teachers of the selected schools taught the concepts covered by this research, the participants were asked to solve the questions. Students were given 1 class hour (40 minutes) to complete the questions.

In this study, ethical principles were followed and ethics committee approval was acquired. Ethics committee was obtained from Karamanoğlu Mehmetbey University, protocol number 23.03.2022, 02-2022/39.

Data analysis

In the analysis of students' statistical thinking, the framework prepared by Jones et al. (2000) was used and deductive coding was performed because constructs and levels were predetermined (Patton, 2002). During the analysis, it was seen that some changes were needed in this framework. In this context, the following changes were made in the framework and these changes were included in the data analysis, and both deductive and inductive analyses were used (Patton, 2002). First, the presence of students who did not respond in the preliminary analysis of the obtained data attracted attention. Thus "No answer" level was added. The expression described for Level 3 regarding to the construct of describing data "In reading the data literally, gives a confident and complete description and demonstrates awareness of graphing conventions" was changed into "In reading the data literally, gives a confident and complete description or demonstrates awareness of graphing conventions". Another change occurred related to the construct of organizing and reducing data. Here, the expression "typicality" in the framework was changed to "average" because the term "average" was used in the questions asked. Moreover, definitions were made to reveal students' knowledge and skill levels about variation. The last change was related to the construct of analyzing and interpreting data. Since "What does the display not say about the data?" was not involved among the questions, the related definitions in the framework were excluded. In addition, in the reading beyond the data construct, the expressions "they ignore the context for level 3" and "they take context into account for level 4" were added (see Appendix).

First of all, the responses of the students in each question were coded according to the framework. Then, the statistical thinking levels of the students were determined by making a general analysis of the codes found in the responses of the students. Afterwards, the students' statistical thinking levels for each question were shown with the help of descriptive statistics (e.g., percentages).

Validity and Reliability

A number of measures were taken to ensure the external and internal validity of the study (Fraenkel & Wallen, 2006). Mortality can be considered as a threat in the study. However, the collection of data at once prevented this threat. In addition, in order to ensure maximum participation, the researcher contacted both the administrators and mathematics teachers of the three schools determined. It was ensured that the students did not participate in any other activity on the day of the test. In addition, the mathematics teachers reminded the students that this test would be administered at the designated time. In order to prevent another threat, data collection location, the data collection tool was administered to all the students in their own classrooms. In addition, it is thought that the administration of the data collection tool only once and using a framework to analyze students' answers prevented the instrumentation threat. In addition to the collection of all the data by the researcher, not allowing any interaction other than the explanations made at the beginning of the application during the application was also one of the measures taken against the instrumentation threat. This study was conducted on fourth grade students attending three different public primary schools in a city in the Central Anatolian Region of Turkey. In addition to the use of the same mathematics curriculum, the same textbook is used in these schools. Therefore, it is thought that the results of the current study can be generalized to public schools with similar environments and to students with similar characteristics. All these can be considered as factors that ensure the external and internal validity of the study (Fraenkel & Wallen, 2006). A researcher working in this field was asked to evaluate the obtained data in terms of statistical thinking constructs and levels. The inter-coder reliability was found to be 0.89. For example, in the 5d question, in which the variability of the data set was questioned, there was a disagreement on whether the students' statistical thinking levels were Level 3 or Level 4. At this point, differences were discussed until a consensual coding was reached.

Results

Based on the data obtained, the results regarding the statistical thinking levels of primary school 4th grade students in the context of describing data, organizing and reducing data, representing data with analyzing and interpreting data are presented in this section.

Students' statistical thinking levels related to the construct of describing data

When the knowledge of the students about describing data was evaluated in general, it was noted that the students were more successful in reading data, followed by the evaluation of the similarities/differences of the data and graphs. The thing in which they were observed to be least successful emerged when they were asked to express what the graphs were telling.

The knowledge and skills of the students regarding the construct of describing data were revealed with the answers obtained for the questions 1a, 1b, 2a, 2b, 2c and 3a, and the statistical thinking levels of the students are expressed in Figure 5.

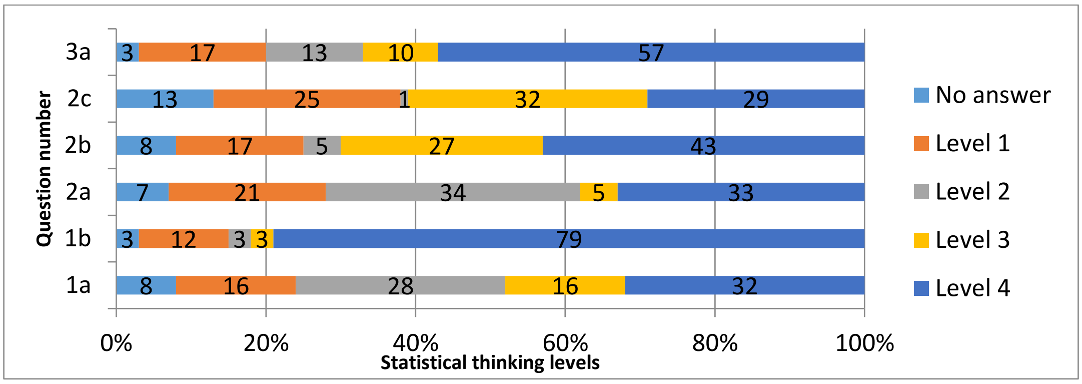
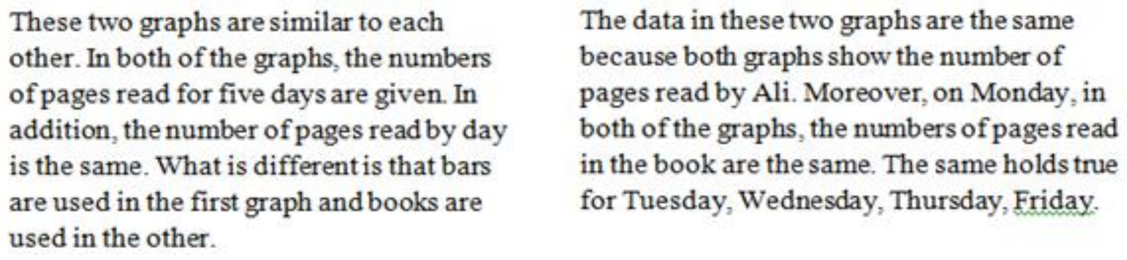


Figure 5. Students' statistical thinking levels related to the construct of describing data.

In questions 1a and 2a, the students were expected to evaluate different forms of representations and express what the graphics tell. It was observed that the percentage of the students who did not respond was similar (1a-8%, 2a-7%). The students at Level 1 constituted approximately 20% of all the students (1a-16%, 2a-21%). The students at Level 1 used very unfocused expressions and did not have any awareness about graphing conventions (e.g. *a graph tells something about book reading"*). The students at Level 2, on the other hand, constituted approximately one third of all the students (1a-28%, 2a-34%). These students were more successful in describing the data (e.g. *"Ali read 10 pages some days, 5 pages some days"-1a*), and they exhibited some awareness of graphing conventions (e.g. taking columns into consideration, reading x and y axes). However, they made incomplete explanations while reading the data. When the answers of the students at Level 3 were examined, it was found that they constituted 16% of the students for 1a and 5% of the students for 2a. At this point, it was noticed that the students differed in the context of the question. The students at Level 3 either thoroughly read the data or included graphing conventions in their explanations. They were placed at Level 3 because they did not include two points at the same time. For example, for question 1a, the student gave the following response *"He read 10 pages on Monday, 15 pages on Tuesday, 10 pages on Wednesday, 20 pages on Thursday, and 5 pages on Friday"*. It was observed that the student read the data in the graph completely, and that he/she did not make any explanation about graphing conventions. It was noted that the proportions of the students at Level 4 were similar to each other and constituted approximately one-third of all the students (1a-32%, 2a-33%). It was mentioned that the students at this level were able to fully describe the data and express their awareness of graphing conventions (e.g. graph title, axes).

In questions related to the construct of describing data (2b-2c), students were expected to evaluate the similarities/differences of different representations created from the same data. While the students who did not answer were 8% in the question 2b, this rate was 13% in the question 2c. The students at Level 1, on the other hand, made up about 20% (17%) of all the students for the question 2b and 25% for the question 2c. It was revealed that these students were insufficient in evaluating the data and graphics (e.g. the data are different from each other-2c). The percentage of the students at Level 2 is very low (2b-5%, 2c-1%). Although these students had certain awareness in evaluating both graphics and data, they stated that data and graphics represent the same things with justifications consisting of rules (e.g. *"the data are the same, they are shown with bars in the first and books in the second"*-2c). The students at Level 3 corresponded to approximately one third of all the students (2b-27%, 2c-32%). It was observed that the students at this level were mostly correct in their evaluations of data and graphics, emphasized more than one point (e.g. *taking axes and frequencies into consideration*), but their explanations were mentioned to remain more limited. The students at Level 4, on the other hand, emphasized more than one point, and their explanations were more comprehensive. The students at Level 4 made up about half of all the students (43%) for the question 2b and one-third (29%) for the question 2c (Figure 6).


These two graphs are similar to each other. In both of the graphs, the numbers of pages read for five days are given. In addition, the number of pages read by day is the same. What is different is that bars are used in the first graph and books are used in the other.

The data in these two graphs are the same because both graphs show the number of pages read by Ali. Moreover, on Monday, in both of the graphs, the numbers of pages read in the book are the same. The same holds true for Tuesday, Wednesday, Thursday, Friday.

Figure 6. Student answers at Level 4 for the questions 2b and 2c.

In questions 3a and 1b, which focused on reading data, it was noted that the rate of students not answering was quite low (3a, 1b 3%). It was mentioned that the students were more successful in reading the graphs than in evaluating the data in their raw form. The percentages of the students at Level 1 were similar to each other and corresponded to 12% for the question 1b and 17% for the question 3a. These students gave idiosyncratic answers (e.g. "*he read 12 pages*"). It was noted that the percentages of the students at Levels 2 and 3 for the question 1b were quite low (Level 2 %3, Level 3 %3), and for the question 3a, these percentages were higher (Level 2 13%, Level 3 10%). Although the students at Level 2 and Level 3 have deficiencies in reading data, it was detected that they have awareness of reading data. It can be said that the proportion of the students at Level 4 to all the students is high (1b-79%, 3a-57%). (e.g. "*Ahmet has 5, Ayşe has 7, Nazlı has 4 materials. Veli does not have any materials*"-3a).

Students' statistical thinking levels related to the construct of organizing and reducing data

The statistical thinking levels of the students for the construct of organizing and reducing data were examined in the context of their answers about the center and spread of the data. In questions that sought to evaluate the concept of average based on different contexts, it was observed that students' levels of statistical thinking varied and that the proportion of students being at the higher statistical thinking levels (Level 3-4) ranged from one-fourth to half of the students. Moreover, it was determined that the majority of the students at this level associated the concept of average with arithmetic mean. In the question that requested an assessment of the spread of the data, this proportion constituted approximately half of the students. In the question in which the concept of variation was examined, it was noted that only one out of every four students was at Level 3-4. The knowledge and skills of the students regarding the construct of organizing and reducing data were revealed through the answers obtained for the questions 1a, 1b, 2a, 2b, 2c and 3a, and the statistical thinking levels of the students are expressed in Figure 7.

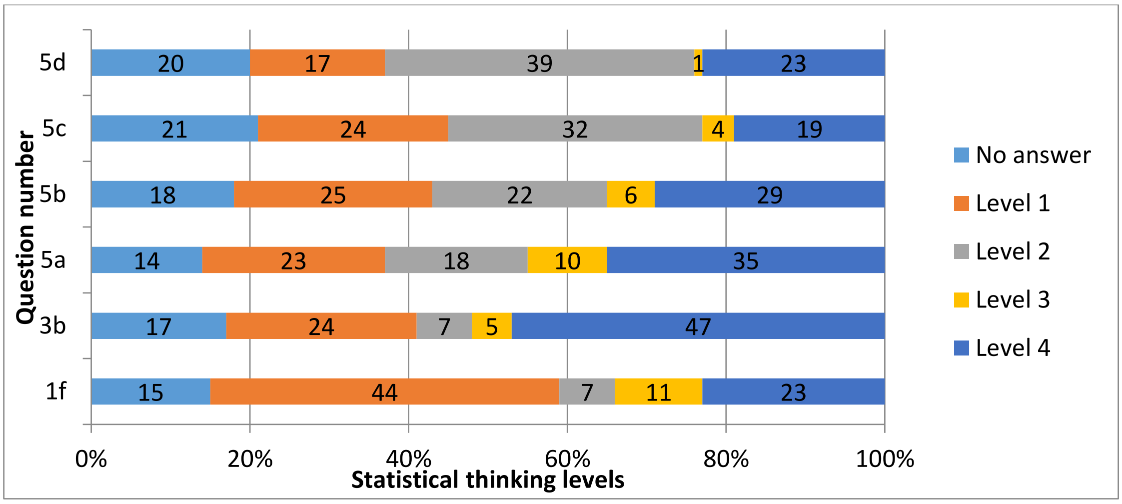


Figure 7. Students' statistical thinking levels related to the construct of organizing and reducing data

In the questions 1f, 3b, 5b, 5c, in which how students perceive the concept of average is investigated, the first thing that draws attention is that for each question, approximately one-fifth of the students left the question unanswered (1f-15%, 3b-17%, 5b-18%, 5c-21%). While the percentage of the students at Level 1 made up about half of all the students (44%) in the question 1f, it corresponded to about a quarter of all the students in the other questions (3b 24%, 5b 25%, 5c 24%).It was mentioned that students did not make an evaluation in terms of the concept of average (e.g. "*gets 21 TL*"-5c). Students whose statistical thinking levels were at Level 2 had a higher percentage (5b-22%, 5c-32%) in the questions 5b and 5c, and this percentage was lower in the questions 1f and 3b (1f-7%, 3b-7%).). Although the students at this level could not clearly express the average, they were able to state that this concept was between two extreme values in the data set (e.g. "the average amount of pocket money received by Veli is between 1 TL and 10 TL"-5c). The question with the highest percentage of students at Levels 3 and 4 is the question 3b (level 3 5%, level 4 47%). In the questions 1f and 5b, it was noted that the sum of the percentages of the students was close to each other and less than in the question 3b (1f-level 3 11%, level 4 23%), (5b-level 3 6%, level 4 29%). In the question 5c, it was detected that this percentage was lower (level 3 4%, level 4 19%). Although the students at Level 3 have the necessary knowledge and skills about the average, it was observed that there are deficiencies in their explanations. For example, for the question 5c, when the answer given by the student "*He gets 5 TLpocket money. Somewhere between the amounts of the pocket money he receives*" is examined, it is clear that although he/she showed awareness that the average could be between two extreme values by emphasizing the concept of range implicitly, he/she did not explain why he/she chose 5 TL.. It was seen that the students at Level 4 had awareness of the concept of average and were able to explain it (Figure 8 and 9).

20+15+10+10+5=60, he read a total of 60 pages. As he read for five days, I need to equally divide it into five days. 60:5=12; he read 12 pages each day.	When I sum all the materials they have, 5+7+0+4=16. When I look at the materials, I can see that there are 4 red pens, 4 pencils, 4 glues and 4 rubbers. I shared all the materials among Ahmet, Ayşe, Veli and Nazlı. Thus, each student has 1 red pen, 1 pencil, 1 glue and 1 rubber.

Figure 8. Student answers at Level 4 for the questions 1f and 3b

It must be 3tl on average because when I look at the amounts of the pocket money I can see that Ahmet receives 3tl most of the days. And when we divide the sum of 5tl an 1tl in two different days, we again have 3tl.	The total amount of pocket money received by Veli for 5 days is 20tl. If I divide this total amount into five days, then it is seen that he gets 4tl. That is, he gets 4tl pocket money each day.

Figure 9. Student answers at Level 4 for the questions 5b and 5c

The students at Level 4 mostly associated the concept of average with arithmetic mean (65%). In addition, there were some students who tried to explain it with the concept of mode and median (35%).

It was mentioned that 14% of the students did not answer the question 5a, which asked them to evaluate how the data were distributed. Almost a quarter (23%) of the students are at Level 1. The students at Level 2, on the other hand, constituted 18% of all the students. It was observed that the students at Level 1 could not make an evaluation of how the data were distributed (e.g. "*Ahmet received 15 TL, Veli received 20 TL*"). Although the students at Level 2 used expressions to make sense of the distribution, they were not at the desired level (e.g. "*they received the same amount of money sometimes and different amount of money other times*"). The students at Levels 3 and 4 constituted approximately 50% of all the students. In particular, it was detected that the percentage of the students at Level 4 was 35%. Although the students at Level 3 tried to explain how the data were distributed, it was observed that these explanations were incomplete. It was mentioned that some students at Level 4 used the concept of centre informally (e.g. "*the amount of pocket money Ahmet received was around 3 TL*"), while some students made comments using the concept of range (e.g. "*Veli did not receive any pocket money on Tuesday, but received 10 TL pocket money on Friday. The amount of pocket money of Veli varied between 1 TL and 10 TL*").

In the question 5d, in which the concept of variation is questioned, the first thing that draws attention is that one-fifth of the students did not give any answer. In addition, the percentage of the students at Level 1 (17%) and Level 2 (39%) was more than half of all the students (56%). When the answers of these students were examined, it was noted that the students at Level 1 did not have any awareness of the concept of variation, while the students at Level 2 tended to focus only on certain values (e.g. the most). About a quarter of the students were at Level 3 and Level 4 of statistical thinking (Level 3 1%, Level 4 23%) (Figure 10).

When the data are examined, it is seen that there is a great variety in the amounts of the pocket money received by Veli

Ahmet received amounts of pocket money close to each other. Veli did not receive any pocket money on Tuesday. He received 10tl pocket money on Friday. He sometimes received a high amount of money, sometimes a very low amount of money. Thus, we can say that there is a great variation in the amounts of pocket money received by Veli.

Figure 10. Student answers at Levels 3 and 4 for the question 5d

When the answers of the students at Level 3 were examined, it was observed that although the students were aware of the concept of variation, they could not explain it. It was seen that the students at Level 4 were able to make correct evaluations about the concept of variation.

Students' statistical thinking levels related to the construct of representing data

The statistical thinking levels of the students for the construct of representing data were examined in the context of their answers about completing the missing graph and showing the data with different representations. It was seen that when the students were asked to complete the uncompleted graph, it was determined that about nine out of ten students were more successful in general and their statistical thinking level was at Level 4. On the other hand, it was observed that the students did not exhibit the same statistical thinking level in the question in which they were asked to represent the data with a different representation. One out of every five students left the question blank, and the statistical thinking level of about four out of ten students was Level 4. Regarding representing data construct, most of the students were at level 4 in "complete graph" while students were at all levels in "organize and present data in another way".

The students' knowledge and skills regarding the construct of representing data were revealed by the answers obtained for the questions 4a and 4b, and the statistical thinking levels of the students are expressed in Figure 11.

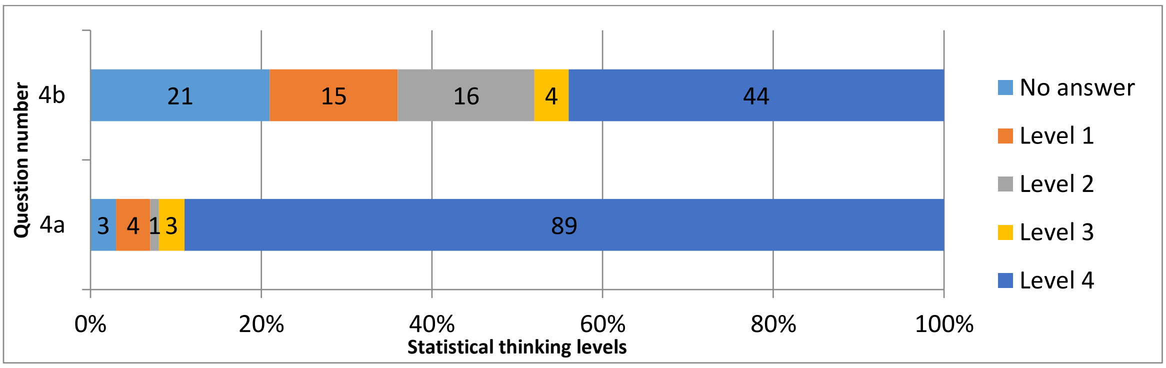


Figure 11. Students' statistical thinking levels related to the construct of representing data

In the first question (4a) related to the construct of representing data, the students were asked to complete an uncompleted graph. It can be said that the students are generally successful in this question. It was mentioned that students who did not respond (3%) and who were at Levels 1 and 2 had a low percentage (Level 1-4%, Level 2-1%) (Figure 12).

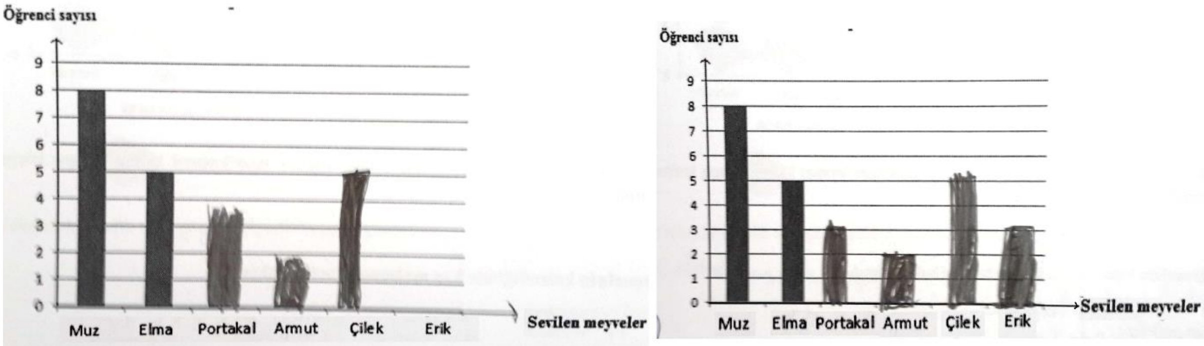


Figure 12. Student answers at Levels 1 and 2 for the question 4a

It was detected that the students at Level 1 made personal evaluations and that although the students at Level 2 made correct evaluations about two columns, they formed the other two columns incorrectly or could not form. While the percentage of the students at Level 3 is quite low (3%), the percentage of the students at Level 4 is considerably high (89%). While the students at Level 3 got only one column missing or wrong, the students at Level 4 were able to get all of them right.

Although the students were generally successful in completing the uncompleted graph, it was observed that they were less successful in expressing the data with different representations. While one fifth of the students (21%) did not answer this question, approximately one third of the students were at Level 1 (15%) and Level 2 (16%). It was mentioned that the students at Level 1 used a form of representation that did not represent the data set. It was observed that the students at Level 2, on the other hand, did not make any attempt to organize the data, although they used a different form of representation (e.g. tally chart). Students at Level 3 constituted 4% of all the students and the students at Level 4 constituted 44% of all the students (Figure 13).

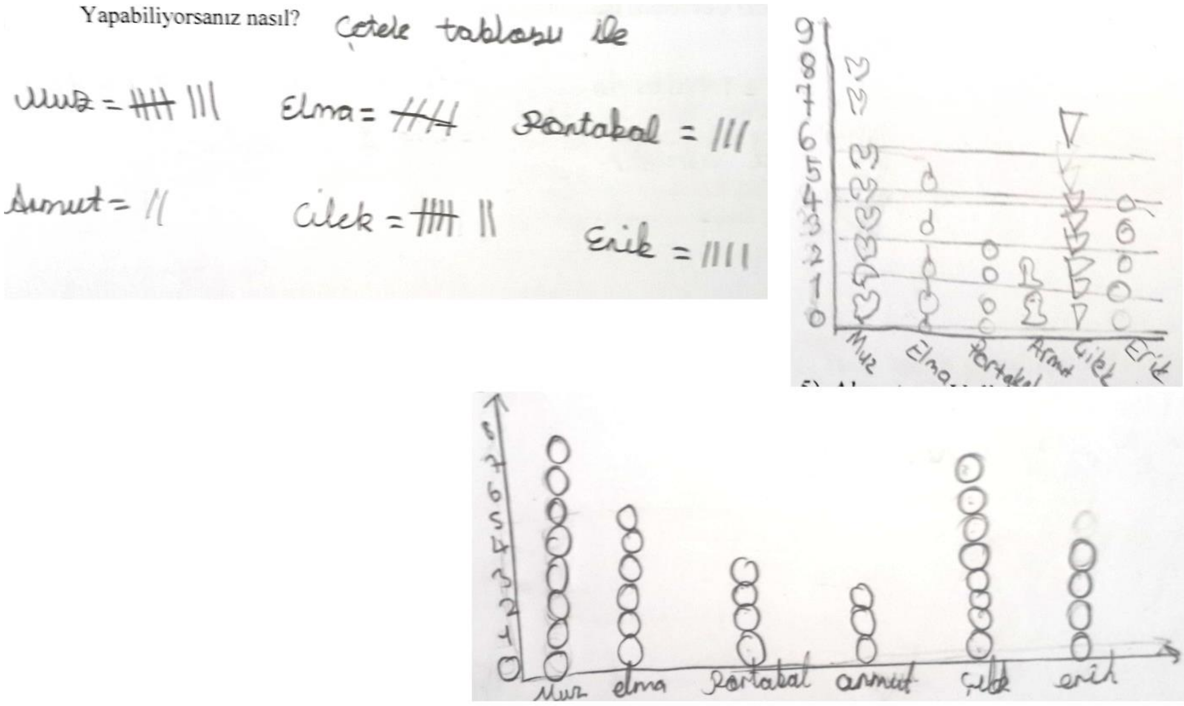


Figure 13. Student answers at Levels 3 and 4 for the question 4b

It was observed that the students at Level 3 were able to express the data in a different form of representation. The students at Level 4, on the other hand, were able to show the data with more than one form of representation.

Students' statistical thinking levels related to the construct of analyzing and interpreting data

When the statistical thinking levels of the students regarding the construct of analyzing and interpreting data were examined, it was mentioned that they were more successful in reading between the data (compare and combine the data) than in reading beyond the data (predict the data).

The knowledge and skills of the students regarding the construct of analyzing and interpreting data were revealed with the answers obtained for the questions 1d, 1e, 1g and 1h, and the statistical thinking levels of the students are shown in Figure 14.

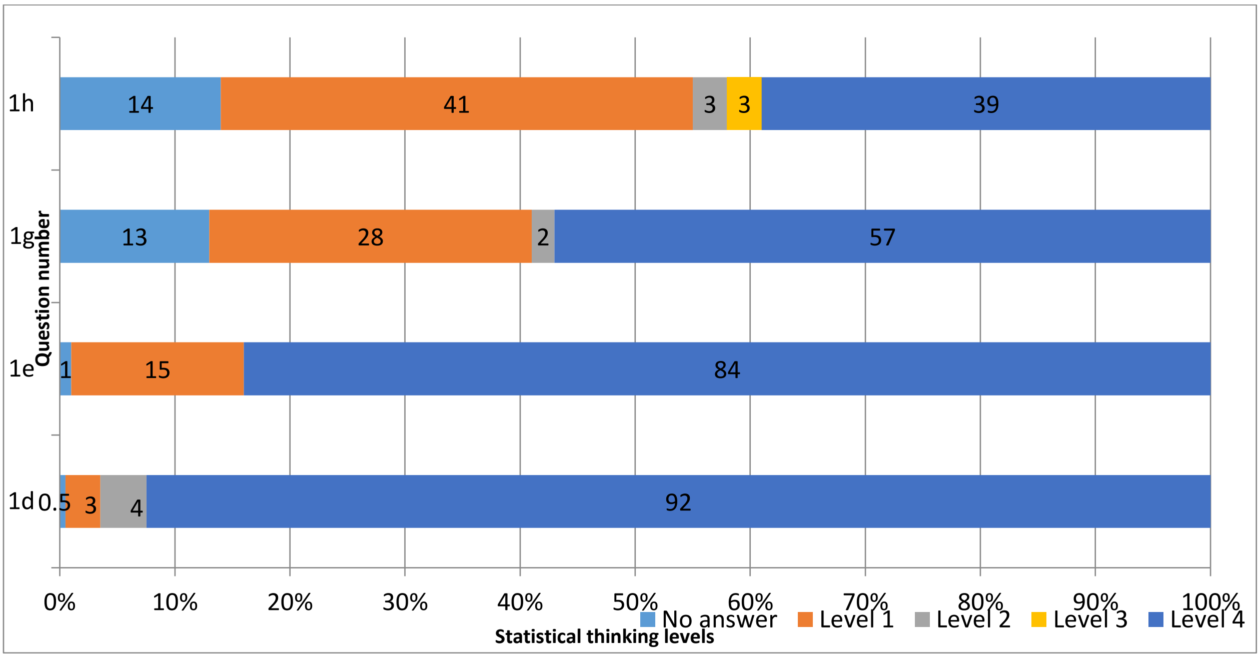
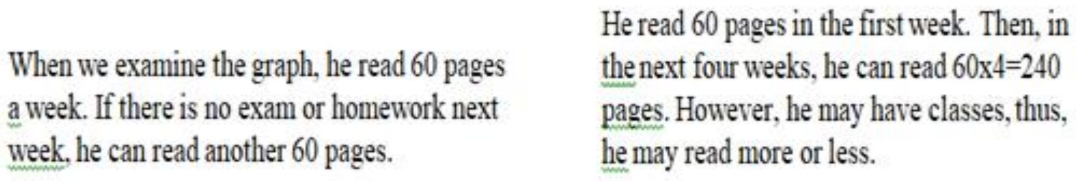


Figure 14. Students' statistical thinking levels related to the construct of analyzing and interpreting data

The question 1d is related to the comparison of the data while the question 1e is related to the combination of the data. It was seen that the success percentages of the students in these two questions were higher than in the other two questions. The first remarkable finding is that the number of students who did not respond is very low (1d-0.5%, 1e-1%). In addition, the difference between the percentages of the students at Level 1 was noteworthy (1d-3%, 1e-15%). It was also seen that the students at this level gave invalid answers (e.g. "*24+25=49*"). While there were no students at Level 2 for the question 1e, the percentage of the students at Level 2 was found to be 4% for the question 1d. It was seen that the students at this level evaluated the data from a single perspective (e.g. "*Thursday 20 pages maximum*"). While there were no students at Level 3 for the questions 1d and 1e, it was noted that the majority of the students were concentrated at Level 4. It was mentioned that the students at this level could both compare and combine data. In the questions 1g and 1h, focused on reading beyond the data, it was observed that the students were concentrated at lower levels of statistical thinking. In both questions, more than 10 percent of the students did not answer the questions (1g-13%, 1h-14%). In addition, it was detected that the percentage of the students at Level 1 was high. About a quarter (28%) of the students for the question 1g and about half (41%) for the question 1h are at Level 1. When the student answers were examined, it was observed that the students at Level 1 made irrelevant predictions and did not take the context into account. In addition, it was mentioned that the percentage of students at Level 2 in both questions was quite low (1g-2%, 1h-3%). It was observed that these students did not make vague or inconsistent predictions and did not make any effort in terms of associating with the context (e.g.

"60+60+60=180"). It was detected that there were no students at Level 3 for the question 1g and the percentage of the students at Level 3 was 3% for the question 1h. It was seen that although students at this level were more successful in making predictions, they did not make an effort to relate their predictions to the context (e.g. "*It is seen as 60 per week in the graph. Thus, in four weeks, 60x4=240*"). It was seen that the students at Level 4 differed in both questions. While more than half of the students (57%) were at Level 4 for the question 1g, this percentage remained at 39% for the question 1h (Figure 15).


When we examine the graph, he read 60 pages a week. If there is no exam or homework next week, he can read another 60 pages.

He read 60 pages in the first week. Then, in the next four weeks, he can read 60x4=240 pages. However, he may have classes, thus, he may read more or less.

Figure 15. Student answers at Level 4 for the questions 1g and 1h

It was observed that the students at this level not only made logical predictions, but also associated these predictions with the context.

Discussion and Conclusion

In the current study, it was aimed to reveal the statistical thinking levels of primary school 4th grade students. It was observed that the students' statistical thinking levels regarding the constructs of describing, representing and analyzing and interpreting data were higher than their statistical thinking level regarding the construct of organizing and reducing data. It can be thought that the main reason for this is that the questions in the construct of organizing and reducing data include concepts that students have not learned formally. Researchers who draw attention to this point (e.g., Kinnear, 2013; Leavy & Hourigan, 2018) have investigated what conditions can support the development of statistical thinking of younger students in formal school environments.

It can be said that the ratio of students at Level 3 and 4 regarding describing data construct and the ratio of students at Level 3 and 4 regarding analyzing and interpreting data construct are similar. Students' encountering tasks that would focus on analyzing and interpreting data during the teaching process may have paved the way for such a result. In this sense, it is inconsistent with the studies in the literature. Studies indicate that students are more successful in describing data than in analyzing and interpreting (Jones et al., 2000; Pereira-Mendoza & Mellor, 1991).

The statistical thinking levels of the students regarding the reading of the data and the use of different representations based on the same data in the construct of describing data, the percentages of the students at Level 3 and Level 4 varied between 38% and 82%. However, in the questions that serve regarding reading the data, differences in the percentages of the students at Levels 3 and 4 were considerable (1b-level 3-3%, level 4 79%; 3a-level 3 10%, level 4-57%). In the question 1b, the students were expected to read the data from the graph, and in the question 3a, they were expected to read from the raw data. At this point, it can be said that the students were more successful in reading data from graphs than reading raw data. Although there are such differences, it can be said that the students generally have adequate knowledge and skills regarding the construct of describing data. The emphasis on the objectives related to describing data in the primary school mathematics curriculum in Turkey may have supported students to have a high level statistical thinking skill regarding this construct. In this connection, when the literature is reviewed, it is seen that primary, middle and high school students generally do not have difficulty in describing data (Jones et al., 2000; Koparan & Güven, 2014; Mooney, 2002; Pereira-Mendoza & Mellor, 1991; Shaughnessy et al., 1996). For example, Koparan and Güven (2014) found that 7th and 8th grade students are mostly at level 4 in terms of statistical thinking. In some studies, it has been detected that middle school students are at lower levels in describing data (Altaylar & Kazak, 2021; Koparan & Güven, 2014). Despite these results, the percentage of the students who preferred not to answer and whose statistical thinking level is Level 1 was found to be higher in the questions in which the meaning of various graphs (bar-object graphs) was tested. Although they are more successful in reading the data one by one, they have lower levels of statistical thinking when it comes to evaluating the graphs holistically and this might be because of the fact that they are less experienced. The fact that they had not encountered such a question before in the given tasks may have also caused difficulties for the students.

In the questions related to the construct of organizing and reducing data, the statistical thinking levels of the students regarding the concepts of average, distribution and variation were examined. When the statistical thinking levels of the students regarding the concept of average were examined, the first point that drew attention was that for each question about one-fifth of them could not answer. The percentage of students in Levels 1 and 2 constituted approximately half of the students for the questions 1f, 5b and 5c (1f 51%, 5b 47%, 5c 56%). The first reason for the high number of students at these levels can be attributed to the fact that they did not formally learn the concept of average. Similar emphasis is also made in the literature, arguing that students have difficulty in making sense of the concepts they have not learned formally (English & Watson, 2015; Makar, 2016). Another striking point is that while the percentage of students in Level 1 made up about half of the students (44%) in the question 1f, it corresponded to about a quarter of all the students in the other questions. The students were expected to interpret the concept of average based on graphics in the question 1f, raw data in the question 3b, and tables in the questions 5b and 5c. At this connection, the point that the students had the greatest difficulty was interpreting the concept of average over the graph. The reason for this difficulty can be associated with several factors. The first of these, as mentioned above, is that the students did not formally learn the concept of average. Another can be thought of as educational experiences. The fact that they had a training focused only on drawing graph may have caused them to have difficulties in evaluating the graphics. The percentage of students at Levels 3 and 4 was the highest in the question 3b (Materials in the pen holder). The fact that students have a high level statistical thinking skill for this question can be attributed to two main reasons. The first of these is that the students worked on raw data, while the second is that the given context is likely to be encountered in daily life. The question 3b was followed by the questions 5b (Amounts of pocket money) (35%) and 1f (number of pages Ali read) (34%). In the question 5c, the percentage of the students at Levels 3 and 4 was approximately one fourth (23%) of all the students. Although the questions 5b and 5c were asked for the same context, it can be thought that the reason for this difference in level percentage is the presence of 0 in the data in the question 5c. The fact that the students could not reason correctly about how the presence of 0 in the data set would affect the data set may have paved the way for such a result. However, it is thought that one point should be emphasized here. There is also 0 in the data set involved in the context of the question 3b, where the percentage of high-level statistical thinking is the highest. However, it is thought that there are two main reasons why the students interpreted the average more accurately in this data set. The first of these is the context used, while the other is thought to be the form of representation (table-raw data). From this point of view, it can be thought that students' informal understanding of the concept of average will be higher in different contexts and different forms of representations. When the literature was examined, it was revealed that students have difficulties in how to evaluate 0 in the data set (Strauss & Bichler, 1988).

It was mentioned that the students who could define the concept of average accurately, mostly tended to use arithmetic mean and use the concepts of mode and median less. It can be thought that the first reason underlying this finding is that students are more likely to experience arithmetic mean in daily life. Besides, in Turkish, these two words can be used interchangeably (mean-aritmetik ortalama, average-ortalama). This may have caused the tendency of using the concept of arithmetic mean by the students. When the literature was examined, similar results were found and it was observed that students and teachers mostly associated the average with the arithmetic mean (Watson & Moritz, 2000). While finding the average, very few of the students tended to comment and mostly preferred to use operations. Similar results were obtained in the literature, and it was noted that the students found the average by using operations (Mokros & Russell, 1995). Although most of the students tended to use operations while finding the average, some students made statements indicating that they were aware of the average being between the maximum and minimum values in the data set and that they were aware that its meaning is something like the equilibrium point. It was revealed that about one-fifth of the students could not answer the questions focused on the variability of the data and how they were distributed. In the question 5a about how the data are distributed, 41% of the students were found to be at Levels 1-2, while 45% of them were found to be at Levels 3-4. In the question 5d, in which the variability of the data was questioned, 56% of the students were at Levels 1-2 and 24% of them were at Levels 3-4. At this point, it can be said that students have various difficulties regarding the concepts of distribution and variation. When the reasons for the difficulties of the students are examined, the first reason can be considered as the fact that statistical inquiries are not included in the teaching process, although they are not formally included in the curriculum because the concept of variation lies on the basis of statistics (Moore, 1990). In the GAISE II report, attention is drawn to the importance of including these concepts in statistical problem solving at the pre-k-12 level of students (Bargagliotti et al., 2020). In addition to these difficulties, the percentage of the students who can interpret the concepts of variation and distribution in the data set is considerable.

When the primary school mathematics curriculum in Turkey is evaluated, it is seen that the concepts of average, variation and distribution are not included in the curriculum, which shows that the results obtained from the students' data are not surprising. Despite these difficulties, it was also found that the students have some informal understanding. This result shows that these concepts can be formally included in the teaching processes. Similar studies also show that students in the younger age groups have an understanding of the concepts such as variation, average, statistical inference which form the basis of statistics (e.g., Makar, 2014, 2016; Paparistodemou & Meletiou-Mavrotheris, 2008). The results obtained can provide teachers with insights while structuring the teaching process regarding these concepts (Makar, 2016). In addition, instructional objectives related to these concepts can be added.

In the question in which the students were asked to complete the uncompleted graph regarding the construct of representing data, 92% of the students were found to be at Levels 3 and 4. In the question in which the data at hand were asked to be represented with a different form of representation, the percentage of students at Levels 3 and 4 remained at 48%. Although there are objectives related to representing data in the curriculum (e.g. uses different representations to present the data obtained), the difficulty of using different forms of representation may be due to the fact that they do not have knowledge of how the relations between different types of representation are. When the literature is examined, it is seen that there are studies that have reached similar results (Koparan & Güven, 2014; McGatha et al., 2002).

When the students' levels of statistical thinking regarding the construct of analyzing and interpreting data were examined, it was detected that the percentage of the students at Levels 3 and 4 were high in both comparing (92%) and combining (84%). In addition, it was seen that the number of students who did not answer was quite low. However, they were observed to be less successful in predicting comments. The percentage of the students who did not respond to the questions related to predicting was higher than the percentages of the students who did not respond to the questions related comparing and combining. For the question 1g, which focuses on predicting, approximately one-third of the students (29.4%) were at Levels 1 and 2, while for the question 1h, this percentage was almost half (44%). The percentage of the students at Levels 3 and 4 was 57.6% for the question 1g and 42% for the question 1h. The reasons why students' predicting levels are lower can be evaluated under several headings. It can be thought that the first of these stems from the fact that the students have not encountered tasks requiring making inference and interpretation before. Another reason is that both in the teaching processes and in the textbooks, it can be said that the interpretation of reading between the data is given weight and less space is given to reading beyond the data, which includes making predictions (Jones & Jacobbe, 2014; Jones et al., 2015). In studies conducted in the literature, it is stated that students have difficulties in making inferences and predicting from data (Jones et al., 2000; Koparan & Güven, 2014; Mooney, 2002).

Limitations and Implications

This study is limited to the data collected from one city of Turkey. The framework used for the analysis of the data was arranged accordingly. As pointed out by some other researchers (e.g., Jones et al., 2000; Shaughnessy, 1992), suggestion can be that further research should explore whether the framework is appropriate for students from other cultural and linguistic backgrounds. In this context, it can be suggested to conduct studies with participants from different countries and different contexts in order to make more in-depth interpretations about how students evaluate the given questions and what knowledge and skills they have. In addition, teaching experiments can be done in classrooms to evaluate the statistical thinking of primary school students and to test the applicability of this framework. This can offer teachers opportunities to reveal and evaluate students' statistical thinking.

Moreover, the results of this study revealed that students were at different statistical thinking levels in the questions presented in different contexts. Thus, students' statistical thinking levels can be examined by preparing questions with different contexts in future studies. Similar/different aspects of the results that emerge according to the given contexts can be analyzed and interpreted in depth.

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Appendix

Statistical Thinking Framework

Construct	No answer	Level 1: Idiosyncratic	Level 2: Transitional	Level 3: Quantitative	Level 4: Analytical
Describing data (D)	No explanation	-In reading the data literally, gives a description that is unfocused and includes idiosyncratic or irrelevant information; has no awareness of graphing conventions (e.g., title, axis labels) -Does not recognize when two different displays represent the same data OR indicates some recognition but uses idiosyncratic or irrelevant reasoning -Considers irrelevant or subjective features when evaluating the effectiveness of two different displays of the same data	-In reading the data literally, gives a description that is hesitant and incomplete but demonstrates some awareness of graphing conventions -Recognizes when two different displays represent the same data but uses a justification based purely on conventions -Focuses only on one aspect when evaluating the effectiveness of two different displays of the same data.	-In reading the data literally, gives a confident and complete description or demonstrates awareness of graphing conventions -Recognizes when two different displays represent the same data by establishing partial correspondences between data elements in the displays -Focuses on more than one aspect when evaluating the effectiveness of two different displays of the same data	-In reading the data literally, gives a confident and complete description and demonstrates awareness of graphing conventions -Recognizes when two different displays represent the same data by establishing precise numerical correspondences between data elements in the displays -Provides a coherent and comprehensive explanation when evaluating the effectiveness of different displays of the same data

Organizing and reducing data (O)	No explanation	-Does not group or order the data or gives an idiosyncratic or irrelevant grouping -Does not recognize when information is lost in reduction process -Is not able to describe data in terms of representativeness or "average" -Cannot describe data in terms of spread; gives idiosyncratic or irrelevant responses -Explains the variation of the data set with idiosyncratic or irrelevant responses	-Gives a grouping or ordering that is not consistent OR groups data into classes using criteria they cannot explain -Recognizes when data reduction occurs but gives a vague or irrelevant explanation -Gives hesitant and incomplete descriptions of data in terms of "average" -Invents a measure, usually invalid, in an effort to make sense of spread -Explains the variation of the data set by focusing on only some values (e.g. the most, the least)	-Groups or orders data into classes and can explain the basis for grouping -Recognizes when data reduction occurs and can explain the reasons for the reduction -Gives valid measures of average" that begin to approximate one of the centers (mode, median, or mean); reasoning is incomplete -Uses an invented measure or description that is valid, but the explanation is incomplete -Has awareness about the variation of the data set but the explanation is incomplete	-Groups or orders data into classes in more than one way and can explain the basis for these different groupings -Recognizes that data reduction can occur in different ways and gives complete explanations for the different reductions -Describes "average" of data in terms of common measures of center such as the median or the mean -Uses the range or an invented measure that has the same meaning as the range -Has awareness about the variation of the data set and can explain this
Representing data (R)	No explanation	-Constructs an idiosyncratic or invalid display when asked to complete a partially constructed graph associated with a given data set -Produces an idiosyncratic or invalid display that does not represent or reorganize the data set	-Constructs a display that is valid in some aspects when asked to complete a partially constructed graph associated with a given data set -Produces a display that is partially valid but does not attempt to reorganize the data	-Constructs a valid display when asked to complete a partially constructed graph associated with a given data set; may have difficulty with ideas like scale or zero categories -Produces a valid display that shows some attempt to reorganize the data	-Constructs a valid display when asked to complete a partially constructed graph associated with a given data set; Works effectively with scale and zero categories -Produces multiple valid displays, some of which reorganize the data

Analyzing and interpreting data (A)	No explanation	-Makes no response or gives an invalid or incomplete response when asked to "read between the data" -Makes no response or gives an invalid or incomplete response when asked to "read beyond the data"	-Gives a valid response to some aspects of "reading between the data" but is imprecise when asked to make comparisons -Gives a vague or inconsistent response when asked to "read beyond the data"	-Gives multiple valid responses when asked to "read between the data" and can make some global comparisons " -Tries to use the data and make sense of the situation when asked to "read beyond the data"; but does not take the context into consideration.	-Gives multiple valid responses when asked to "read between the data" and can make coherent and comprehensive comparisons -Gives a response that is valid, complete, and consistent when asked to "read beyond the data" and takes the context into account