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Original Article

Investigating Kindergarten Children's Performance and Strategies Used in Tasks Regarding Part-Whole Relations of Numbers

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Abstract

The present study aims to examine kindergarten children's performance regarding the part-whole relations and the strategies they use. The qualitative case study method was used to gain an in-depth understanding of the researched subject. Participants in the study were 43 children (21 girls and 22 boys) attending the kindergarten in six public schools run by the Ministry of National Education (MoNE). The age of the participants varied between 61 and 80 months. In order to determine children's performance regarding the part-whole relations of numbers, interviews were held with each of them for 20 minutes. Because of the pandemic, the interviews were conducted online, using the Zoom application, in the spring semester of the Academic Year 2020-2021. Interview questions prepared in the form of individual PowerPoint slides, with different visuals on each slide, were presented to the children by the researcher in the form of a game. Children were asked 3 types (finger patterns, visual forms, hidden objects) of tasks including part-whole relations of numbers ranging from 1-7. Findings indicated that about half of the children were able to give correct answers in the tasks presented to them. Further, the types of tasks used to present the number representing the whole also did matter. One of the most interesting findings of the present study was that children who used the strategy of part-whole relations performed better compared to those who used the counting strategy. As a result, it is recommended preschool teachers, and also curriculum developers enrich the teaching process with activities that encourage thinking about numbers in terms of their parts.

Keywords

Kindergarten
Part-whole relations
Number pairs
Compose-decompose
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Introduction

Interpreting numbers in the context of part-whole relations is arguably the most important skill that children should acquire in their early childhood mathematics education (Baroody, 2000; Fischer, 1990; Hunting, 2003; Kullberg & Björklund, 2020; Resnick, 1984; Sophian & McCorgray, 1994). Different terms are used in the literature to refer to this skill, including part-part-whole relations (Baroody, 2000; Cheng, 2012; Ekdahl, 2021; Kullberg & Björklund, 2020; Sinnakaudan, Kuldas, Hashim, & Ghazali, 2016; Sophian & McCorgray, 1994), part-whole reasoning (Hunting, 2003), part-whole structure, part-whole knowledge (Sophian & McCorgray, 1994), and part-whole schema (Resnick, 1984; Wolters, 1983). This study prefers the term "part-whole relations," which will be used throughout this article.

At the most basic level, part-whole relations refer to understanding how parts are related to the whole (Baroody, 2000). In other words, it refers to a child's ability to interpret numbers as a combination of other numbers (Resnick, 1984). This skill means understanding that numbers consist of other numbers and each number can be divided into smaller parts, and using this understanding when necessary (Sprenger & Benz, 2020). The concepts of part and whole describe the relationship between a set and a subset, represented by numbers. The representation of 5 marbles as "4 and 1" (or 1 and 4), "3 and 2" (or 2 and 3), or "2 and 2 and 1" are examples of this sort of reasoning (Ekdahl, 2021).

The part-whole relation is considered to be very important in terms of developing number sense and the emergence of more flexible strategies in arithmetic operations such as addition and subtraction. A child's ability to compose and decompose numbers depends on being able to structure numbers conceptually and interpret part-whole relations of numbers (Cheng, 2012; Novakowski, 2007; Sinnakaudan et al., 2016). This, in turn, requires the child to be able to identify the whole as well as the parts that make up the whole, understand that the whole is larger than the parts, and know that a number does not represent the quantity of a single set (Kullberg & Björklund, 2020; Novakowski, 2007; Sinnakaudan et al., 2016). Part-whole relations make it possible to interpret the semantic structures of different addition and subtraction problems in terms of parts and wholes, and allow children to be more flexible when solving these problems (Fischer, 1990). As children master representing numbers in different ways (Cheng, 2012), they would be able, upon encountering an addition operation such as 17 + 5, to decompose the number 5 as "3 and 2" (or divide it into possible subsets) and transform the operation into 20 + 2. Understanding part-whole relations of numbers makes it easier to understand mathematical concepts such as the commutative property, the relationship between subtraction and addition, and the concept of digits (Ekdahl, 2021; Sophian & Vong, 1995).

One important aspect in early mathematical development is subitizing. Subitizing is defined as a basic ability to recognize small numbers without counting (Clements, Sarama, & MacDonald, 2019). Many frameworks describing numeracy, counting, and arithmetic skills assume that subitizing is a starting point for cardinality, unitizing, and arithmetic skills. Since conceptual subitizing requires focusing on both the unit and the whole at the same time, it helps children to reveal decomposing and composing strategies in arithmetic operations. For example, presenting the number 5 on subitizing cards in different sequences may be effective in children's interpretation of the number 5 as 3+2, 2+1+2 or 1+4 (Clements et al., 2019; Kullberg & Björklund, 2020; Paliwal & Baroody, 2020).

One of the important debates in the literature on part-whole relations concerns the relationship between counting, on the one hand, and parts and wholes on the other. Hunting (2003) argued that the two skills were related to one another, but noted that it was not clear how counting skills supported part-whole skills, particularly in early childhood. As is well-known, children use many different counting strategies such as "counting all" or "counting on from first or larger," which is a more advanced counting strategy, when they are asked to do additions or combine sets. According to Kullberg and Björklund (2020), these approaches to counting do not encourage children to experience part-whole relations. The transition from the strategy of "counting on" to the strategy of "parts and whole" is an important milestone in the development of a child's mathematical thinking (Young-Loveridge, 2002). Eventually, children stop using the strategy of counting on altogether, and develop a completely different method by composing and decomposing numbers and making them easier to calculate. Use of this strategy requires having an idea of part-whole relations of numbers (Cheng, 2012; Young-Loveridge, 2002). After a certain point, continuing to support counting strategies for addition and subtraction operations, which are easier and require less effort, can hinder the development of more advanced mathematical skills and thus the development of number sense in children. Studies show that children who use counting strategies make a lot less effort to understand part-whole relations of numbers compared to children who utilize the decomposing strategy. They have been observed, even at an advanced age, to keep using counting strategies for simple addition problems (Cheng, 2012; Sinnakaudan et al., 2016).

Another debate in the literature concerns the age group that can understand the part-whole relations. Some researchers argue that the part-whole relations cannot be learned prior to elementary school. Piaget (1965, as cited in Baroody, 2000; Hunting, 2003) found that children typically did not develop the skill to distinguish between parts and wholes until the age of six or seven. The cognitive development of children limits the mathematical concepts that they are able to learn, and therefore, teaching about the part-whole relations at an early age can be very difficult. However, starting at the age of 5, children can intuitively perceive parts and wholes when the set of numbers is small (in the range of 1 to 5) (Baroody, 2000; Hunting, 2003; Kamii, 1982). On the other hand, some studies show that children can develop part-whole skills long before they go to elementary school, at an early age (Baroody, 2000; Cheng, 2012; Fischer, 1990; Sophian & McCorgray, 1994; Sophian & Vong, 1995). To help five-year old children understand part-whole relations of numbers 2 to 10, Cheng (2012) trained them by assigning multiple classification tasks. The goal in this training was to help children understand that numbers can consist of different pairs of numbers, allowing them to develop more advanced strategies such as decomposition for addition tasks. Findings of the study showed that it was possible to teach five-year-olds strategies such as decomposition so that they could make additions more effectively. Similarly, in another study conducted with children aged 58 to 71 months (kindergarten), early instruction that emphasized part-whole relations of numbers was found to help children develop the concept of number, and improved their performance in verbal addition and subtraction problems (Fischer, 1990). In other similar studies, Sophian and McCorgray (1994), Sophian and Vong (1995) found that five-year-olds had a basic understanding of part-whole relations and used them to solve verbal arithmetic problems, but four-year-olds were unable to utilize this skill especially in the case of problems with unknown beginning points. The conclusion to be drawn from these studies is that the skill to consider part-whole relations of numbers in solving verbal arithmetic problems develops in the preschool period, and plays an important role in problem-solving by 5 years of age (Sophian & Vong, 1995).

Recent studies do not provide conclusive evidence regarding whether preschool children can learn the part-whole relations, but they also show that instruction that focuses on this concept makes them perform better in their later years, particularly in addition and subtraction operations. Effective planning of instruction requires having a consistent and coherent explanation of children's cognitive development and its limitations (Cheng, 2012; Sophian & McCorgray, 1994).

When the preschool education program in Turkey is considered in terms of mathematics objectives, it is observed that the objectives are limited to number knowledge, number principles (such as one-to-one correspondence, cardinality) and counting skills. It is noteworthy that there are no objectives in the curriculum emphasizing the relations between numbers (part-whole relationship, subitizing) and number sense. The objective specified in the curriculum for the part-whole relationship is an objective that includes the concept of fraction (Bozkurt, Şapul, & Şimşekler Dizman, 2020; MoNE, 2013). As stated in Şapul's (2019) study comparing Turkey and Singapore preschool education curriculum, this objective includes specifying the whole and parts of an object. On the other hand, in the Nurturing Early Learners (NEL) national preschool curriculum booklet of Singapore, which is one of the countries that usually ranks first in international exams, the part-whole relationship is used to mean that a number consists of smaller parts, emphasizing the relationship between numbers (Ministry of Education, 2013).

In internalizing the concept of number, children are expected to be able to establish relationships between numbers beyond the number knowledge specified in the preschool education curriculum. In order for children to establish these relationships, they need to be able to construct unit that are important in the development of arithmetic skills and to understand numbers as composite units in a part-whole relationship (Baroody, 2016; Björklund, Ekdahl, Kullberg, & Reis, 2022; Kullberg & Björklund, 2020). Children who use part-whole relations of numbers in arithmetic problems experience numbers as composite units. Understanding numbers as composite units is a prerequisite for decomposing or re-grouping strategies in arithmetic operations. Children who cannot establish these relationships construct the numbers in all problems by counting single units. In other words, they cannot discover effective strategies in which number relations (based on the combination of their units) are applied (Kullberg & Björklund, 2020). In this sense, it is important to know how children establish and make sense of the part-whole relationship in order to contribute to the development of arithmetic skills.

In studies on part-whole structure (or part-whole reasoning) in children (Ekdahl, 2021; Hunting, 2003; Kullberg & Björklund, 2020; Lüken, 2012; Tsamir, Tirosh, Levenson, Tabach, & Barkai, 2015) researchers utilize different tools or tasks such as word problems, objects hidden under boxes, covers, or glasses, and finger patterns. These studies find that the type of tasks used in different contexts and the ways in which they are implemented affect the strategies used by children. Therefore, this study aims to examine how kindergarten children aged five to six structure the part-whole relations in different task contexts and types (with unknown parts or wholes). Within the general scope of the study, the answers to the following questions were sought:

1. What is the performance of kindergarten children in tasks regarding the part-whole of numbers?
2. What sorts of strategies do kindergarten children use in tasks regarding the part-whole relations of numbers?

Method

Research Design

This study aims to examine kindergarten children's performance regarding the part-whole relations and the strategies they use. The qualitative method of case study was used to gain an in-depth understanding of the subject matter. Merriam (1998) defines case study as an in-depth description and examination of a limited system. Accordingly, the goal in the study is to provide an in-depth description and examination of the performances of a group of kindergarten children regarding the part-whole relations and the strategies they use. We do not aim to generalize beyond the case under study.

Participants and Setting

Participants in the study consisted of 43 children (21 girls and 22 boys) attending the kindergarten in six public schools run by the Ministry of National Education, catering to families of medium socioeconomic status in the Urla district of the province of İzmir/Turkey. Children participating in the study were selected using criterion sampling, which is a purposive sampling method. The age of the participants varied between 61 and 80 months, with an average age of 72.5 months. Criteria used in selecting the children to interview included being older than 60 months, attending the kindergarten in the Academic Year 2020-2021, and volunteering to participate in an online interview. Because the kindergarten curriculum used in Turkey does not contain any learning outcomes concerning the part-whole relations, participants did not have formal prior experience regarding this relationship. Because of the pandemic, children have not been able to receive regular face-to-face education in schools for the last two years. Schools offered mandatory distance education in the spring semester of the Academic Year 2019-2020, and a mix of face-to-face and distance education in the Academic Year 2020-2021.

Data Collection

All required ethics board approvals and Ministry of National Education permissions were obtained prior to collecting data for the study. Before conducting the interviews, the parents of all the children attending the kindergarten at selected schools were contacted via school administrators and teachers, and detailed information was provided via e-mail. The message explained the ethical approvals obtained for the study and the scope of the study. Parents volunteering to have their children participate in the study have filled out the parent approval forms attached to the email and returned them back to the researchers.

Because of the pandemic, the interviews were conducted online, using the Zoom application, in the spring semester of the Academic Year 2020-2021. Because the children had experience of conducting activities with their teachers in an online environment, conducting the interviews online did not present a problem. Interviews were conducted by a member of research team who had teaching experience, with the children accompanied by their parents. Interviews were conducted individually with each child and lasted an average of 20 minutes.

Preliminary meetings were held with the children to develop rapport and ensure that they would be comfortable during the interview. During these preliminary meetings, also conducted in the form of online video calls, the researcher introduced herself to the child, had a small chat, talked about the character called Zıpzıp mentioned in interview questions, and invited the child to participate in an online interview to help this character out in various games. No further information was provided during this meeting regarding the main interview.

The interviews began by having a chat with the children about their favorite topics. Then, children were introduced to the Zıpzıp character, who is very curious and loves to play games. This character appeared in the games in the form of visuals, and the researcher held a puppet-like toy figure of the character in her hand throughout the game. Participants were told Zıpzıp could only get help from children. Children helped Zıpzıp by responding to questions and explaining their thinking. That game required that the children would not get any help from people around them (parents, siblings, etc.). However, an effort was made to motivate the children for the game by providing them with heart stickers as they responded (regardless of whether their responses were correct or incorrect).

After explaining the general rules in this manner, interview questions prepared in the form of individual PowerPoint slides, with different visuals on each slide, were presented to the children by the researcher in the form of a game. Conducting the interviews in the form of a game and having children help the Zıpzıp character allowed them to develop a rapport with the character, and happily participate in the interview, without getting bored. Children who got distracted or lost their focus during the interviews were not forced to respond to the questions; instead, the researcher had a little chat with them, and when they felt ready, asked them to continue playing the game.

Data Collection Instrument

To develop the data collection instrument, first, studies in the literature on the part-whole relations examined, and the contexts and tools used by researchers to investigate preschool children's performance regarding the part-whole relations were explored (Cheng, 2012; Ekdahl, 2021; Fischer, 1990; Hunting, 2003; Kullberg & Björklund, 2020; Novakowski, 2007; Sinnakaudan et al., 2016; Tsamir et al., 2015). It was found that these studies made frequent use of finger patterns, tasks involving hidden objects, and displaying objects or pictures that represent quantities. Accordingly, three types of questions were created, with two questions each (totally 6 question). Interviews with the children lasted an average of 20 minutes. Since the focus and attention spans of children in this age group are considered, the number of questions is limited.

The range of numbers used in these questions was 1 to 7. In the kindergarten period, children are expected to work with numbers in the range of 1-10 regarding part-whole relations of numbers. However, while determining the number range of this study, care was taken to ensure that all probabilities in the part-whole relations of the last number targeted in finger patterns, which is one of the three question types directed to children, can be shown with two hands. For example, it would not be possible to express some of the parts of the number 8 as 2 and 6, and 3 and 5, with the fingers of each hand pointing at one part. Therefore, this number is not preferred in finger patterns. In order to ensure consistency in other question types, the number range is limited to 1-7.

In questions of Type I on the data collection instrument (Appendix 1), children were asked to show the numbers 5 and 6 using the fingers of both their hands. The goal here is to have children show the parts that make up the numbers 5 and 6 (the pairs of 1 and 4, and 2 and 3 for the number 5, and the pairs of 1 and 5, 2 and 4, and 3 and 3 for the number 6). There are also two questions of Type II. In these questions, children were assigned a task that required them to identify the parts of the numbers 5 and 7. For example, in the first Type II question, cards that had different numbers of apple visuals representing the parts of the number 5 were shown. There were a total of eight cards: two cards with 1 apple, two cards with 2 apples, two cards with 3 apples, and two cards with 4 apples. Cards that represented the same number had apples arranged differently. All the cards were shown at once, and participants were asked which two of these cards should be given to Zıpzıp if it wants to eat 5 apples. In posing the questions, the researcher avoided using expressions such as adding and combining that would suggest making arithmetic operations. The pairs of cards that participants selected were removed from the screen, and they continued to identify pairs of numbers that constituted the target number using the remaining cards on the screen. The second Type II question, on the other hand, used cards that contained different numbers of apple visuals representing the parts of the number 7. This question also used multiple cards representing the same quantity in order to avoid lowering the probability that the children would give the correct answer. The last type of question (Type III) on the data collection instrument contained a task involving hidden objects. In the first Type III question, participants were shown 5 counters, and told that someone has hidden some of these counters in their hand. The next visual showed a closed hand and 4 counters. The goal was to have the children recognize the numbers 1 and 4, one of the pairs making up the number 5, in the context of a hidden object task. The second Type III question also began with a visual of 5 counters, and participants were again told that someone had hidden some of the counters in their hand. The next visual showed a closed hand and 2 counters, and the children were asked to identify how many counters were hidden. The goal was to have the children recognize the numbers 2 and 3 this time, another one of the pairs making up the number 5, in the context of a hidden object task.

Within the scope of this study, the performances of the children include the correct answers they gave to the questions asked to them. In addition, the strategies used by the children while answering these questions were also examined. Therefore, the children were first asked the questions in the data collection instrument, and then, the question "Can you explain how you think?" has been asked in order to determine their strategies. In addition, while determining the strategies used by the children while answering the questions, observations (such as counting with their fingers or lip movements) were also used.

Validity and Reliability

To ensure reliability, all the interviews were conducted by the same researcher, who made an effort to avoid providing any cues during the interviews. Moreover, to produce a more controlled environment, parents who accompanied the children were asked, prior to the interviews, to avoid helping their children or saying anything during the interviews that could serve as clues or reveal the correct strategies. The answers given by the children to the questions in the data collection instrument were scored with the rubric created by the researchers. At the stage of determining the strategies used by the children, all researchers determined the strategies individually based on the children responses and behaviors (finger counting, etc.) in the video recordings. In rare cases where coding discrepancies occured, a consensus was reached after having rewatched the video recording of the child in question along with a duly developed dialogue on the issue.

The questions used in the data collection instrument were adapted from the samples in the relevant literature. The study of Kullberg and Björklund (2020) in questions involving finger patterns, Ekdahl (2021) in hidden tasks, and Tsamir et al. (2015) in questions involving object or picture representations corresponding to quantities were utilized. Thus, in terms of validity, the scope of the data collection instrument has been established in a way that is consistent with the relevant literature. In addition, the questions to be used were examined by 4 kindergarten teachers, who have over 10 years teaching experience and expert judgments were obtained from the teachers. The teachers found the questions used in the data collection instrument appropriate in terms of clarity, suitability for the level of the child, and reflecting the part-whole relationship.

Pilot Study

Prior to data collection, a pilot study was conducted with 5 children ages 60 to 72 months in the fall semester of the Academic Year 2020-2021. The pilot study was also conducted by the researcher who conducted the main interviews, using the Zoom platform. Other researchers have monitored the interviews online, with their microphones and cameras turned off, and took notes. Parents were informed about the participation of other researchers and their approvals were obtained.

During the pilot study, researchers did not encounter any situations that would require amending the basic structure of the interview questions. Issues to be paid attention to and expressions to be emphasized by the researcher were identified to make the questions easier to understand by the children. The pilot study showed that, in the first Type I question, some of the children struggled with using the fingers on both their hands to show the number 5. They showed two separate fives using their both hands. Make the question clearer; a decision was made to have the researcher show the number 4 using the fingers of both her hands as an example.

The pilot study also showed that in questions of Type II, children were confused when the pairs of numbers they selected did not disappear from the screen as they identified the pairs that made up the numbers 5 and 7. Children forgot the previous selections they made, and ended up selecting the same pair of numbers multiple times, or failed to add new pairs. Therefore, following the pilot study, a decision was made to remove the pairs of cards that participants selected, and have them continue to identify pairs of numbers that constitute the target number using the remaining cards on the screen.

In the pilot study for questions of Type III, children were observed to have a tendency to start counting as soon as the 5 counters appeared on the screen. A decision was made for the researcher to state the number of counters as soon as the visual appeared in the main interviews, by saying "you see 5 counters here," so that children would not need to count. In the pilot study, to make sure that every participant understood the question, the researcher has hidden some of the physical counters in their hand, and explained the question by showing with concrete materials.

Data Analysis

Data obtained from the interviews with the children were first summarized in the form of tables. Recordings of each interview were examined and children's behaviors when responding to the questions (for example counting by their fingers) and explanations regarding the questions were entered in the relevant tables. Then, this dataset was analyzed in two stages. In the first stage, the total scores of each participant on the interview questions were calculated, and these scores were used to assess participants' performances regarding the part-whole relations. The rubric used in assigning the scores is presented in Appendix 1. According to the rubric, children can receive a maximum score of 10 points: two points each from questions of Type I, two points each from questions of Type II, and one point each from questions of Type III.

The second stage of data analysis involved identifying strategies used by children in their responses. The focus was on whether they used part-whole relations or the strategy of counting when identifying the parts that made up the whole. Using part-whole relations can be defined, in basic terms, as a strategy that involves intuitive elements, is related to the development of number sense, and requires a higher-level thinking. Cases where the numbers were not perceived to be composed of smaller numbers, and the focus was on counting to reach the target number were coded as instances of using the counting strategy. Situations where the children did not explain their thinking and it was not possible to infer their reasoning by observing their behaviors were coded as unknown. Children's explanations provided in Table 1 can be examined to gain a better understanding of the strategies used.

Table 1. Description of the Strategies and Sample of Children's Responses

Type of Strategy	Strategy Description	Sample Children's Responses
Using part-whole relations	Composing and decomposing numbers by paying attention to the part-whole relations	(Responding very quickly.) 5 and 1 makes 6. 2 and 4 makes 6. 3 and 3 makes 6. (Question 2)
Counting	Counting on or counting all without paying attention to the part-whole relations	Gets close to the screen and counts the apples on the cards one by one, using fingers. Asked to explain his reasoning, he says "I counted my fingers." "First, I counted this. Then, I counted this." (Counting all) (Question 3) First, she selects the card with 2 apples. Then, she counts the apples on other cards, going 3, 4, 5, 6, and 7, until she reaches the number 7. (Counting on) (Question 4)
Unknown	Inconsistent answers without a clear strategy	At first, he says 9. The researcher explains the question one more time. He selects 3 and 4. He then says 9 again. The researcher explains one more time. He selects 4 and 4. He then selects 1 and 3. (Question 3)

Results

Findings regarding the first research question

(What is the performance of kindergarten children in tasks regarding the part-whole relations of numbers?)

To answer the first research question, first, children's responses to each question were examined. Table 2 provides descriptive statistics for children's performance on each question.

Table 2. Children's Performance in Tasks Regarding the Part-Whole Relations of Numbers

	Type I Questions		Type II Questions		Type III Question	
Points	**Question 1 f (%)**	**Question 2 f (%)**	**Question 3 f (%)**	**Question 4 f (%)**	**Question 5 f (%)**	**Question 6 f (%)**
0	19 (44.1 %)	19 (44.1 %)	13 (30.2 %)	18 (41.8 %)	25 (58.1 %)	23 (53.4 %)
1	9 (20.9 %)	9 (20.9 %)	2 (4.6 %)	5 (11.6 %)	18 (41.8 %)	20 (46.5 %)
2	15 (34.8 %)	15 (34.8 %)	28 (65.1 %)	20 (46.4 %)	-	-
$\bar{X}$	0.90	0.90	1.34	1.04	0.41	0.46

Table 2 shows that, for each question, the average performance was about half of the maximum possible score. For example, the average for Question 1, which had a maximum score of 2 points, was 0.90, whereas for Question 5, which had a maximum score of 1, the average was 0.41. The average of the total scores, on the other hand, was 5.09 out of a maximum score of 10 points. These findings indicate that children had an average performance regarding the part-whole relations.

Table 2 also reports the frequencies and percentages of the scores received by the children on each question. In questions of Type I, close to half of all participants (44.1%) were unable to show the numbers 5 and 6 using their fingers to represent parts. 20.9% of the children were able to show the parts of the number 5 in one way only (either as 1 and 4, or as 2 and 3), and 34.8% were able to show the parts in two different ways (as 1 and 4, and as 2 and 3). Similar percentages were observed when the target number was 6. The notable finding here is that close to half of all children were unable to use their fingers to represent parts of the numbers 5 and 6 in Type I questions. Given the nature of the task, children were expected to think about the parts of the numbers, and simultaneously use their fingers to show these parts. In other words, they were expected to come up with the parts on their own. It might have been challenging to recognize and structure the parts of the whole. However, a significant percentage (34.8%) of the children were able to divide the numbers in question into their constituent parts in at least two different ways. Children who were able to divide the numbers into their parts in a single way were the smallest group (20.9%). These findings indicate that once children were able to think of numbers in terms of their constituent parts, they performed better in considering all possibilities instead of focusing on a single possibility.

Table 2 also shows, regarding performance on Type II questions, that 30.2% of the children were unable to give the correct answer to Question 3. 65.1% of the children, on the other hand, were able to divide the number 5 into its constituent parts using tangible objects in Question 3. What is notable here is that, although Questions 1 and 3 were about part-whole relations regarding the same number, children's performances on these two questions were significantly different. Only 34.8% of the children were able to divide the target number into its constituent parts in two different ways (1 and 4, and 3 and 3) in Question 1, whereas this figure was 65.1% in Question 3. The difference may have emerged because Question 1, which required children to use their fingers to show parts of the number was more abstract, whereas Question 3, which expected them to reach the whole by selecting apple cards was more tangible for the children. The same finding applies to Questions 2 and 4 as well. In Questions 2 and 4, the task was to identify the parts of the numbers 6 and 7, respectively. In Question 2, only 34.8% of the children were able to identify at least two of the pairs that make up 6, whereas in Question 4, which required identifying parts of 7 by selecting from among apple cards, this figure rose to 46.4%. Moreover, looking at performance on Questions 3 and 4, both of which were questions of Type II, the percentage of children who were able to identify at least two of the pairs that make up the number 5 (65.1%) was significantly higher than the corresponding percentage in the case of the number 7 (46.4%). This is probably because the quantity represented by the number 5 is easier to perceive compared to the quantity represented by the number 7. Looking at the performances on Questions 5 and 6, of Type III, the numbers of children who gave correct answers and those who gave incorrect answers were close to one another. For example, 41.8% of the children gave a correct answer to Question 5, and 58.1% gave an incorrect answer. Similarly, 46.5% of the children gave a correct answer to Question 6, and 53.4% gave an incorrect answer. Comparing Questions 5 and 6, the performance on Question 5, which asked about the pair 1 and 4 from among the parts that make up the number 5, was lower (41.8%) than the performance on Question 6, which asked about the pair 2 and 3 (46.5%). This might be because children have more difficulty perceiving the part-whole relations when the parts are farther apart from one another.

Table 3 reports performance statistics regarding part-whole relations by question types. The table shows that the maximum total score on questions of Type I and II is 4, whereas the maximum total score on questions of Type III is 2. Similar to the finding regarding performance on individual questions, children had an average performance on different types of questions. Comparing Type I and Type II questions, children had a higher performance on Type II questions ($\bar{x}$ =2.62) than on Type I questions ($\bar{x}$ =1.81). In Type III questions, on the other hand, the mean performance ($\bar{x}$ =0.88) was close to the half of the total maximum score of 2. Thus, children had an average performance on this question type as well.

Table 3. Children's Overall Performance Regarding the Part-whole Relations by Question Types

Question Type	N	Min	Max	$\bar{X}$
Type I	43	0	4	1.81
Type II	43	0	4	2.62
Type III	43	0	2	0.88

Findings regarding the second research question

(What sorts of strategies do kindergarten children use in tasks regarding the part-whole relations of numbers?)

This study examined children's understanding of part-whole relations by focusing on the strategy of counting and strategy of part-whole relations. Accordingly, as the children worked on the tasks presented to them, they were observed to see if they were aware of the part-whole relations or displayed a counting behavior. Although the percentages were different for each question, overall, the strategy of counting was used more frequently than the strategy of part-whole relations. Table 4 shows that in Questions 1, 3, 5, and 6, which had the target number of 5, the percentages of who used the part-whole relations strategy were close to one another (around 25%). In Questions 2 and 4, which had the target numbers 6 and 7, these percentages dropped to 20.9% and 16.2%, respectively. This indicates that children tend to use the counting strategy more often when the target number is larger.

The strategy of part-whole relations was used the most often in Questions 3 and 6. Question 3 requires children to use visuals to show parts of the number 5. Thus, when the target number was not large (e.g. when it was 5), and the parts were presented to children in a more tangible manner, using visuals, the tendency to use the strategy of part-whole relations was stronger. Question 6 also had the target number of 5. However, unlike the Question 3, this question also offered a visual presentation of one of the parts that made up the target number. The children tended to use the part-whole relations strategy at the similar rate (27.9%) in these two types of tasks.

The counting strategy was used most frequently in responses to Question 4. In this question, children were asked to identify which of the parts presented visually made up the number 7. The bigger tendency to count in this question might be attributed to the fact that the target number was larger (larger than 5) and the parts were presented visually, making it easier to count.

Table 4. Strategies Used by Children in Tasks Regarding the Part-whole Relations of Numbers

	Type 1 Questions		Type 2 Questions		Type 3 Questions	
Strategies	**Question 1 f (%)**	**Question 2 f (%)**	**Question 3 f (%)**	**Strategies**	**Question 1 f (%)**	**Question 2 f (%)**
Using part-whole relations	11 (25.5%)	9 (20.9%)	12 (27.9%)	7 (16.2%)	11 (25.5%)	12 (27.9%)
Counting	14 (32.5%)	17 (39.5%)	16 (37.2%)	18 (41.8%)	17 (39.5%)	13 (30.2%)
Unknown	18 (41.8%)	17 (39.5%)	15 (34.8%)	18 (41.8%)	15 (34.8%)	18 (41.8%)

The strategies used by the children were coded as unknown in the range of 34-41%. Responses coded unknown either did not have a clear thinking behind them or were inconsistent. The researcher who conducted the interview returned to the question several times for the children who had difficulty in explaining their thinking processes and asked the same question again in different ways. In this way, she tried to help the child express his/her thinking process. However, no meaningful response was received from these children to determine the strategy. They only used expressions such as "Because I thought so", "I don't know", "It just occurred to me". The fact that some children explain their thinking processes very clearly, while others fail to do so, can also be explained by the difference in self-expression skills among preschool children.

Table 5. Arithmetic Means of Children's Total Scores by the Strategies Used

Strategies	Question 1	Question 2	Question 3	Question 4	Question 5	Question 6
Using part-whole relations	9.27	9.22	8.66	9.28	9.81	8.16
Counting	5.21	5.29	5.81	6.72	5.76	5.76
Unknown	2.44	2.7	1.46	1.88	2.66	2.55

Considering the arithmetic mean of the total scores of the children according to the strategies they use, a remarkable result can be reached. Table 5 shows that across all questions, children who used the part-whole relations strategy had a higher mean score compared to those who used the counting strategy. For example, in Question 1, children who used the part-whole relations strategy had a mean score of 9.27, whereas those who used the counting strategy had a mean score of 5.21. Similarly, children who used unknown strategies had the lowest performance in terms of the mean scores across all questions. This shows that using the part-whole relations strategy increases the likelihood that children would give more correct answers, or, in other words, children who give more correct answers use the part-whole relations strategy more frequently.

Discussion, Conclusion and Suggestions

This study, which aimed to examine kindergarten children's performance regarding part-whole relations of numbers, found that children participating in the study had an average performance. About half of the children were able to give correct answers in the tasks presented to them, whereas the other half were unable to divide numbers into their constituent parts. Some of the studies in the literature focus on the age at which the part-whole relations start to develop (Baroody, 2000; Cheng, 2012; Fischer, 1990; Hunting, 2003; Kamii, 1982; Sophian & McCorgray, 1994; Sophian & Vong, 1995). Five-year old participants in the present study were found to be able to use the part-whole relations up to the number 7. However, in questions of Type II, children had a better performance in terms of using the part-whole relations when the whole was 5, compared to the case when the whole was 7. One conclusion to be drawn from these findings is that in this age group, children have difficulty recognizing part-whole relations when the numbers involved are large, whereas they perform quite well when the number representing the whole is 5.

The types of tasks used to present the number representing the whole also matter (Ekdahl, 2021; Hunting, 2003; Kullberg & Björklund, 2020; Lüken, 2012; Tsamir et al., 2015). In this study, the task with the highest performance was a Type II question, with the number 5 representing the whole, and the task presented using visuals. On a question of Type I, which also had the number 5 representing the whole but did not provide visuals (instead asking children to create parts using their fingers) performance was about half what it was on the question of Type II. Compared to the questions in the hidden object tasks, the result of being more successful in the apple question, in which the whole is known, the parts are not known, and the visuals are presented, is similar to the findings of the study of the Tsamir et al. (2015). As one of the possible reasons for this result, Tsamir et al. (2015) interpreted in their study that children saw the apples on the cards and thus could count them in the apple question.

Two important strategies used by the children participating in the present study were the strategy of counting and the strategy of part-whole relations. Most children who used the counting strategy by thinking either counting all or counting on. One observation during the interviews, particularly in the case of questions that presented visuals, was that children selected the first card by counting, and tried to reach the whole by counting on, but had difficulty completing the process. Similar to the findings of our study, Kullberg and Björklund (2020) in a study examining how preschool children establish the part-part-whole relationship while solving arithmetic tasks, found that most children use counting strategies.

Studies have presented the transition from the counting on strategy to the part-whole strategy as an indicator of mathematical development (Cheng, 2012; Young-Loveridge, 2002). One of the most interesting findings of the present study was that children who used the strategy of part-whole relations performed better compared to those who used the counting strategy. Children who were able to think about numbers in terms of their parts were able to give correct answers to the questions by using part-whole relations. Children who did not think about numbers in terms of part-whole relations, on the other hand, tended to count, but they gave fever correct answers because of the counting errors they made.

For each question type, the tendency to use the counting strategy was stronger in the case of larger numbers. For example, more children resorted to counting when the whole was 6 in questions of Type I, and 7 in questions of Type II. This finding of the study is compatible with the literature that emphasizes children can intuitively perceive parts and wholes when the set of numbers is small (in the range of 1 to 5) (Baroody, 2000; Hunting, 2003; Kamii, 1982). In questions of Type III, on the other hand, children tended to count more when the parts were farther apart from one another (1 and 4 counters as opposed to 2 and 3 counters). This situation may be caused by the structure of the task (hidden object).

This study found that five-year old children attending the kindergarten were able to perceive numbers 1 to 7 with their parts. Children who used the strategy of part-whole relations performed better on the part-whole tasks they were given compared to those who used the counting strategy. These findings indicate that it is very important to provide children in this age group with learning environments that would allow them to make use of the strategy of part-whole relations. One of these learning environments is subitizing activities that can be applied in the classroom. Studiesexplored the role of conceptual subitizing activities for supporting preschool class children's learning of the part-part-whole relations of number (Wästerlid, 2020) and arithmetic skills (Özdem & Olkun, 2021). Studies also, show that children who are given opportunities for part-whole relations can use more effective computational strategies such as composing and decomposing in four operations compared to children who receive instruction based on standart algorithm (Langhorst, Ehlert, & Fritz, 2012). Therefore, we recommend increasing awareness of this issue among kindergarten, also preschool teachers, in particular, and enriching the teaching process with activities that encourage thinking about numbers in terms of their parts. Another recommendation is that people working on curriculum development should avoid an exclusive focus on counting, and include learning objectives that emphasize perceiving numbers with their parts.

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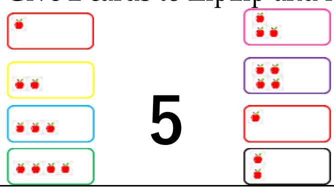
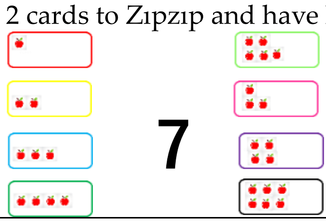
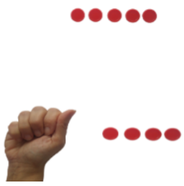
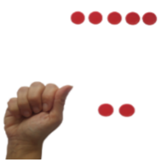
Appendices

Appendix 1. Data Collection Instrument and Rubric

Question	Target Number	Question Text and Visual	Rubric
1	5	**Question 1:** How many fingers are open in the hand shown in this picture? Now, show me five opened fingers as in the example, but some should be from one hand, and some from the other hand. How else can you show 5 fingers? 	If showing the pair **(1,4): 1 point** If showing the pair **(2,3): 1 point** **Total: 2 points**
2	6	**Question 2:** Now, show me 6 opened fingers, but use fingers form both of your hands. How would you do that?	If showing one of the pairs **(1,5), (2,4),** and **(3,3): 1 point** If showing more than one of the pairs **(1,5), (2,4),** and **(3,3): 2 points** **Total: 2 points**
3	5	**Question 3:** Zıpzıp wants to eat 5 apples. Which two cards should you give Zıpzıp so that it can eat exactly 5 apples? You can tell me by their colors. Give 2 cards to Zıpzıp and have him eat 5 apples. 	If showing the pair **(1,4): 1 point** If showing the pair **(2,3): 1 point** **Total: 2 points**
4	7	**Question 4:** Zıpzıp wants to eat 7 apples. Which two cards should you give Zıpzıp so that it can eat exactly 7 apples? You can tell me their colors. Give 2 cards to Zıpzıp and have him eat 7 apples. 	If showing one of the pairs **(1,6), (2,5),** and **(3,4): 1 point** If showing more than one of the pairs **(1,6), (2,5),** and **(3,4): 2 points** **Total: 2 points**
5	5	 **Question 5:** There are 5 counters here. All of a sudden, a secret hand comes, and takes some of these counters. How many counters would we find if we opened the hand?	Correct answer: **1 point** Incorrect answers: **0 points** **Total: 1 point**
6	5	 **Question 6:** There are 5 counters here. All of a sudden, a secret hand comes, and takes some of these counters. How many counters would we find if we opened the hand?	Correct answer: **1 point** Incorrect answers: **0 points** **Total: 1 point**